Open Education Resources Book

High School Equivalency

Math

HEP

WEST HILLS
COMMUNITY COLLEGE DISTRICT
HIGH SCHOOL EQUIVALENCY PROGRAM

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High School Equivalency Program

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Information on what is on the HiSET Exam:

For information on what is on the HiSET exam, refer to the website link below:

https://hiset.ets.org/about/content

Chapter 1: Number and Operations on Whole Numbers

Four Operations

The four basic operations of math are addition, subtraction, multiplication, and division.

Addition (+)

Adding whole numbers is probably very familiar to you. You have been adding whole numbers almost as long as you have been in school. Here is a problem that probably looks familiar.

4+5=

In this problem, you are adding four and five. You have four whole things plus five whole things, which is a total of nine whole things. The numbers that you are adding are called **addends**. The answer to an addition problem is the **sum**.

This first problem was written horizontally. In the past, you may have seen addition problems written vertically.

When you add whole numbers, it can be less confusing to write them vertically according to place value. Think about the first example.

4+5=9

If you wrote that vertically, first you would line up the numbers.

  4

+5

  9

When you have more digits, write the problem vertically by lining up each digit according to place value.

Subtraction (−)

Subtraction is the opposite of addition. This means that if you add two numbers and get a total, then you can subtract one of the original numbers from the total and end up with the other original number. When you add two numbers you get the **sum**, when you subtract two numbers, you get the **difference**.

Let's look at an example.

$15-9=6$

First, rewrite the problem vertically. Remember to line up the digits according to place value.

$$\begin{array}{r} 15 \\ \underline{-9} \\ 6 \end{array}$$

The difference is 6.

Multiplication (×)

Did you know that addition and multiplication are related? Let's explore what this means through an example.

$5\times6=30$

You can use your timetables to complete this problem using mental math, but let’s look at what it means to multiply 5 by 6. 5×6 means that you need five groups of six.

You can also think of this as adding 5 six times.

$5+5+5+5+5+5=30$

Looking at this way, you can see that multiplication is a shortcut for repeated addition.

$5\times6=30$

In this problem, 5 and 6 are the **factors.** Factors are numbers that are multiplied by each other. The **product** is the answer to a multiplication problem, in this case, the product is 30. You can say that 5 and 6 are factors of 30 because the product of 5 and 6 is 30

Division (÷)

The opposite operation of multiplication is division. To multiply means to add groups of matching things together, to divide means to split up into matching groups.

Let's look at an example.

$72 \div 9 = 8$

In this problem, 72 is the **dividend** - it is the number being divided. The **divisor** is the number of parts that the dividend is being split into, in this case, 9. The answer to a division problem is called the **quotient.** One way to complete this problem and find the quotient is to recall multiplication facts and work backwards.

To divide 72 by 9, start asking "What number multiplied by 9 equals 72?"

$9 \times 8 = 72$

If 8 groups of 9 equal 72, then of course 72 can be split into 8 groups of 9.

$72 \div 9 = 8$

The quotient is 8.

Activity

What do banking, shopping, meteorology, geography, and accounting have in common?

All of these professions deal with positive and negative numbers. Positive numbers go above 0. For example, if you have money, temperatures above 0, and height above sea level these would be considered "positive." On the other hand, negative numbers go below 0. This could be expressed in owing money, temperatures below 0, and depth below sea level.

Sometimes we have to add positive and/ or negative amounts, and sometimes we have to find the difference. What do you think we are doing in these situations?

Situation	What we're doing	Say it with numbers
You have $25 and find $5.	*Adding two positive numbers.*	$25 + 5 = 30$
You have $25 and lose $5		
You owe $25 and earn $5.		

You owe $5 and borrow another $25.		

Attributions:

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Chapter 2: Number and Operations on Number Sense

Classifying Numbers

Integers include the set of whole numbers and their opposites: {... -3, -2, -1, 0, 1, 2, 3 ...}. As you work with integers you are working with both **positive** and **negative** numbers.

You can use integers when adding, subtracting, multiplying, and dividing. When working with integers and operations, it's important to keep track of whether the integer is positive or negative. While you can use a calculator to solve these problems, the more you can do in your head the better. (Much like multiplication.)

Adding Integers

Let's look at an example. $-5 + (-6) = -11$

Notice that the number increases but in the negative direction. If you think about this on a number line, you can see how your number becomes more negative.

First, draw your number line and place your dot on -5 as this is where your problem starts.

Next, move the blue dot six places to the left (left because it is negative).

The answer is -11.

RULE: When the numbers have the same sign, add the numbers and keep the sign.

Let's look at another example. $-623 + 215$

To add two numbers with opposite signs, you are really subtracting and using the sign that occurs with the larger number. Think of it as starting at -623 on the number line and moving 215 spaces to the right. You won't make it all the way to zero. You will still be in negative numbers.

First, find the difference between the two numbers. $623 - 215 = 408$

Next, since 623 is larger and is negative (see the original problem), the answer is also negative.

The answer is -408.

Remember

1. When the numbers have different signs, find the difference and keep the sign of the bigger number.

Subtracting Integers

Let's look at an example using the subtraction rule. $-412 - 244$

Notice that a negative number is going down even more. This is like owing $412 and borrowing $244 more. **We never actually subtract integers. Instead, we use the "keep-change- change" method.**

KEEP – CHANGE – CHANGE (subtraction only)

- Keep the first number- change subtraction to addition- change the sign of the last number

  $-412 - 244$ becomes $-412 + (-244)$

- Next, use the rules of addition to solve the problem.

  $-412 + (-244) = -656$

Tip

Parenthesis are used here to separate the + from the -. They are not telling you to multiply or do anything first.

When you see a minus next to a negative, think about it as two parts of a plus sign.

Let's look at another example. $54 - (-789)$

- First, use "keep-change-change". $54 + (+789)$
- Next, use the rules of addition to evaluate. $54+789 = 843$

Word Problems

Most tests contain a lot of word problems. Now that you understand how integers work, can you apply your knowledge to solving real life problems?

Remember that most word problems have something to do with money, temperature, and sea level, but not all. Here are some sample word problems.

Activity

Let's practice! Solve the following exercises and choose the best answer. You can use a calculator.

Add the following integers.

1. $6 + 7 =$
2. $5 + (-8) =$
3. $8 + (-8) =$
4. $6 + (-10) =$
5. $8 + (-2) =$
6. $9 + (-4) =$
7. $-14 + (-7) =$
8. $-12 + (-14) =$
9. $-13 + (-10) =$
10. $-18 + (-30) =$

Subtract the following integers.

1. $-9 - 5 =$
2. $-8 - 7 =$
3. $-12 - 8 =$
4. $6 - 9 =$
5. $10 - 15 =$
6. $18 - (-5) =$
7. $12 - (-4) =$
8. $23 - (-9) =$
9. $-5 - (-2) =$
10. $-8 - (-5) =$

Word Problems

1. You are creating a new salad dressing by combining different types of oils. You decide to modify the amount of each oil by adding 7 milliliters more of olive oil and decreasing the walnut oil by 11 milliliters, what is the overall difference in the end amount of oil? Set up the problem:

2. Roman Civilization began in 509 B.C. and ended in 476 A.D. How long did Roman Civilization last? Set up the problem:

3. A submarine was situated 450 feet below sea level. If it descends 300 feet, what is its new position? Set up the problem:

4. You have your own landscaping business. It is the end of the month and time to pay bills. There is a balance of $5,425 in the checkbook, and the total amount billed is $6,330. What will your balance be after paying the bills? Set up the problem:

5. Ahmed's school said they would cancel classes if the temperatures dropped below $-3°$ Celsius, the temperature is currently $5°C$. How much colder does it have to get before school is cancelled? Set up the problem:

6. Wine has a freezing point of 18°F Ethanol, on the other hand, has a freezing point of -174°F. What is the difference between the freezing point of wine and the freezing point of ethanol? Set up the problem:

7. The GED testing center is preparing a report showing the change in the number of students who complete GED tests each month. 14 students passed last month and 12 passed this month. What number reflects the change in the number of students passing the GED this month?
   a. 5
   b. 1
   c. 0
   d. -2
   e. -4

8. Chicken needs to be cooked to an internal temperature of 165°F to be safe to eat. Your chicken is frozen at the temperature of -3°F. How many degrees does it need to be warmed up in order to be safe to eat?
   a. 168°F
   b. -8°F
   c. -16°F
   d. 16°F
   e. 168°F

Answers!

Add the following integers.

1. $6+7=$ 13
2. $5+(-8)=$ -3
3. $8+(-8)=$ 0
4. $6+(-10)=$ -4
5. $8+(-2)=$ 6
6. $9+(-4)=$ 5
7. $-14+(-7)=$ -21
8. $-12+(-14)=$ -26
9. $-13+(-10)=$ -23
10. $-18+(-30)=$ -48

Subtract the following integers.

1. $-9-5=$ -14
2. $-8-7=$ -15
3. $-12-8=$ -20
4. $6-9=$ -3
5. $10-15=$ -5
6. $18-(-5)=$ 23
7. $12-(-4)=$ 16
8. $23-(-9)=$ 32
9. $-5-(-2)=$ -3
10. $-8-(-5)=$ -3

Word Problems

1. 7 – 11 = -4 milliliters
2. 476 – (-509) = 985 years
3. -350 + -200= -550, or 550 below sea level
4. -6,330 + 5,425 = -$905
5. 5 – (-3) = 8 degrees colder
6. 18 – (-174) = 192 degrees difference
7. 12 – 14 = -2 or 2 fewer students
8. 165 – (-3) = 168

Multiplying and Dividing

The rules for multiplying and dividing are refreshingly simple!

If the signs are the same, the answer is positive.

positive x positive = positive; $(+)\ (+)$

negative x negative = positive; $(-)\ (-)$

3 x 4 = 12 **-3 x - 4 = 12**

If the signs are different, the answer is negative.

negative x positive = negative; $(-)\ (+)$

positive x negative = negative; $(+)\ (-)$

-3 x 4 = -12 $\quad$ 3 x $-4 = -12$

Example 1: -12 (5)

Notice that this is a multiplication problem. **Parentheses** are used around a single value to show multiplication. There is no operation between the numbers.

First, multiply the two values. **12 × 5=60**

Next, determine the sign. Remember, if the signs are different the answer is negative.

The answer is -60.

Example 2:

First, divide the two values. **= 3**

Next, determine the sign. Remember, if the signs are the same the answer is positive.

The answer is $+3$, or 3

Example 3: Multiply. $(8)\ (-5)\ (10)$

First, multiply the three values. $8 \times 5 \times 10 = 400$

Next, add the sign. **If there are an odd number of negative numbers your answer will be negative.** If there are an even number of negative numbers your answer will be positive.

Or you can multiply from left to right. 8 x $-5 = -40$, -40 x $10 = -400$

Either way, *the answer is* -400.

Practice Activity

Let's practice! Multiply the following integers.

Multiply the following exercises.

1. $-6(-8) =$
2. $5(-10) =$

3. $3(-4) =$
4. $-3(4) =$
5. $8(-9) =$
6. $-9(12) =$
7. $8(-11) =$
8. $(-5)\ (-9) =$
9. $-7(-8) =$
10. $(-12)\ (12) =$

Divide the following exercises.

11. $-12 \div 2 =$
12. $-18 \div -6 =$
13. $-24 \div 12 =$
14. $-80 \div -4 =$
15. $-60 \div -30 =$
16. $28 \div 4 =$
17. $-36 \div 4 =$
18. $-45 \div 9 =$
19. $-75 \div 25 =$
20. $-68 \div -2 =$

Word Problems

21. A person has a debt of $200. Five friends offer to pay off all the debt. How much does each person need to pay to pay off the debt?

22. Ashley needed 20 dollars for the show. She withdrew seven times that amount. How much money did she withdraw?

23. A sprinkler was -8 feet below ground level. Mr. S has a machine that digs - 2 feet at a time. How many times does he need to dig to reach the sprinkler?

24. After being exposed to an electromagnetic shrinking machine, Joe shrank a total of 120 centimeters over 5 seconds. His height decreased by the same amount each second. Joe wants to figure out how much his height changed each second.

25. Grandma Millie is shrinking! Her height decreases by ¼ cm each year. She wants to predict the total change in her height over the next 3 years.

26. A total of 300 residents left town over the past 5 years. The same number of residents left each year. The mayor wants to know what the town's change in population is each year.

Answers Key

1. 48
2. -50
3. -12
4. -12
5. -72
6. -108
7. -88
8. 45
9. 56
10. -144
11. -6
12. 3
13. -2
14. 20
15. 2
16. 7
17. -9
18. -5
19. -3
20. 34
21. -40 dollars
22. -140 dollars
23. 4 digs
24. -120 ÷ 1.5 = -80 He Shrank 80cm per second
25. – ¼ x 3 = -3/4 She will shrink ¾ cm in three years
26. -300 ÷ 5 = -60 Sixty people left each year.

Order of Operations

Why is there an order?

Look at the expression 3 + 2 x 4.

To solve it you must decide what to do first: add 3 + 2 or multiply 2 x 4.

- If you add first, you get 5 x 4 = 20
- If you multiply first, you get 3 + 8 = 11

Both answers cannot be right!

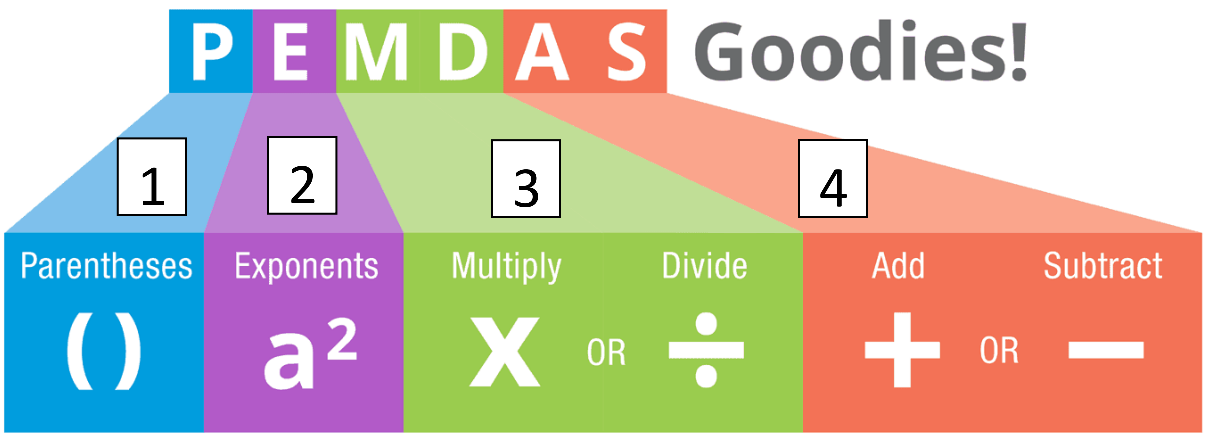
Having a standard order of operations is important because it makes sure all mathematicians are reading and solving problems the same way. Without a standard order of operations, formulas for calculations in finance and science would be useless.

If you type the problem into your calculator exactly as it is written on the page, the calculator will perform the order of operations for you. For this handout, solve by hand first and then check your work using the calculator. This will give you practice with both. Answers are at the end of the handout.

The Order of Operations

Grouping symbols:

1. **P**arenthesis & absolute value
2. **E**xponents: powers and roots
3. **M**ultiplication
4. **D**ivision
5. **A**ddition
6. **S**ubtraction



Example:

Use the order of operations to simplify the following expression:

1. $(7-2)\times 4\div 2-3.$

First, check for parentheses. Since there are parentheses, they must be done first.

2. $(5)\times 4\div 2-3$

Next, look for exponents (little numbers written a little above the others). Since there are no exponents, skip to the next math verb.

Multiplication and division are equally important and must be done from left to right.

3. $5 \times 4 \div 2 - 3$
4. $20 \div 2 - 3$
5. 10 – 3

Finally, addition and subtraction are equally important and must be done from left to right.

6. $10 - 3 = 7$

The answer is 7.

Always working from left to right!

Practice with the basic operations + − x ÷

1. 8 x 5 −9 + 3 =
2. 6 −15 ÷ 3 =
3. −10 ÷ 2 + 1 =
4. 4 + 5 x 2 x 3 =

Parentheses () [] {}

Parentheses are used for a number of purposes:

1. to show multiplication, ex 3(4) is the same as 3x4,
2. to organize your work, ex −5 + (-5), and
3. to show that **the work inside them needs to be done first**: ex. 32 ÷ (3 + 5) becomes 32 ÷ 8 = 4

*Without the (), you might divide 32 ÷ 3 first, then add 5.

Practice with Parentheses

1. 3(4 – 7) – (– 6) =
2. 1 – (9 – 4) ÷ 5 =
3. 7 – 3(4 – 5) =

Sometimes there are several parentheses in a problem. When this happens, start with the inner most parenthesis and work your way out. (Some textbooks use [] and {} to show different levels of parenthesis, but I have not seen this on High School Equivalency tests. Whether or not those signs are used, always work from the inner most to the outer most parentheses.

Fraction Bars

Fraction bars are a signal to **divide**. Ex. $\frac{4}{2}$ means 4 ÷ 2.

When working with fraction bars, **do all the work above and below the fraction bar** before dividing the top by the bottom.

Example: $\frac{5+7}{2} \quad = \frac{12}{2} \quad = 6$

You can use your calculator to solve fractions by hitting the $\frac{n}{d}$ button, typing the top of your fraction, clicking down on the grey oval on the top of your calculator to move to the bottom of the fraction, and typing the bottom of your fraction. Click right on the grey oval to get out of the fraction.

Practice Using Fraction Bars

1. $\frac{3+(2x5)-4}{8\div4+1} =$

2. $\frac{(3\times2-5)+(8\times2=6)}{|-4|+(10\div2+1)+(30\div6+3)} =$

3. $\frac{-9(2)-(3-6)}{1-(-2+1)-(-3)} =$

4. $7+2\div\frac{1}{3} \quad =$

Exponents (Also Known as Powers)

In math, an **exponent** shows a number multiplied by itself.

The expression 4^2 is read "4 to the second power." It means that 4 is multiplied by itself two times: 4 x 4

4 is called the **base** 2 is called the **exponent**

To solve powers, multiply the base **by itself** the number of times shown by the exponent.

$2^1 = 2$ $4^2 = 4 \text{ x } 4 = 16$ $6^3 = 6 \text{ x } 6 \text{ x } 6 = 216$

** special case: anything to the power of 0 equals 1 ex. $83^0 = 1$ (unless the base is 0)

Rules of Exponents For nonzero real numbers a and b and integers m and n	
Product rule	$a^m \cdot a^n = a^{m+n}$
Quotient rule	$\frac{a^m}{a^n} = a^{m-n}$
Power rule	$(a^m)^n = a^{m \cdot n}$
Zero exponent rule	$a^0 = 1$
Negative rule	$a^{-n} = \frac{1}{a^n}$
Power of a product rule	$(a \cdot b)^n = a^n \cdot b^n$
Power of a quotient rule	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

perfect square trinomial	$(x+a)^2 = (x+a)(x+a) = x^2 + 2ax + a^2$
difference of squares	$(a+b)(a-b) = a^2 - b^2$
sum of cubes	$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
difference of cubes	$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Note: On your calculator, the "x^2" button will square any number. If you want to multiply a negative number to a power, you must put parentheses around the negative sign and the number. Ex $(-2)^3$ For exponents larger than 2, type in your base, then hit the green "2nd" button, the "^" button, and the exponent.

Practice Using Exponents

1. $(5 - 3)^4 =$
2. $5 - 3^4 =$
3. $4 - 2^3 + 3^2 - 16 =$

Roots

The opposite of raising a number to the **second** power is finding the **square root**.

The square root symbol is called a **radical sign** √. To find the square root, ask "what number multiplied by itself equals this number?" On the calculator press "2nd" then "x^2" before typing your number.

It is also useful to memorize the common squares:

$\sqrt{1} = 1$ $\sqrt{4} = 2$ $\sqrt{9} = 3$ $\sqrt{16} = 4$ $\sqrt{25} = 5$

$\sqrt{36} = 6$ $\sqrt{49} = 7$ $\sqrt{64} = 8$ $\sqrt{81} = 9$ $\sqrt{100} = 10$

Sometimes you will see something like 8–√3. This means a number of times itself three times equals 8. On the calculator, press the number before the radical, then "2nd" then "^" and enter. These are rare.

Practice using Roots

$\sqrt{144} - 116 - \sqrt{4} =$

$\frac{\sqrt{81}+39}{\sqrt{16}} =$

$(\sqrt[3]{8} \times 2)^2 =$

Practice Activity

1. $2(-6 + 2) \div 4 =$
2. $7 - 3(\sqrt{16} - 5) =$
3. $8 - (-4)^2 - 5 =$
4. $4 - 7 + 1^2 + 2 =$
5. $-3^3 - 6(-2) - 2 =$
6. $5 \times 3 - (-3)^3 =$
7. $-8(2 - 6) \div 2 =$
8. $4(6 - 9) \div 6 =$
9. $-8(2 - 5) \div (-4) =$
10. $8 - 3 \times 2 - 33 \div 11 =$
11. $9 - 3(6 \div 2) =$
12. $(-3)^2 - (-2)^2 - 1 =$
13. $7 \times 2 - 5 \times 3 =$
14. $20 \div 4 - 14 \div 2 =$
15. $2^3 - 6 \times |-2| + 3 =$
16. $(-3)^2 \times (5 - 7)^2 - (-9) \div 3 =$
17. $1^3 - 6 \div (-3) =$
18. $4 \times 5 - 10 - 2(1 - 2) + 5 =$
19. $(-1) \times (2 - 6)^2 \div 8 + 8 - 3 \times 4 =$
20. $5 - (-3)^2 - 6 =$
21. $10 \div 5 - (-2)^2 =$

Answers:

Basic operations:

1. $8 \times 5 - 9 + 3 = 34$
2. $6 - 15 \div 3 = 1$
3. $-10 \div 2 + 1 = -4$
4. $4 + 5 \times 2 \times 3 = 34$

Parentheses:

1. $3(4 - 7) - (-6) = -3$
2. $1 - (9 - 4) \div 5 = 0$
3. $7 - 3(4-5) = \mathbf{10}$

Multiple Parentheses:

1. $(-9 - (2 - 5)) \div (-6) = \mathbf{1}$
2. $(-7 - 5) \div [-2 - 2 - (-6)] = \mathbf{-6}$
3. $2(-3) + 3 - 6(-2 - (-1 - 3)) = \mathbf{-15}$

Fraction bars:

1. $\frac{3+(2x5)-4}{8\div4+1} = \mathbf{3}$
2. $\frac{(3\times2-5)+(8\times2-6)}{4+(10\div2+1)+(30+6+3)} = \frac{11}{18}$
3. $\frac{-9(2)-(3-6)}{1-(-2+1)-(-3)} = \mathbf{-3}$
4. $\mathbf{7 + 2 \div \frac{1}{3} = 13}$

Powers:

1. $(5 - 3)^4 = \mathbf{16}$
2. $\mathbf{5} - 3^4 = \mathbf{-76}$
3. $4 - 2^3 + 3^2 - 16 = -11$

Roots:

$\sqrt{144} - 116 - \sqrt{4} = -106$

$\left(\sqrt[3]{8} \times 2\right)^2 = 16$

Attributions:

Chapter 3: Decimals Numbers

Introduction

We will begin working with types of numbers that are extensions of whole numbers and related to fractions. These numbers are called decimals or decimal numbers. Decimals contain numbers to the right of the decimal point. The place value chart below identifies the first few decimal places. Use this chart to help you with the examples below

BILLIONS			MILLIONS			THOUSANDS			ONES			DECIMALS			
100	10	1	100	10	1	100	10	1	100	10	1	tenths	hundredths	thousdandths	ten-thousandths

Practice Exercises:

For the following three problems, give the decimal name of the posi-tion of the given number in each decimal fraction.

1. 3.941
   9 is in the position.
   4 is in the position.
   1 is in the position.
2. 17.1085
   1 is in the position.
   0 is in the position.
   8 is in the position.
   5 is in the position.
3. 652.3561927
   9 is in the position.
   7 is in the position.

Exercise Answers

1. Tenths; hundredths, thousandths
3. Hundred thousandths; ten millionths

Scientific notation

Scientific notation is a way of representing a very large or very small number without having to write all the zeros at the beginning or end of the number.

When a number is written in **scientific notation** it is written as a product of a number between 1 and 10 (greater than or equal to 1 and less than 10) multiplied by a power of 10. Large numbers (numbers greater than 1) are written with a positive power of ten. Small numbers (numbers between 0 and 1) are written with a negative power of ten. The specific power of 10 indicates just how big or how small the number is.

Here are the same quantities as before written in scientific notation.

- $595{,}000{,}000 = 5.95 \times 10^{8}$
- $0.000000000017 = 1.7 \times 10^{-11}$

Notice that the first number is very large, and it has a positive exponent on the 10. The second number is very small, and it has a negative exponent on the 10.

Also notice that when written in scientific notation, both numbers are the product of a decimal number less than 10 and a power of 10.

Here are the steps for writing a number in scientific notation.

Ex. Write 328,500,000,000 in scientific notation.

1. Move the decimal point so that it is to the right of the **first non-zero digit** of the number. The result should be a number that is between 1 and 10. This will be the first part of your number in scientific notation.

328,500,000,000 becomes **3.285**00000000

2. Count how many spaces you need to move your decimal point in step 1. The number of spaces will be your power of 10. If you move the decimal point to the left, your exponent will be positive. If you move your decimal point to the right, your exponent will be negative.

3.28500000000 ← I moved the decimal 11 spaces to the left.

3. The number in scientific notation is the decimal number from step 1 multiplied by 10 to the power from step 2.

3.285 x 10^{11}

Here is another example.

Write 0.000000595 in scientific notation.

1. Start by finding the first non-zero digit and put a decimal point to its right. Here, the 5 at the beginning of the number is the first non-zero digit.

0.000000595 becomes *0000005.95* which is equal to 5.95

Notice that you don't need to write the zeros at the beginning of the number.

2. Next, count how many spaces you needed to move the decimal point to get from *0.000000595* to 5.95.

0.000000595 → *becomes* 5.95: Move the decimal point 7 spaces to the right.

3. Now, put everything together. Your number in scientific notation is 5.95 multiplied by 10 to the power of -7. Because you have a very small number and you moved the decimal point to the right in the first step, your exponent will be negative.

5.95×10^{-7}

The answer is *0.000000595* = 5.95×10^{-7}

Practice Questions

Write the following in scientific notation

1. 450,000,000
2. 0.000000067
3. 0.0000000056731
4. 24,010,000,000
5. 960,000,000,000,000,000
6. 0.0000001245
7. 36,000,000
8. 0.00098
9. 0.000000034
10. 345,000,000

Answers

1. 4.5×10^{8}
2. 6.7×10^{-9}
3. $5.6 \text{ x } 10^{-9}$
4. $2.4 \text{ x } 10^{10}$
5. $9.6 \text{ x } 10^{17}$

6. 1.245×10^{-7}
7. 3.6×10^{7}
8. 9.8×10^{-4}
9. 3.4×10^{-8}
10. 3.45×10^{8}

Attributions:

Chapter 4: Fractions

Tessa made a pan of brownies for the sixth grade social. She cut the brownie pan into sixteen brownies. She sold 12 out of 16 brownies at the bake sale.

What fraction of the brownies did she sell? What fraction did she not sell?

In this concept, you will learn to write a fraction in simplest form.

Finding Fractions in Simplest Form

Some fractions can describe large quantities. Simplifying a fraction can make it easier to understand its value. To **simplify** a fraction, divide the numerator and the denominator by a common factor. Sometimes you will also hear simplifying called *reducing* a fraction. A fraction that has been simplified by the greatest common factor is in **simplest form.** Remember that the **greatest common factor** (GCF) is the greatest factor that two or more numbers have in common.

Finding Common Multiples

A **multiple** is the product of a quantity and a whole number. Here are some multiples for the quantity of 3, multiplied by different whole numbers.

$3\times1=3$, $3\times2=6$, $3\times3=9$, $3\times4=12$, $3\times5=15$, $3\times6=18$

Listing out these products is the same as listing out multiples.

3, 6, 9, 12, 15, 18 ...

You can see that this is also the same as counting by threes. The dots at the end mean that these multiples can go on and on and on. Every number has an **infinite** number of multiples.

List six multiples for 4.

To do this, think of taking the quantity 4 and multiplying it by 1, 2, 3, 4, 5...

$4\times1=4$, $4\times2=8$, $4\times3=12$, $4\times4=16$, $4\times5=20$

Our answer is 4, 8, 12, 16, 20, 24...

A **common multiple** is a multiple that two or more numbers have in common. List the multiples of the numbers to find the common multiples.

Find common multiples for 3 and 4.

First, write out the first few multiples for the numbers and then identify the multiples the two numbers have in common.

For 3: 3, 6, 9, **12**, 15, 18, 21, **24**, 27, 30, 33, **36**

For 4: 4, 8, **12**, 16, 20, **24**, 28, 32, **36**, 40, 44, 48

Some of the common multiples of 3 and 4 are 12, 24, and 36.

Examples

Here is a fraction $\frac{48}{60}$

Simplify the fraction to better understand its value.

First, find a common factor for the numerator and denominator.

48 – 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

60 – 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

Then, divide by the GCF common factor. The GCF for 48 and 60 is 12

$$\frac{48\div12}{60\div12}=\frac{4}{5}$$

The simplest form of $\frac{48}{60}=\frac{4}{5}$

Comparing fractions with different denominators can be difficult. Some fraction may look similar, but not be equivalent fractions. You can compare fractions with different denominators by comparing them in their simplest form.

Here are two fractions. $\frac{3}{6}$ and $\frac{4}{8}$

Let's simplify $\frac{3}{6}$. To do this, divide the numerator and denominator by the GCF. The GCF of 3 and 6 is 3.

$$\frac{3 \div 3}{6 \div 3} = \frac{1}{2}$$

Let's simplify $\frac{4}{8}$. To do this, divide the numerator and the denominator by the GCF. The GCF of 4 and 8 is 4.

$$\frac{4 \div 4}{8 \div 4} = \frac{1}{2}$$

Both $\frac{3}{6}$ and $\frac{4}{8}$ are equivalent to $\frac{1}{2}$. Therefore, $\frac{3}{6}$ and $\frac{4}{8}$ are also equivalent fractions.

Example 1

Earlier, you were given a problem about Tessa and her brownies.

Tessa sold 12 out of 16 brownies at the bake sale. Simply the fraction for the number of brownies she sold and did not sell.

First, write the fraction for the number of brownies that were sold.

$$\frac{12}{16}$$

Then, simplify the fraction by dividing both by the GCF. The GCF is 4

$$\frac{12}{16} = \frac{12 \div 4}{16 \div 4} = \frac{3}{4}$$

Tessa sold $\frac{3}{4}$ of the brownies

Now, write the fraction for the number of brownies that were not sold.

$$\frac{4}{16}$$

Then, simplify the fraction by dividing both by the GCF. The GCF is 4.

$$\frac{4}{16} = \frac{4 \div 4}{16 \div 4} = \frac{1}{4}$$

She did not sell $\frac{1}{4}$ of the brownies.

Example 2

Simplify the fraction $\frac{27}{36}$.

First, find the GCF of 27 and 36. The GCF for 27 and 36 is 9.

27 – 1, 3, 9, 27

36 – 1, 2, 3, 4, 6, 9, 12, 18, 36

Then, divide both the numerator and the denominator by 9.

$$\frac{27 \div 9}{36 \div 9} = \frac{3}{4}$$

The simplest form of $\frac{27}{36}$ is $\frac{3}{4}$

Example 3

Simplify the fraction $\frac{4}{20}$.

First, find the GCF of 4 and 20. The GCF for 4 and 20 is 4

Then, divide both the numerator and the denominator by 4.

The simplest form of $\frac{4}{20}$ is $\frac{1}{5}$ $\frac{4}{20} = \frac{4 \div 4}{20 \div 4} = \frac{1}{5}$

Example 4

Simplify the fraction $\frac{8}{16}$.

First, find the GCF of 8 and 16. The GCF for 8 and 16 is 8

Then, divide both the numerator and the denominator by 8.

The simplest form of $\frac{8}{16}$ is $\frac{1}{2}$. $\frac{8}{16} = \frac{8 \div 8}{16 \div 8} = \frac{1}{2}$

Activity

Simplify the following fractions

1. $\frac{11}{44}$
2. $\frac{5}{20}$
3. $\frac{5}{15}$
4. $\frac{2}{12}$
5. $\frac{18}{20}$
6. $\frac{12}{24}$
7. $\frac{3}{12}$
8. $\frac{4}{20}$
9. $\frac{20}{45}$
10. $\frac{18}{20}$

Answers

1. $\frac{1}{4}$
2. $\frac{1}{4}$
3. $\frac{1}{3}$
4. $\frac{1}{6}$
5. $\frac{9}{10}$
6. $\frac{1}{2}$
7. $\frac{1}{4}$
8. $\frac{1}{5}$
9. $\frac{4}{9}$
10. $\frac{9}{10}$

A fraction can be written as a percent if it has a denominator of 100. Sometimes, you will be given a fraction with a denominator of 100 and sometimes you will have to rewrite the fraction to have a denominator of 100 before you can write it as a percent. Example,

$$\frac{9}{100}$$

This fraction is already written with a denominator of 100, so you can just change it to a percent.

$$\frac{9}{100}$$

A **proportion** is two equal ratios. If a fraction does not have a denominator of 100, you can write a fraction equal to it that does have a denominator of 100 and then solve the proportion.

Let's look at an example.

Write $\frac{3}{5}$ as a percent

First, notice that the denominator is not 100. Therefore, you need to create a new fraction equivalent to this one with a denominator of 100.

Next, set up a proportion.

$$\frac{3}{5} = \frac{x}{100}$$

Then, you can cross multiply to find the value of *x*.

$$5x = 300$$

$$X = 60$$

$$\frac{3}{5} = \frac{60}{100}$$

Now you have a fraction with a denominator of 100, and you can write it as percent. The answer is that the fraction $\frac{3}{5}$ is equal to 60%

Improper Fractions

An **improper fraction** is a fraction where the numerator is larger than the denominator. An improper fraction can be written as a mixed number. A **mixed number** is composed of a whole number and a fraction.

To change an improper fraction to a mixed number, divide the numerator by the denominator. This will tell you the number of wholes. If there is a remainder, it is the fraction part of a mixed number.

Example 1

Here is an improper fraction $\frac{18}{4}$.

There are 18 parts and the whole has only been divided into 4 parts. Remember that when the numerator is larger than the denominator, there is more than one whole.

1 whole = $\frac{4}{4}$

Convert $\frac{18}{4}$ to a mixed number.

First, divide the numerator by the denominator.

18÷ 4= 4 R2

Then, write the quotient as a mixed number with the remainder as a fraction. The remainder is the numerator of the fraction

$\frac{18}{4} = 4\frac{2}{4} = 4\frac{1}{2}$

The improper fraction $\frac{18}{4}$ is expressed as $4\frac{1}{2}$.

Example 2

Express this improper fraction as a mixed number $\frac{24}{5}$

First, divide the numerator by the denominator. 24÷ 5 = 4 *R4*

Then, write the quotient as a mixed number with the remainder as a fraction. The remainder is the numerator of the fraction. $\frac{24}{5} = 4\frac{4}{5}$

The improper fraction $\frac{24}{5}$ is expressed as $4\frac{4}{5}$

Example 3

Express this improper fraction as a mixed number $\frac{21}{3}$

First, divide the numerator by the denominator 21 ÷ 3= 7

This fraction has no remainder and is not a mixed number.

The improper fraction $\frac{21}{3}$ is equal to 7.

Review

Convert each improper fraction to a mixed number. Simplify when it is necessary.

1. $\frac{22}{3}$
2. $\frac{44}{5}$

3. $\frac{14}{3}$
4. $\frac{7}{2}$
5. $\frac{10}{3}$
6. $\frac{47}{9}$
7. $\frac{50}{7}$
8. $\frac{60}{8}$
9. $\frac{43}{8}$
10. $\frac{19}{5}$

Answers

1. $7\frac{1}{3}$
2. $8\frac{4}{5}$
3. $4\frac{2}{3}$
4. $3\frac{1}{2}$
5. $3\frac{1}{3}$
6. $5\frac{2}{9}$
7. $7\frac{1}{7}$
8. $7\frac{1}{2}$
9. $5\frac{3}{8}$
10. $3\frac{4}{5}$

SECTION 1: In the following exercises, simplify.

1. $-\frac{63}{84}$

2. $-\frac{90}{120}$

3. $-\frac{14a}{14b}$

4. $-\frac{8x}{8y}$

Multiplying Two Fractions

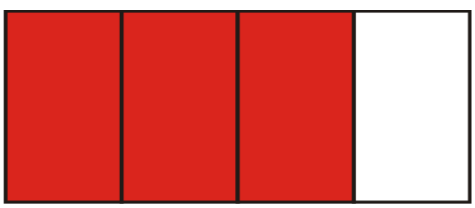
Simon has $\frac{3}{4}$ of a pie left over from last night's dinner. He wants to take half of the remaining pie to his friend's house. How much pie is Simon taking with him?

In this concept, you will learn how to multiply two fractions.

Multiplying fractions can be a little tricky to understand. When adding fractions, you are finding the sum. When you subtract fractions, you are finding the difference. When **multiplying** a fraction by a whole number, you are finding the sum of a repeated fraction or a repeated group.

When you multiply two fractions, it means that you are looking for a part of a part. Here is a multiplication problem with two fractions. $\frac{1}{2} \times \frac{3}{4} =$

The **product** is one-half of three-fourths. Here is a diagram. $\frac{3}{4}$



Three-fourths of the whole is shaded. To find one-half of the three-fourths, divide the entire diagram in half.

The diagram is evenly divided into 8 parts. The shaded parts are divided into 6 parts. The gray shaded part represents half of the three-fourths. Therefore, $\frac{1}{2}$ of $\frac{3}{4} = \frac{3}{8}$

You can't always draw pictures to figure out a problem, so you can multiply fractions using a few simple steps.

To multiply two fractions, multiply the numerator by the numerator and the denominator by the denominator.

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

Here is an example.

$$\frac{1}{2} \times \frac{3}{4} = \frac{1 \times 3}{2 \times 4} = \frac{3}{8}$$

The product is $\frac{3}{8}$. The answer is the same as the one found earlier.

Let's look at another example. $\frac{3}{6} \times \frac{1}{9}$

First, multiply the numerator by the numerator and the denominator by the denominator.

$$\frac{3 \times 1}{6 \times 9} = \frac{3}{54}$$

Next, simplify the fraction $\frac{3}{54}$ by dividing by the greatest common factor (GCF). The GCF of 3 and 54 is 3.

$$\frac{3 \div 3}{54 \div 3} = \frac{1}{18}$$

The product is $\frac{1}{18}$

To solve this problem, you multiplied and then simplified. Sometimes, you can simplify before multiplying. Let's look at the problem again. $\frac{3}{6} \times \frac{1}{9}$

here are two ways you can simplify this problem before multiplying.

1. **Simplify any fractions that can be simplified.**

Here three-sixths can be simplified to one-half. The new problem would be

$$\frac{1}{2} \times \frac{1}{9} = \frac{1}{18}$$

2. **Cross simplify the fractions.**

To **cross-simplify**, simplify on the diagonals by using greatest common factors to simplify a numerator and an opposite denominator.

Credit: thebittenword.com

Look at the numbers on the diagonals and simplify any that you can. Now, 1 and 6 cannot be simplified, but 3 and 9 have the GCF of 3.

$3 \div 3 = 1$

$9 \div 3 = 3$

Next, substitute the new numbers for the old ones and multiply. $\frac{1}{6} \times \frac{1}{3} = \frac{1}{18}$

Notice that you can simplify two different ways but will always end up with the same answer.

Example 1:

Earlier, you were given a problem about Simon and his pie.

Simon is taking half of $\frac{3}{4}$ of a pie to his friend's house. Multiply one-half times $\frac{3}{4}$ to find the amount of pie Simon is taking with him.

$$\frac{1}{2} \times \frac{3}{4}$$

First, multiply the fraction. Find the product of the numerators over the product of the denominators. $\frac{1 \times 3}{2 \times 4} = \frac{3}{8}$

The fraction is in simplest form.

Simon is taking $\frac{3}{8}$ of a pie with him to his friend's house.

Example 2:

Find the product: $\frac{3}{7} \times \frac{2}{3}$ Answer in simplest form.

First, multiply the numerator by the numerator and the denominator by the denominator. $\frac{6}{21}$

Then, simplify the fraction. Divide 6 and 21 by the GCF of 3.

$$\frac{6 \div 3}{21 \div 3} = \frac{2}{7}$$

The product is $\frac{2}{7}$.

Example 3:

Find the product: $\frac{6}{9} \times \frac{1}{3}$Answer in simplest form.

First, simplify the fraction $\frac{6}{9}$ and rewrite the problem. The GCF of 6 and 9 is 3.

$$\frac{6}{9} = \frac{2}{3}$$

$$\frac{2}{3} \times \frac{1}{3}$$

Then, multiply the numerator by the numerator and the denominator by the denominator.

$$\frac{2 \times 1}{3 \times 3} = \frac{2}{9}$$

The product is $\frac{2}{9}$

Section 2: In the following exercises multiply.

1. $\frac{3}{4} \times \frac{2}{5}$

2. $-\frac{4}{7} \times \frac{7}{7}$

3. $\frac{5}{2} \times \frac{3}{6}$

4. $\frac{6m}{1} \times -\frac{4}{11}$

5. $\frac{8}{9} \times \frac{2}{5}$

6. $\frac{9}{1} \times \frac{2}{3}$

Division Fractions

When dividing whole numbers and fractions, you first change the operation to **multiplication** and then change the divisor to its **reciprocal**. The same rule applies to dividing a fraction by another fraction. Here is a division problem.

$$\frac{1}{2} \div \frac{1}{3}$$

Start by applying the first part of the rule and change the sign to multiplication. Then apply the second part of the rule, the reciprocal of one-third is three over one.

$$\frac{1}{2} \div \frac{1}{3} \quad = \quad \frac{1}{2} \times \frac{3}{1}$$

Then, multiply the fractions.

$$\frac{1}{2} \times \frac{3}{1} = \frac{3}{2}$$

Next, simply the fraction. Convert the improper fraction to a mixed number.

$$\frac{3}{2} = \ 1\frac{1}{2}$$

Example:

Divide: $4\ \frac{3}{10} \div \frac{1}{2}$

First, change the mixed number to an improper fraction.

$$4 \times 10 + 3 = 43$$

$$4\ \frac{3}{10} = \frac{43}{10}$$

Next, flip the second fraction in order to multiply.

Therefore, $\frac{1}{2}$ becomes $\frac{2}{1}$.

Then, multiply.

$$\frac{43}{10} \times \frac{2}{1} = \frac{86}{10}$$

Then, simplify your answer as a mixed number.

$$\frac{86}{10} = 8\frac{6}{10} = 8\frac{3}{5}$$

The answer is $8\frac{3}{5}$.

SECTION 1: In the following exercises, simplify.

1. $-\frac{63}{84}$

2. $-\frac{90}{120}$

3. $-\frac{14a}{14b}$

4. $-\frac{8x}{8y}$

SECTION 2: In the following exercises, multiply.

1. $\frac{3}{4} \times \frac{2}{5}$

2. $-\frac{4}{7} \times \frac{7}{7}$

3. $\frac{5}{2} \times \frac{3}{6}$

4. $\frac{6m}{1} \times -\frac{4}{11}$

5. $\frac{8}{9} \times \frac{2}{5}$

6. $\frac{9}{1} \times \frac{2}{3}$

SECTION 3: In the following exercises, divide.

1. $\frac{2}{3} \div \frac{1}{6}$

2. $\left(-\frac{3x}{5}\right) \div \left(-\frac{3y}{3}\right)$

3. $\frac{4}{5} \div 3$

4. $8 \div \frac{8}{3}$

5. $\frac{5}{8} \div \left(-\frac{b}{a}\right)$

SECTION 4: In the following exercises, perform the indicated operation.

1. $3\,\frac{1}{5} \cdot\ 1\frac{7}{8}$

2. $-5\frac{7}{12} \cdot\ 4\frac{4}{11}$

3. $8 \div 2\frac{2}{3}$

4. $8\,\frac{2}{3} \div 1\frac{1}{2}$

SECTION 5: In the following exercises, add.

1. $\frac{3}{8} + \frac{2}{8}$

2. $\frac{4}{5} + \frac{1}{5}$

3. $\frac{2}{5} + \frac{1}{5}$

4. $\frac{15}{32} + \frac{9}{32}$

5. $\frac{x}{10} + \frac{7}{10}$

SECTION 6: In the following exercises, Subtract.

1. $\frac{8}{11} - \frac{6}{11}$

2. $\frac{11}{12} - \frac{5}{12}$

3. $\frac{4}{5} - \frac{4}{5}$

4. $-\frac{31}{30}-\frac{7}{30}$

5. $\frac{3}{2}-\left(\frac{3}{2}\right)$

6. $\frac{11}{15}-\frac{5}{15}-\left(-\frac{2}{15}\right)$

SECTION 7: In the following exercises, find the least common denominator.

1. $\frac{1}{3}$and$\frac{1}{2}$

2. $\frac{1}{3}$and$\frac{4}{5}$

3. $\frac{8}{5}$ and $\frac{11}{20}$

4. $\frac{3}{4}, \frac{1}{6}$, and$\frac{5}{10}$

SECTION 8: In the following exercises, change to equivalent fractions using the given LCD.

1. $\frac{1}{3}$and$\frac{1}{5}$, LCD=15

2. $\frac{3}{8}$and$\frac{5}{6}$, LCD=24

3. $-\frac{9}{16}$and$\frac{5}{12}$, LCD=48

4. $\frac{1}{3},\frac{3}{4}$and$\frac{4}{5}$, LCD=60

SECTION 9: In the following exercises, perform the indicated operations and simplify.

1. $\frac{1}{5}+\frac{2}{3}$

2. $\frac{11}{12}-\frac{2}{3}$

3. $-\frac{9}{10}-\frac{3}{4}$

4. $-\frac{11}{36}-\frac{11}{20}$

5. $-\frac{22}{25}+\frac{9}{40}$

6. $\frac{y}{10}-\frac{1}{3}$

7. $\frac{2}{5}+\left(-\frac{5}{9}\right)$

8. $\frac{4}{11}\div\frac{2}{7d}$.

9. $\frac{2}{5}+\left(-\frac{3n}{8}\right)\left(-\frac{2}{9n}\right)$

10. $\frac{\left(\frac{2}{3}\right)^2}{\left(\frac{5}{8}\right)^2}$

11. $\left(\frac{11}{12}+\frac{3}{8}\right)\div\left(\frac{5}{6}=\frac{1}{10}\right)$

Answers:

SECTION 1:

2.$-\frac{3}{4}$

4.$-\frac{x}{y}$

SECTION 2:

2.$-\frac{4}{7}$

4.$-\frac{24m}{11}$

6. 6

SECTION 3:

2.$-\frac{3}{4}$

4.$-\frac{x}{y}$

SECTION 4:

2.$-\frac{268}{11}$

4. 8

SECTION 5:

1.$\frac{5}{8}$

3.$\frac{3}{5}$

5.$\frac{x+7}{10}$

SECTION 6:

1. $\frac{5}{8}$

2. $\frac{1}{2}$

3. $\frac{3}{5}$

4. $-\frac{19}{15}$

5. $\frac{x+7}{10}$

6. $\frac{8}{15}$

SECTION 7:

2. 15

4. 60

SECTION 8:

2.$\frac{9}{24}$and$\frac{20}{24}$

4.$\frac{20}{60}$,$\frac{45}{60}$ and $\frac{48}{60}$

SECTION 9:

2. $\frac{1}{4}$

4. $-\frac{77}{90}$

6. $\frac{3y-10}{30}$

8. $\frac{14d}{11}$

10. $\frac{256}{225}$

Fractions – Word Problems Video: https://youtu.be/MMLCiLNkEww

Attributions:

Chapter 5: Percent

Introduction

Definition Of Percent

Numbers written in percent form represent amounts out of 100. The word "*percent*" actually means "per 100" (Think of it as "per cent" and there are 100 cents in $1). The following example will help us start thinking about numbers in percentage form.

Example 1: At a recent "Rats on Rafts" rock concert (there actually is a 2012 band with this name! ©), 50% of the attendees were under 18, 25% were 18 – 24 and the rest were over 24. If 22,140 people attended the concert, how many were in each age group?

Decimals, Fractions, Percents

Decimals, fractions, and percent are connected. The following table shows howto convert from one type to the other.

Percent to Decimal	50%=.50	Remove % sign. Divide by 100.
Percent to Fraction	$50\% = \frac{50}{100} = \frac{1}{2}$	Remove % sign. Place over 100. Reduce fraction.
Decimal to Percent	.050=50%	Multiply by 100. Include % sign.
Fraction to Percent	$\frac{1}{2} = .50 = 50\%$	Divide. Multiply by 100. Include % sign.

Example 2: Complete the missing parts of the table. Round to THREE decimal places as need. Simplify all fractions. Show all work.

Fraction	Decimal	Percent
		32%
	0.040	
$\frac{3}{4}$		
	0.625	
		150%
$1\frac{3}{7}$		

Practice Activity

1. Complete the missing parts of the table. Round decimal part to FOUR decimal placesas needed. Simplify all fractions. Show all work.

Fraction	Decimal	Percent
a. $\frac{1}{9}$		
b.	0.0625	
c.		80%

Creating And Solving Percent Equations

When working with a problem involving percent, the most straightforward way to solve it is by setting up a *percent equation*. The information we just covered on the previous page will help solve percent equations once they are set up. The information below provides translation guidance for words or phrases that are part of percent statements.

- The percent (usually represent as a decimal)
- Multiplication (replace the word "of" with multiplication)
- The unknown (usually represented by the word "what" and replace with "x")
- Equals (replace the word "is" with "=")

Let's see how these translations are used when setting up the three main types of percent problems.

TYPE I: Unknown is A% of B

Example 4: Determine the missing number in each of the following. Round to two decimals as needed.

a. What is 12% of 20?

b. What is 30.45% of 450?

c. 12% of 600 is what number?

d. What number is 0.5% of 8?

Practice Activity

2. Determine the missing number in each of the following. Round two decimals as needed.

a. What is 15% of 324? b. 25.12% of 132 is what number?

TYPE II: A% Unknown is B

Example 4: Determine the missing number in each of the following. Round to TWO decimals as needed. Show all work.

a. 60% of what number is 15? b. 25 is 12.25% of what number?

c. 175% of what number is 325.16? d. 20 is 0.14% of what number?

Practice Activity

3. Determine the missing number in each of the following. Round to TWO decimals as needed. Show all work.

a. 40% of what number is 20? b. 105 is 15.15% of what number?

TYPE III: Unknown % of A is B

Example 5: Determine the missing number in each of the following. Round to TWO decimals as needed. Show all work.

a. What % of 140 is 3.8? b. What percent of 620 is 136.4?

c. What % of 25 is 0.05? d. 240 is what percent of 100?

Practice Activity

4. Determine the missing number in each of the following. Round to TWO decimals as needed. Show all work.

a. What % of 12 is 8? b. 105 is what percent of 123?

Applications Of Percent's – Types I, II, III

Try to recognize the percent problem as one of our three types and set up the percent equation to solve the missing part. Use a modified version of our problem-solving process by circling the given information and underlining the goal in each problem.

Example 6: (TYPE I)

At a restaurant, the bill comes to $51.23. If you decide to leave a 14% tip, how much is the tip and what is the final bill? Round to the nearest cent.

Example 7: (TYPE II)

Joyce paid $33.00 for an item at the store that was marked as 45 percent off the originalprice. What was the original price? Round to the nearest cent.

Example 8: (TYPE III)

Trader Joe's sold 8233 bags of tortilla chips recently. If 5178 of these bags were fat free, find the percent that were fat free. Round your answer to the nearest whole percent.

Practice Activity

5. To win the election as president of the United States of America, a person must obtain 270 out of 538 possible votes from the electoral college. What percentage of the overall electoral votes is this? Round to 4 decimals initially. Be sure to set up your percent statement and equation as in the examples.

ANSWERS TO PRACTICE EXERCISES - correct numbers as above

1a: 1/9 = 0.1111=11.11%

1b: 1/16 = 0.0625 = 6.25%

1c: 4 5 = 0.8000 = 80%

2a: x = 0.15(324), x = 48,60

2b: x = 0.2512(132), x =33.16

3a: 0.40(x) = 20, x = 50

3b: 105 =.1515(x), x = 693.07

4a: 12(x)= 8, x = 0.67 = 67%

4b: 105 = 123(x), x = 0.85 = 85%

5: Percentage statement: 270 is what percentage of 538?

Percentage equation 270 = x (538), x =.5019 = 50.19%

Attribution:

Chapter 6: Number Relationships

Introduction

In this chapter, we will be learning about and solving problems with a special kind of fraction called a *ratio.*

The table below shows the specific objectives that are the achievement goal for this chapter.

Objectives
Write and simplify *ratios*
Write and simplify *rates*
Compute *unit rates*
Solve proportions using *cross-products*
Solve applications using *proportional reasoning*

Key Terms

The key terms listed below will help you keep track of important mathematical words and phrases that are part of this lesson. Look for these words and circle or highlight them along with their definition or explanation as you work through the chapter.

- Ratio
- Rate
- Unit Rate
- Proportion
- Cross Product
- Cross Product Method
- Proportional Reasoning

Ratios & Rates

Write a ratio to compare two different quantities. Units are important and are always included if they are present to begin with. The examples below demonstrate the different notations you may see when writing ratios.

Example 1: Write "8 feet to16 feet" as a ratio in simplest form.

Example 2: Write "6:18" as a ratio in simplest form.

EXPLANATION: https://www.youtube.com/watch?v=gWq2CYsm9dM

If the quantities you are comparing have different units, then your ratio is known as a *rate*. Units are especially important here and should absolutely be included.

Example 3: Write "12 miles in 10 hours" as a ratio in simplest form.

Example 4: In a small bag of mixed nuts, 15 were peanuts, 20 were almonds, and 5 were Brazil nuts. Write the ratio of peanuts to almonds in simplest form.

EXPLANATION: http://youtu.be/kGbddCcWAFU

Practice Activity

1. Use the information to write a ratio in simplest form. Indicate if the ratio is also a rate.

a. 5 feet:10 feet

b. 12 geese to 15 ducks

Unit Rates

A unit rate is a special kind of rate in which the denominator of the ratio is equal to 1. This kind of rate allows for easier comparison of different rates as seen in the example below. As with rates, units are essential and must be included.

Example 5: Which is faster, "12 miles in 10 hours" or "10 miles in 8 hours"? Use unit rates to compare.

EXPLANATION: http://youtu.be/bk6xQt1PS9U

Example 6: Determine which bag of Cheetos is the better buy. Bag A: $4.99 for 20.50 oz Bag B: $4.29 for 12.50 oz

EXPLANATION: http://youtu.be/tnCPrgKpC3Q

Example 7: Write each of the following as a unit rate:

a. There are 5280 feet in a mile

b. There are 60 seconds per each minute

c. Gasoline costs $3.49 a gallon

EXPLANATION: http://youtu.be/XgZ-ljhYAtE

Practice Activity

2. Amazon.com recently advertised the following choices for ibuprofen tablets (200mg). Use unit rates to determine which is the better buy.

Option 1: 360 pills for $15.45 Option 2: 300 pills for $12.98

Proportions & Proportional Reasoning

In Example 3, we were given the rate, "12 miles in 10 hours" which we simplified to "6 miles in 5 hours". Let's see how we might write that as a formal mathematical statement of equality:

$$\frac{12\ miles}{10\ hours} = \frac{6\ miles}{5\ hours}$$

The statement above is called a *proportion* because it sets two rates (or ratios) equal. Because the above rates are equivalent, the equality statement is true.

Suppose, however, that the following problem was posed:
"If George walks 6 miles in 5 hours, how far would he walk in 10 hours?" We will use the concept of variable from Lesson 6 to set up the following proportion:

$$\frac{x\ miles}{10\ hours} = \frac{6\ miles}{5\ hours}$$

The distance George walks in 10 hours is our unknown value and is represented by the variable x. Technically, in this problem, we know that our solution for x is 12. But how would we determine that? First, because our ratios of units are the same (miles/hours) we can simplify our statement this way:

$$\frac{x}{10} = \frac{6}{5}$$

Then, we can use one *cross-product* to rewrite as follows:

$$\frac{x}{10} = \frac{6}{5}$$

Multiply across the = sign bottom to top

$$x = \frac{6x10}{5}$$

And finally, we can write our final solution a $x = \frac{60}{5} = 12$

The final answer to our original question, "if George walks 6 miles in 5 hours, how far would he walk in 10 hours" is that George could walk 12 miles in 10 hours. We solved this problem using proportional reasoning, one of the most-used problem-solving techniques in mathematics.

The following examples will illustrate additional ways to work with and solve proportions using the cross-product method.

Example 8: Use the cross-product method to determine the value for t in each of the following proportion problems. Round any decimals to the hundredths place.

a. $\frac{3}{4} = \frac{t}{40}$ b. $\frac{t}{2} = \frac{3}{5}$

EXPLANATION: http://youtu.be/oNQi5CHtbZo

Example 9: Use the cross-product method to determine the value for x in each of the following proportion problems. Round any decimals to the hundredths place.

a. $\frac{6}{12} = \frac{18}{x}$ b. $\frac{23}{x} = \frac{4.1}{5.6}$

EXPLANATION: http://youtu.be/zU3FMt1fnBM

Example 10: Use the cross-product method to determine the value for r in each of the following proportion problems. Round any decimals to the hundredths place.

a. $\frac{r}{5} = 3$ b. $\frac{\frac{1}{2}}{4} = \frac{8}{r}$

EXPLANATION: http://youtu.be/6-ewX3Oso8Q

Practice Activity

3. Solve the proportions showing all possible steps. Round your answer to the nearest hundredth as needed.

a. $\frac{x}{12} = \frac{3}{6}$ b. $\frac{6}{5} = \frac{10}{p}$

Example 11: Ten gallons of water leak from a hose in 20 hours. At this rate, how much water will leak in 10 days? Practice circling the GIVENS and underlining the GOALS to start your problem-solving process.

EXPLANATION: http://youtu.be/P8cUgmz-JK4

Practice Activity

4. Mary earned $112.50 last week working 12 hours at her part-time job. If she works 15 hours this week and is paid the same rate, how much will she earn? Use proportional reasoning to determine your result. Round to the nearest cent.

ANSWERS TO PRACTICE EXCERCISES

1a: 1 /2 [Notice that the units cancelled. This ratio is NOT a rate]

1b: 4geese 5ducks [Notice that the units did NOT cancel. This ratio IS a rate.]

2: Option 1 is the better buy per pill.

3a: x = 6

3b: p = 8.33

4: Mary will earn $140.63 in 15 hours this week.

SOLVE APPLICATIONS USING PROPORTIONS

In the following exercises, solve the proportion problem.

1. Pediatricians prescribe 5 milliliters (ml) of acetaminophen for every 25 pounds of a child's weight. How many milliliters of acetaminophen will the doctor prescribe for Jocelyn, who weighs 45 pounds?

2. Brianna, who weighs 6 kg, just received her shots and needs a pain killer. The pain killer is prescribed for children at 15 milligrams (mg) for every 1 kilogram (kg) of the child's weight. How many milligrams will the doctor prescribe?

3. At the gym, Carol takes her pulse for 10 sec and counts 19 beats. How many beats per minute is this? Has Carol met her target heart rate of 140 beats per minute?

4. Devin wants to keep his heart rate at 160 beats per minute while training. During his workout he counts 27 beats in 10 seconds. How many beats per minute is this? Has Kevin met his target heart rate?

5. A new energy drink advertises 106 calories for 8 ounces. How many calories are 12 ounces of the drink?

6. One 12-ounce can of soda has 150 calories. If Josiah drinks the big 32-ounce size from the local mini-mart, how many calories does he get?

7. Karen eats 12 cups of oatmeal that counts for 2 points on her weight loss program. Her husband, Joe, can have 3 points of oatmeal for breakfast. How much oatmeal can he have?

8. An oatmeal cookie recipe calls for 12 cups of butter to make 4 dozen cookies. Hilda needs to make 10 dozen cookies for the bake sale. How many cups of butter will she need?

9. Janice is traveling to Canada and will change $250 US dollars into Canadian dollars. At the current exchange rate, $1 US is equal to $1.01 Canadian. How many Canadian dollars will she get for her trip?

10. Todd is traveling to Mexico and needs to exchange $450 in to Mexican pesos. If each dollar is worth 12.29 pesos, how many pesos will he get for his trip?

11. Steve changed $600 into 480 Euros. How many Euros did he receive per US dollar?

ANSWERS USING PROPORTIONS

1. 9 ml

3. 114 beats/minute. Carol has not met her target heart rate.

5. 159 Cal

7. 34 cups

9. $252.50

11. 0.8 Euros

Explanation: Multi-Step Ratio Problems https://youtu.be/M8MMTid33sQ

Attribution:

Chapter 7: Measurements

Measurement – Length

Measurement is also a concept that appears all the time in everyday life. To measure something is to determine a size, amount, or degree of an object using tools or comparison of objects.

Length is the measurement of the distance between two points, two things or two places. To measure length, a unit of measurement is used. There are two different systems for measuring: customary and metric. This concept is about Customary Units of measurement.

The most common customary units of measurement are the inch, the foot, the yard and the mile. These units can be converted between one another; for example, there are 12 inches in 1 foot, 3 feet in 1 yard, 5,280 feet in 1 mile etc. An important skill in measuring is to know when to use each of these units of measurement so less conversions need to occur.

On a ruler, there are 12 inches marked, which makes one whole foot. Larger objects are measured using feet instead of inches because it is a larger unit of measurement. Feet are commonly used in measurement for objects such as the length of a car, height of a door, or the distance between two objects.

Another customary unit of measurement is **yards.** The abbreviation for yards is yd (for example, 7 yd). There are three feet in one yard. Yards can be used to measure larger objects than would be measured with inches or feet.

It makes sense to use inches, feet and yards when measuring short distances or the length of objects or people. We use these customary units of measurement all the time in our everyday life.

To measure longer distances, the customary unit of **miles** is used. A mile is the longest customary unit of measurement. There are 5,280 feet in one mile.

Here are some equivalent tables.

Customary Units of Length

inch (in.)	foot (ft)	yard (yd)	mile (mi)
12	1		
	3	1	
	5,280	1,760	1

Customary Units of Weight

ounces (oz)	pounds (lb)	tons (T)
16	1	
	2,000	1

Customary Units of Capacity

fluid ounces (fl oz)	cups (c)	pints (pt)	quarts (qt)	gallon (gal)
8	1			

16	2	1		
32	4	2	1	
128	16	8	4	1

Customary Units of Time

60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day
7 days = one week
52 weeks = 1 year
12 months = 1 year
36 days = 1 year

Practice Activity

Write the appropriate customary unit of measurement for each item.

1. The height of a sunflower
2. The depth to plant a seed in the soil
3. The height of a tree
4. The area of a garden plot
5. The distance from a garden to the local farm store
6. The length of a carrot
7. A stretch of fencing
8. The length of a hoe
9. The distance between two seedlings planted in the ground
10. The height of a corn stalk
11. A piece of pipe for a water line
12. The depth of a pool
13. The distance across a lake
14. The distance from your home to school
15. The size of a paperclip
16. The measure of the length of thread

Practice Answers

1) inches 2) inches 3) feet 4) square ft. 5) miles 6) inches 7) yards or feet 8) feet 9) inches 10) feet 11) feet 12) feet 13) miles 14) miles 15) inches 16) inches

Converting Customary Units by Adding

When adding units of measurements first add the units that are the same and convert the units to the simplest form.

2 pounds + 15 pounds = 17 pounds

3 yards + 2 yards = 5 yards

Converting Customary Units by Subtracting

When subtracting units of measurements first subtract the units that are the same and convert the units to the simplest form.

5.5 pounds - 3.5 pounds = 2 pounds

7 yards - 2 yards = 5 yards

Converting Customary Units by Multiplying

When converting customary units of measure from a larger unit to a smaller unit, multiply the larger unit by its smaller equivalent unit. You may already be wondering why you need to multiply as opposed to some other operation. Here is an example to demonstrate this.

1 dollar = 100 pennies

There are 100 pennies in one dollar. The dollar is a larger unit than the penny. You need many pennies to equal one dollar. The same is true when working with units of length, weight and capacity. You need more of a smaller unit to equal a larger unit.

Think back to all of the units of length, weight and capacity that you have previously learned about.

Let's look at a conversion problem.

John has a rope that is 10 feet long. How long is his rope in inches?

Notice, you are converting from **feet** to **inches**. A foot is a larger unit than an inch.

10 feet = _____ inches

To solve this problem, multiply the number of feet by the unit equivalence. This will give you the measurement in inches.

1 foot = 12 inches

$10 \times 12 = 120$

The answer is 10 feet is equivalent to 120 inches.

Practice Activity

Convert the following units of measure.

1. 5 tons = ____ pounds
2. 6 feet = ____ inches
3. 9 tons = ____ pounds
4. 8 pounds = ____ ounces
5. 2.5 feet = ____ inches
6. 3.5 tons = ____ pounds
7. 2.25 pounds = ____ ounces
8. 9 cups = ____ fl oz
9. 5 pints = ____ cups
10. 7 pints = ____ cups
11. 8 quarts = ____ pints
12. 1 quart = ____ pints
13. 6 gallons = ____ quarts
14. 7.75 gallons = ____ quarts
15. 8 miles = _____ feet
16. 3 feet = _____ inches
17. 12 miles = _____ feet

Practice (Answers)

1. 10,000 pounds 2. 72 inches 3. 18,000 pounds 4. 128 ounces 5. 30 inches 6. 7000 pounds 7. 36 ounces 8. 72 ounces 9. 10 cups 10. 14 cups 11. 16 pints 12. 2 pints 13. 24 quarts 14. 31 quarts 15. 42,240 feet 16. 36 inches 17. 63,360 feet

Converting Customary Units by Dividing

When converting from a larger unit to a smaller unit, you multiplied by the unit equivalence. To convert from a smaller unit to a larger unit, divide by the unit equivalence.

Let's consider pennies.

5,000 pennies = ____ dollars

The penny is a smaller unit than the dollar. You need more of a smaller unit to equal a larger unit. Divide the number of pennies by the unit equivalence. You know that there are 100 pennies in one dollar.

100 pennies = 1 dollar

Divide 5,000 by 100 to get the equivalent value in dollars.

$5000 \div 100 = 50$

Therefore, 5,000 pennies are equivalent to 50 dollars.

Apply this to your work with converting measurements. Remember to think about the equivalent units of length, capacity and weight when dividing.

Let us look at a conversion problem.

5,500 pounds = ____ tons

A pound is smaller than a ton. To solve this problem, divide the number of pounds by the unit equivalent.

2,000 pounds = 1 ton

5,500 ÷÷ 2,000 = 2.75 or 2 ¾   tons

Notice that you can write the answer in fraction or decimal form. Therefore, 5,500 pounds is equivalent to 2.75 or 2 ¾   tons.

Practice Activity

Convert the following units of measure.

1. 6 quarts = ____ gallons
2. 24 inches = ____ feet
3. 18 inches = ____ feet
4. 4 quarts = ____ gallons
5. 12 pints = ____ quarts
6. 25 pints = ____ quarts
7. 1 quart = ____ gallons
8. 9 quarts = ____ gallons
9. 15 quarts = ____ gallons
10. 99 inches = ____ feet
11. 98 pints = ____ gallons
12. 12,000 pounds = ____ tons
13. 22,000 pounds = ____ tons

14. 5,000 pounds = ____ tons
15. 20,000 pounds = ____ tons

Practice Answers

1. 1.5 gallons 2. Feet 3. 1.5 feet 4. 1 gallon 5. 6 quarts 6. 12.5 quarts 7. 25 gallons 8. 2.25 gallons 9. 3.75 gallons 10. 8.25 feet 11. 12.25 gallons 13. 11 tons 14. 2.5 tons 15. 10 tons

The Metric System

The metric system uses units such as **meter, liter,** and **gram** to measure length, liquid volume, and mass, just as the U.S. customary system uses feet, quarts, and ounces to measure these. In addition to the difference in the basic units, the metric system is based on 10s, and different measures for length include kilometer, meter, decimeter, centimeter, and millimeter. Notice that the word "meter" is part of all of these units.

The metric system also applies the idea that units within the system get larger or smaller by a power of 10. This means that a meter is 100 times larger than a centimeter, and a kilogram is 1,000 times heavier than a gram.

Length, Mass, and Volume

The table below shows the basic units of the metric system. Note that the names of all metric units follow from these three basic units.

Length	Mass	Volume
Basic Units		
Meter	Gram	Liter
Other units you may see		
Kilometer	Kilogram	Dekaliter
Centimeter	Centigram	Centiliter
Millimeter	Milligram	Milliliter

In the metric system, the basic unit of length is the meter. A meter is slightly larger than a yardstick, or just over three feet.

The basic metric unit of mass is the gram. A regular-sized paperclip has a mass of about 1 gram.

Finally, the basic metric unit of volume is the liter. A liter is slightly larger than a quart.

The table below shows the relationship between some common units in both systems.

	Common Measurements in Customary and Metric Systems
Length	1 centimeter is a little less than half an inch.
	1.6 kilometers is about 1 mile.
	1 meter is about 3 inches longer than 1 yard
Mass	1 kilogram is a little more than 2 pounds.
	28 grams is about the same as 1 ounce.
Volume	1 liter is a little more than 1 quart.
	4 liters is a little more than 1 gallon.

Prefixes in the Metric System

The metric system is a base 10 system. This means that each successive unit is 10 times larger than the previous one. The names of metric units are formed by adding a prefix to the basic unit of measurement. To tell how large or small a unit is, you look at the prefix. To tell whether the unit is measuring length, mass, or volume, you look at the base.

Prefixes in the Metric System

Kilogram (kg)	**Hectogram (hg)**	**Dekagram (dag)**	**Gram (g)**	**Decigram (dg)**	**Centigram (cg)**	**Milligram (mg)**
1,000 times **larger** than base unit	100 times **larger** than base unit	10 times **larger** than base unit	Base Unit	10 times **smaller** than base unit	100 times **smaller** than base unit	1,000 times **smaller** than base unit
Measuring Mass in the Metric System						
1,000 grams	**100 grams**	**10 grams**		**.1 gram**	**.01 gram**	**.001 gram**

Practice Activity

Which of the following sets of three units are all metric measurements of length?

A) inch, foot, yard
B) kilometer, centimeter, millimeter
C) kilogram, gram, centigram
D) kilometer, foot, decimeter

Answer: C

Practice:

1. How many milliliters are in 1 liter?
2. Convert 3,085 milligrams to grams

Answers:

1. 1,000 ml 2. 3.085 grams

Factor Label Method

The table below shows some of the unit equivalents and unit fractions for length in the metric system. (You should notice that all of the unit fractions contain a factor of 10. Remember that the metric system is based on the notion that each unit is 10 times larger than the one that came before it.)

Unit Equivalents	Conversion Factors	
1 meter = 1,000 millimeters	$\frac{1m}{1{,}000mm}$	$\frac{1{,}000mm}{1m}$
1 meter = 100 centimeters	$\frac{1m}{100cm}$	$\frac{100cm}{1m}$
1 meter = 10 decimeters	$\frac{1m}{10dm}$	$\frac{10dm}{1m}$
1 dekameter = 10 meters	$\frac{1dam}{10m}$	$\frac{10m}{1dam}$
1 hectometer = 100 meters	$\frac{1hm}{100m}$	$\frac{100m}{1hm}$

1 kilometer = 1,000 meters	$\frac{1km}{1,000m}$	$\frac{1,000m}{1km}$

When applying the factor label method in the metric system, be sure to check that you are not skipping over any intermediate units of measurement!

This problem asked for the difference between two quantities. The easiest way to find this is to convert one quantity so that both quantities are measured in the same unit, and then subtract one from the other.

Determining Metric Units of Length and Converting Metric Measurements

The common metric units of length are **millimeter***,* **centimeter***,* **decimeter, meter,** and **kilometer***.* When making a measurement, you will need to choose which unit to use.

A **millimeter** (mm) is the smallest unit. There are 10 mm in one centimeter, if an object is smaller than one centimeter, you would use millimeters. A scientist measuring something under a magnifying glass might use millimeters to represent a tiny specimen.

A **centimeter** (cm) is the next smallest unit. You can use a ruler to measure things in centimeters. If an object is the length of a ruler or smaller, then it makes sense to use centimeters to measure.

Meters (m) are used to measure everything between the length of a ruler and the distance between things in a room. Most household objects such as tables, rooms, window frames, television screens, etc. would be measured in meters. A **meter** is a little more than 3 feet.

Kilometers (km) are used to measure long distances. If you are looking to figure out the length of a road, the distance between two locations, etc, you would use kilometers.

Here is a chart to help with the conversions.

1 km = 1000 m	1 m = 100 cm	1 cm = 10 mm

Notice that the conversions are all base ten. This means that all metric units can be evenly multiplied or divided by 10.

Now that you know the conversions, you can change one unit to another unit.

5 km = ____ m

Just like customary units, you can convert larger units to smaller units by multiplying. It is 1000 meters in one kilometer.

5 km = ____ m

5×1000=5000 m

The answer is 5000 m.

Here's an example of converting smaller units to larger units.

600 cm = ______ m

You can convert smaller units to larger units by dividing. There are 100 cm in one meter.

600 ÷ 100 = 6

The answer is 6 m.

Practice Activity

Complete the following metric equivalents.

1. 6 km = ______ m
2. 5 m = ______ cm
3. 100 cm = ______ m
4. 400 cm = ______ m
5. 9 km = ______ m
6. 2000 m = ______ km
7. 20 mm = ______ cm
8. 8 cm = ______ mm
9. 900 cm = ______ m
10. 12 m = ______ cm
11. 10 cm = ______ mm
12. 100 cm = _____ mm
13. 6700 m = ______ km
14. 8200 m = ______ km
15. 12,500 m = ______ km

Practice (Answers)

1. 6000 m 2. 500 cm 3. 1 m 4. 4 m 5. 9000 m 6. 2km 7. 2 cm 8. 80 mm 9. 9 m
10. 12000 cm 11. 100 mm 12. 1,000 m 13. 6.7 km 14. 8.2 km 15. 12.5 km

Temperature Scales

You may notice that meteorologists measure heat and cold differently outside of the United States. For example, a TV weatherman in San Diego may forecast a high of 89°, but a similar forecaster in Tijuana, Mexico—which is only 20 miles south—may look at the same weather pattern and say that the day's high temperature is going to be 32°.

The difference is that the two countries use different temperature scales. In the United States, temperatures are usually measured using the **Fahrenheit** scale, while most countries that use the metric system use the **Celsius** scale to record temperatures.

Measuring Temperature on Two Scales

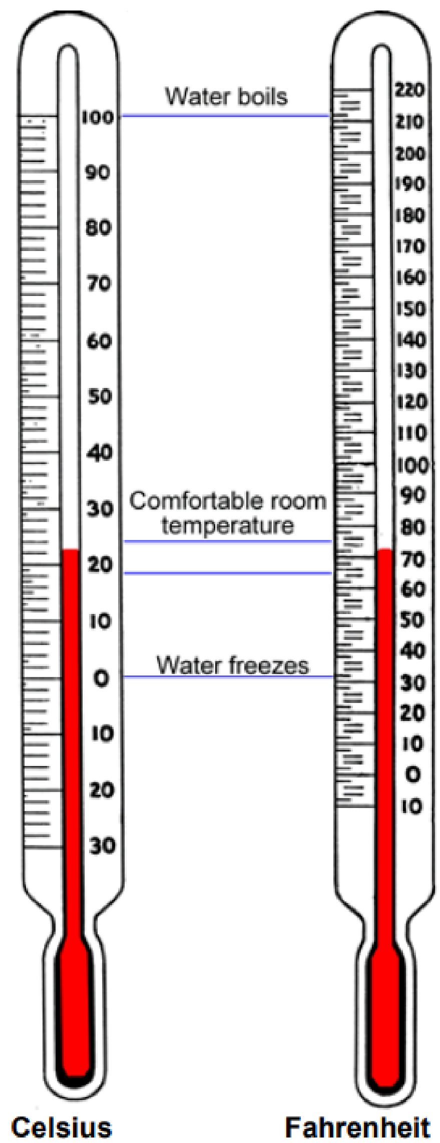
Fahrenheit and Celsius are two different scales for measuring temperature.

A thermometer measuring a temperature of 22° Celsius is shown here.

On the Celsius scale, water freezes at 0° and boils at 100°.

If the United States were to adopt the Celsius scale, forecast temperatures would rarely go below -30° or above 45°. (A temperature of -18° may be forecast for a cold winter day in Michigan, while a temperature of 43° may be predicted for a hot summer day in Arizona.)

Most office buildings maintain an indoor temperature between 18°C and 24°C to keep employees comfortable.



A thermometer measuring a temperature of 72° Fahrenheit is shown here.

On the Fahrenheit scale, water freezes at 32° and boils at 212°. In the United States, forecast temperatures measured in Fahrenheit rarely go below -20° or above 120°. (A temperature of 0° may be forecast for a cold winter day in Michigan, while a temperature of 110° may be predicted for a hot summer day in Arizona.)

Most office buildings maintain an indoor temperature between 65°F and 75°F to keep employees comfortable.

Converting Between the Scales

By looking at the two thermometers shown, you can make some general comparisons between the scales.

Sometimes, it is necessary to convert a Celsius measurement to its exact Fahrenheit measurement or vice versa. Converting temperature between the systems is a straightforward process as long as you use the formulas provided below.

Temperature Conversion Formulas:

To convert a Fahrenheit measurement to a Celsius measurement, use this formula.

$$C = \frac{5}{9}(F - 32)$$

To convert a Celsius measurement to a Fahrenheit measurement, use this formula.

$$F = \frac{9}{5}C + 32$$

The example below illustrates the conversion of Celsius temperature to Fahrenheit temperature, using the boiling point of water, which is 100° C.

The two previous problems used the conversion formulas to verify some temperature conversions that were discussed earlier—the boiling and freezing points of water. The next example shows how these formulas can be used to solve a real-world problem using different temperature scales.

Practice:

1. A cook puts a thermometer into a pot of water to see how hot it is. The thermometer reads 132° , but the water is not boiling yet. Which temperature scale is the thermometer measuring?

2. Tatiana is researching vacation destinations, and she sees that the average summer temperature in Barcelona, Spain is around 26°C .What is the average temperature in degrees Fahrenheit?

Answers:

1. Fahrenheit
   Water boils at 212° on the Fahrenheit scale, so a measurement of 132° on a Fahrenheit scale is legitimate for hot (but non-boiling) water.

2. Tatiana can find the Fahrenheit equivalent by solving the equation $f = \frac{9}{5}(26) + 32$. The result is 78.8°F , which rounds to 79°F .

Attribution:

Chapter 8: Geometry

Solve Geometry Applications

- Step 1. **Read** the problem and make sure you understand all the words and ideas. Draw a figure and label it with the information given.
- Step 2. **Identify** what you are looking for.
- Step 3. **Name** what you are looking for and choose a variable to represent it.
- Step 4. **Translate** into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.
- Step 5. **Solve** the equation using good algebra techniques.
- Step 6. **Check** the answer to the problem and make sure it makes sense.
- Step 7. **Answer** the question with a complete sentence.

1.1: Lines

Geometry (from Greek words meaning earth-measure) originally developed as a means of surveying land areas, In its simplest form, it is a study of figures that can be drawn on a perfectly smooth flat surface, or **plane**. It is this **plane geometry** which we will study in this bock and which serves as a foundation for trigonometry, solid and analytic geometry, and calculus.

The simplest figures that can be drawn on a plane are the point and the line. By a line we will always mean a **straight line.** Through two distinct points one and only one (straight) line can be drawn. The line through points A and B will be denoted by $\overrightarrow{AB}$ (Figure 1.1.1). The arrows indicate that the line extends indefinitely in each direction, The **line segment** from A to B consist of A, B and that part of $\overleftrightarrow{AB}$ between A and B, It is denoted by AB (some textbooks use the notation $\overline{AB}$ for line segment). The ray $\overrightarrow{AB}$ is the part of $\overleftrightarrow{AB}$ which begins at A and extends indefinitely in the direction of B.

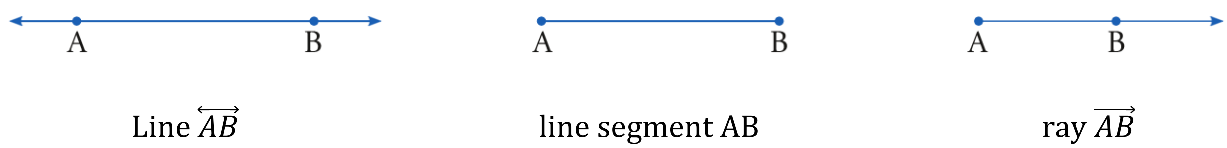


Figure 1.1.1: Line $\overleftrightarrow{AB}$, line segment $\overline{AB}$, and ray $\overrightarrow{AB}$. (CC BY-NC 4.0; Ümit Kaya via LibreTexts)

We assume everyone is familiar with the notion of length of a line segment and how it can be measured in inches, or feet, or meters, etc, The distance between two points A and B is the same as the length of AB.

Two line segments are equal if they have the same length, e.g., in Figure 1.1.2, AB=CD,

Figure 1.1.2: AB=CD. (CC BY-NC 4.0; Ümit Kaya via LibreTexts)

We often indicate two line segments are equal by marking them in the same way, e.g., in Figure 1.1.3, AB=CD and EF=GH.

Figure 1.1.3: AB=CD and EF=GH. (CC BY-NC 4.0; Ümit Kaya via LibreTexts)

Example 1:

Find x if AB=CD:

Figure 1.1.E1: (CC BY-NC 4.0; Ümit Kaya via LibreTexts)

Solution

$$\begin{aligned} AB &= CD \\ 3x-6 &= x \\ 3x-x &= 6 \\ 2x &= 6 \\ x &= 3 \end{aligned}$$

Check:

AB	= CD
3x - 6	x
3(3) - 6	3
9 - 6	
3	

Answer: x=3.

Notice that in Example 1 we have not indicated the unit of measurement. Strictly speaking, we should specify that AB=3x−6 inches (or feet or meters) and that BC=x inches. However, since the answer would still be x=3 we will usually omit this information to save space.

We say that B is the midpoint of AC if B is A point on AC and AB=BC (Figure 1.1.4).

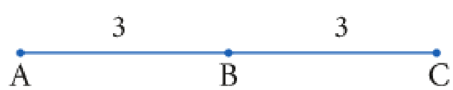


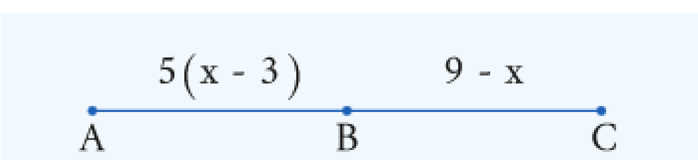
Figure 1.1.4: B is the midpoint of AC. (CC BY-NC 4.0; Ümit Kaya via LibreTexts)

__**Example 2:**__

Find x and AC if B is the midpoint of AC and AB=5(x−3) and BC=9−x,

Solution:

We first draw a picture to help visualize the given information:



Since 3 is a midpoint,

$$\begin{aligned} AB &= BC \\ 5(x-3) &= 9-x \\ 5x-15 &= 9-x \\ 5x+x &= 9+15 \\ 6x &= 24 \\ x &= 4 \end{aligned}$$

Check:

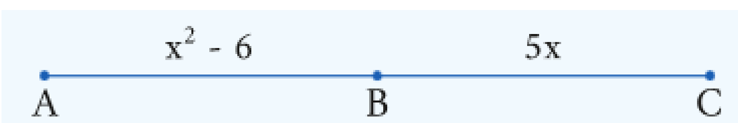
AB	= BC
$5(x-3)$	$9-x$
$5(4-3)$	$9-4$
$5(1)$	5
5	

We obtain AC=AB+BC=5+5=10.

Answer: x=4, AC=10.

Example 3:

Find AB if B is the midpoint of AC :



Solution

$$\begin{aligned} AB &= BC \\ x^2-6 &= 5x \\ x^2-5x-6 &= 0 \\ (x-6)(x+1) &= 0 \end{aligned}$$

$$\begin{aligned} x-6 &= 0 \qquad & x+1 &= 0 \\ x &= 6 \qquad & x &= -1 \end{aligned}$$

If x=6 then AB=$x^2-6=6^2-6=36-6=30$.

If $x=-1$ then $AB=(-1)^2-6=1-6=-5$.

We reject the answer $x=-1$ and $AB=-5$ because the length of a line segment is always positive. Therefore $x=6$ and $AB=30$.

Check:

AB	= BC
$x^2 - 6$	$5x$
$6^2 - 6$	$5(6)$
$36 - 6$	30
30	

Answer: $AB=30$.

Three points are collinear if they lie on the same line.

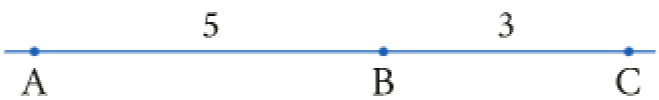


Figure 1.1.5 : A , B , and C are collinear $AB=5$, $BC=3$, and $AC=8$ (CC BY-NC 4.0; Ümit Kaya via LibreTexts)

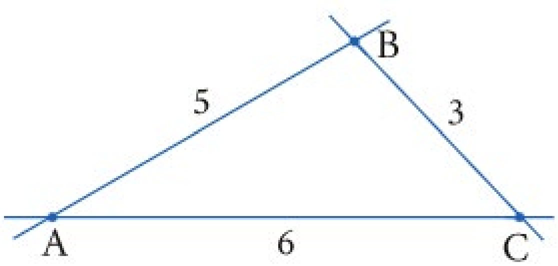
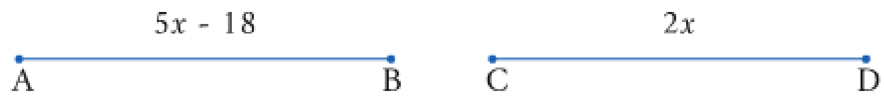


Figure 1.1.6 : A , B , and C are not collinear. $AB=5$, $BC=3$, $AC=6$. (CC BY-NC 4.0; Ümit Kaya via LibreTexts)

A , B , and C are collinear if and only if $AB+BC=AC$.

Practice Exercises

1. Find x if AB=CD.

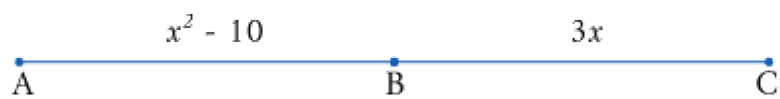


2. Find x if AB=CD.

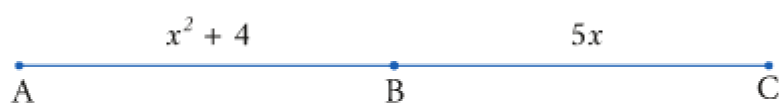
3. Find x and AC if B is the midpoint of AC and AB=3(x−5) and BC=x+3.

4. Find x and AC if B is the midpoint of AC and AB=2x+9 and BC=5(x−9),

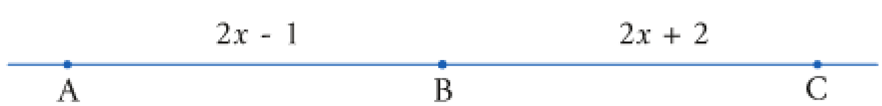
5. Find AB if B is the midpoint of AC:



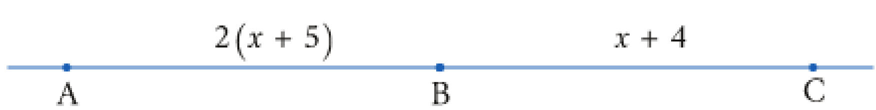
6. Find AB if B is the midpoint of AC:



7. If A, B, and C are collinear and AC=13 find x:



8. If A, B, and C are collinear and AC=26 find x:



Answers to Odd Nubered Problems:

1. 6

3. x=9 , AC=24 .

5. 15

7. 3

1.2: Angles

An **angle** is the figure formed by two rays with a common end point, The two rays are called the sides of the angle and the common end point is called the *vertex* of the angle, The symbol for angle is ∠

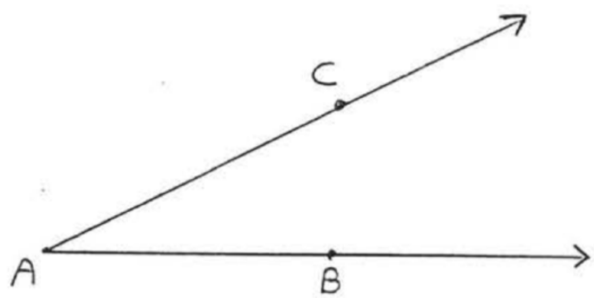


Figure 1.2.1 : Angle BAC has vertex A and sides $\overrightarrow{AB}$ and $\overrightarrow{AC}$

The angle in Figure 1.2.1 has vertex A and sides AB and AC, It is denoted by ∠BAC or ∠CAB or simply ∠A. When three letters are used, the middle letter is always the vertex, In Figure 1.2.2 we would not use the notation ∠A as an abbreviation for ∠BAC because it could also mean ∠CAD or ∠BAD, We could however use the simpler name ∠x for ∠BAC if "x" is marked in as shown,

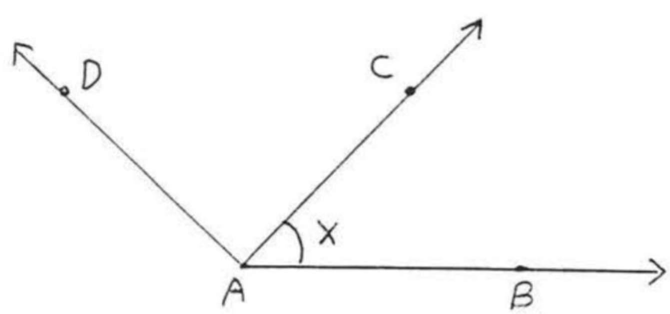


Figure 1.2.2: ∠BAC may also be denoted by ∠x.

Angles can be measured with an instrument called a *protractor.* The unit of measurement is called a *degree* and the symbol for degree is °.

To measure an angle, place the center of the protractor (often marked with a cross or a small circle) on the vertex of the angle, Position the protractor so that one side of the angle cuts across 0, at the beginning of the scale, and so that the other side cuts across a point further up on the scale, We use either the upper scale or the lower scale, whichever is more convenient, For example, in Figure 1.2.3, one side of ∠BAC crosses 0 on the lower scale and the other side crosses 50 on the lower scale. The measure of ∠BAC is therefore 50° and we write ∠BAC = 50°.

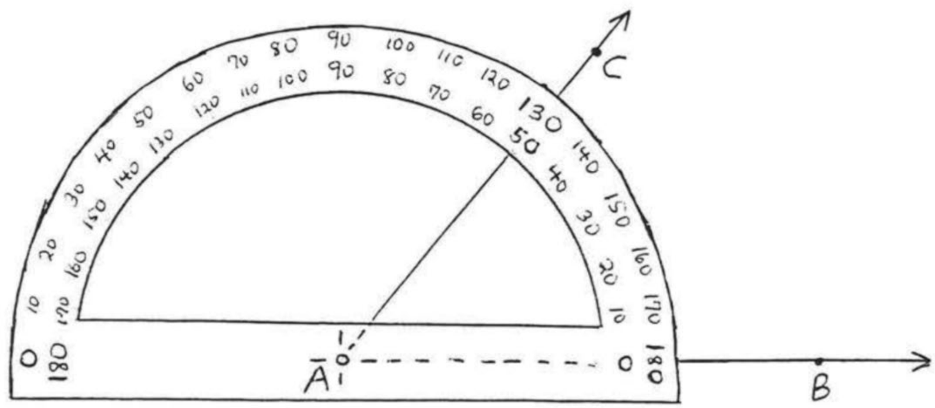


Figure 1.2.3: The protractor shows ∠BAC=50∘

In Figure 1.2.4 , side $\vec{A}$D of ∠DAC crosses 0 on the upper scale. Therefore we look on the upper scale for the point at which $\overrightarrow{AC}$ crosses and conclude that ∠DAC=130°.

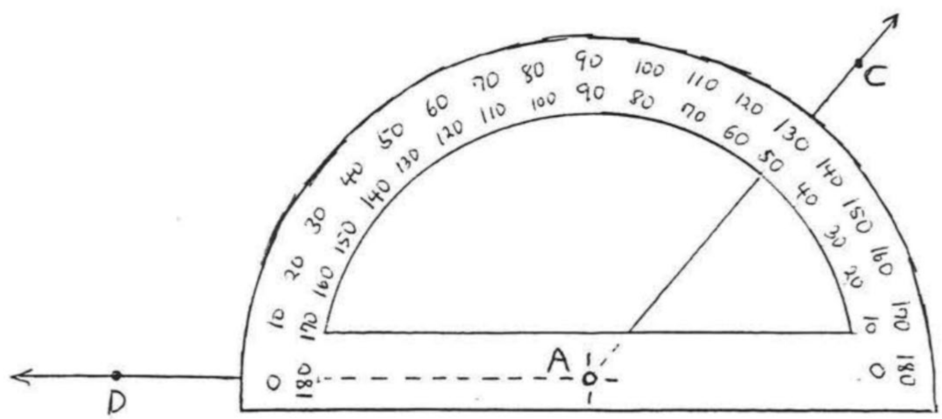


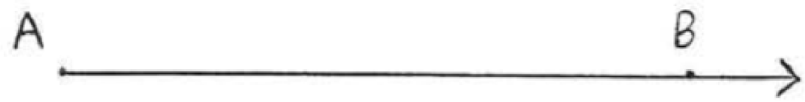
Figure 1.2.4: ∠DAC=130°.

<u>Example 1:</u>

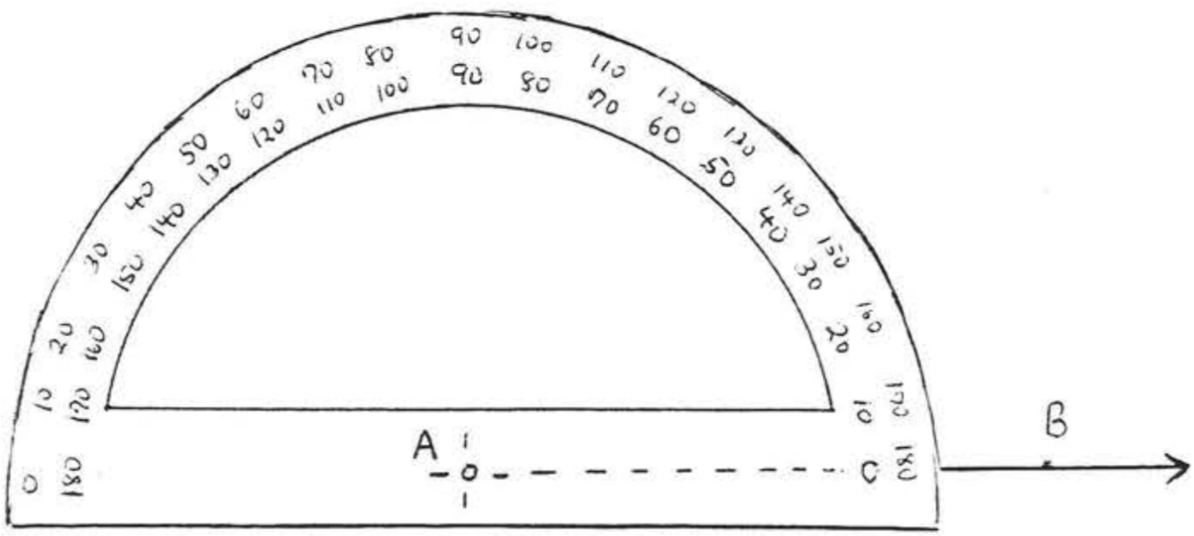
Draw an angle of 40° and label it ∠BAC

Solution

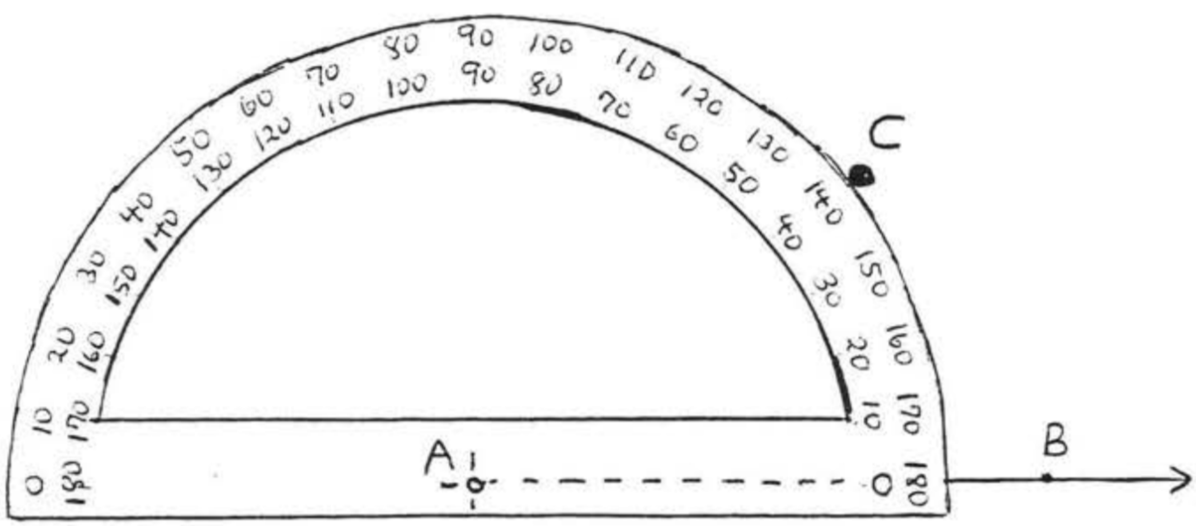
Draw ray $\overrightarrow{AB}$ using a straight edge:



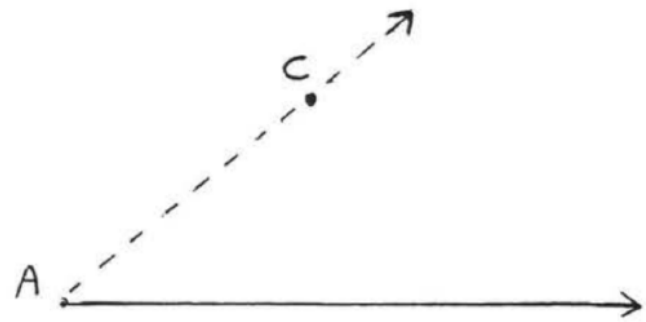
Place the protractor so that its center coincides with A and $\overrightarrow{AB}$ crosses the scale at 0:



Mark the place on the protractor corresponding to 40°Label this point C:



Connect A with C:



Two angles are said to be equal if they have the same measure in degrees. We often indicate two angles are equal by marking them in the same way. In Figure 1.2.5, ∠A=∠B.

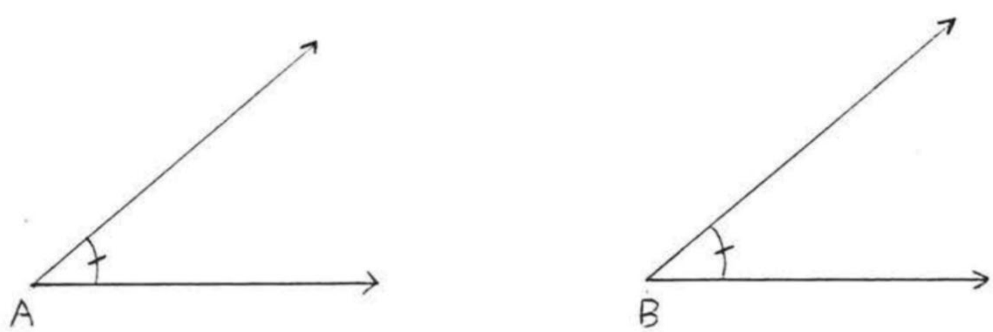


Figure 1.2.5: Equal angles.

An angle bisector is a ray which divides an angle into two equal angles. In Figure 1.2.6 , $\overrightarrow{AC}$ is an angle bisector of ∠BAD . We also say $\overrightarrow{AC}$ bisects ∠BAD .

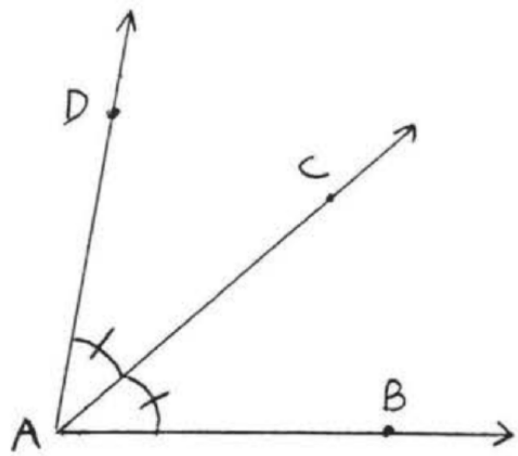
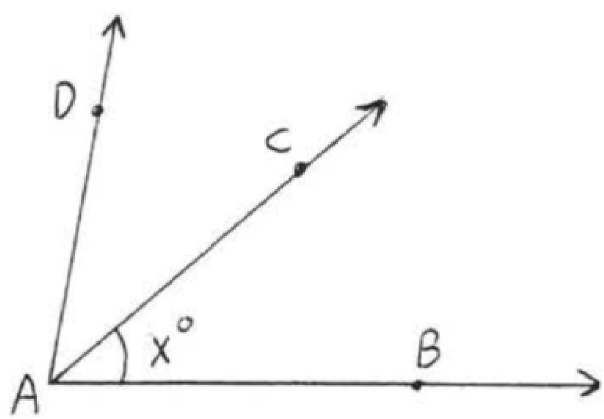


Figure 1.2.6: $\dot{AC}$ bisects $\angle BAD$.

Figure 1.2.6 : $\overrightarrow{AC}$ bisects ∠BAD.

<u>Example 2:</u>

Find x if $\overrightarrow{AC}$ bisects ∠BAD and ∠BAD=80°:



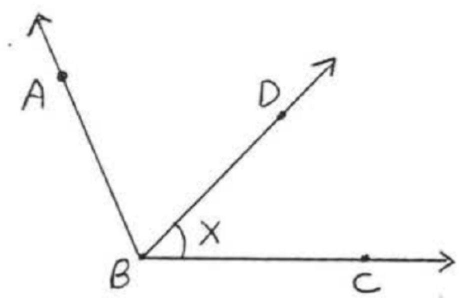
Solution

$$x^\circ = \frac{1}{2}\angle BAD = \frac{1}{2}(80^\circ) = 40^\circ$$

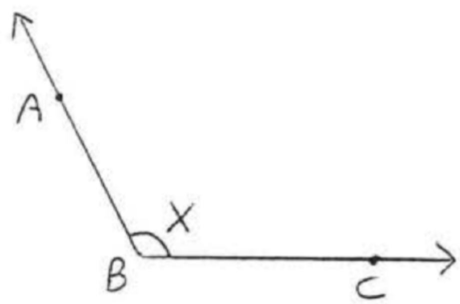
Answer: x=40.

Practice Exercises

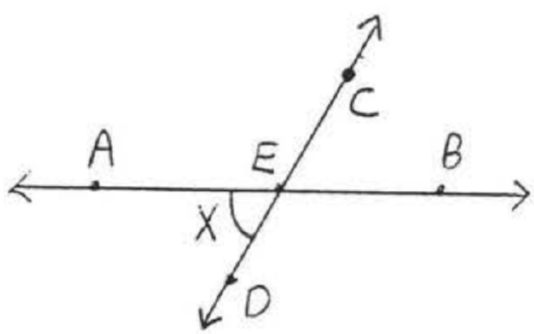
1 - 6. For each figure, give another name for ∠x:



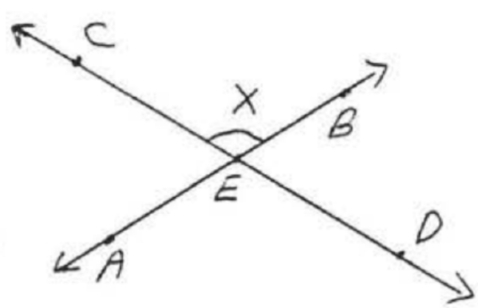
1.



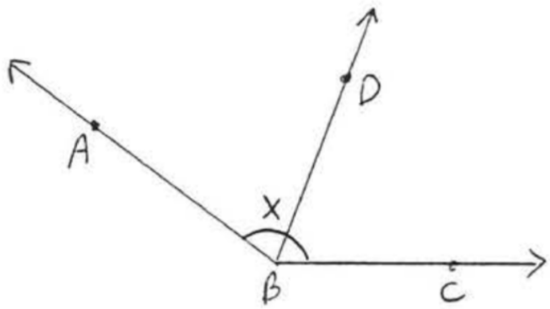
2.



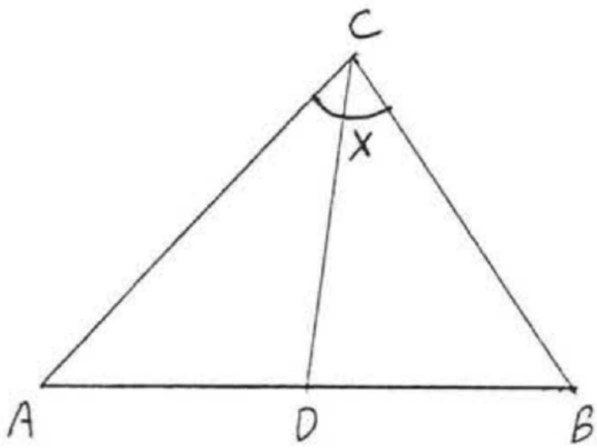
3.



4.

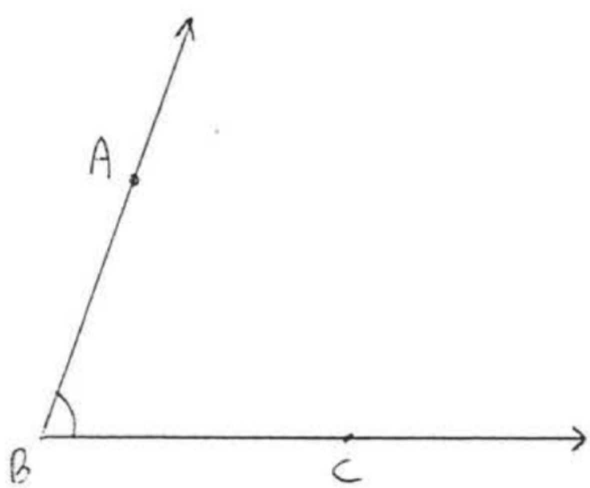


5.

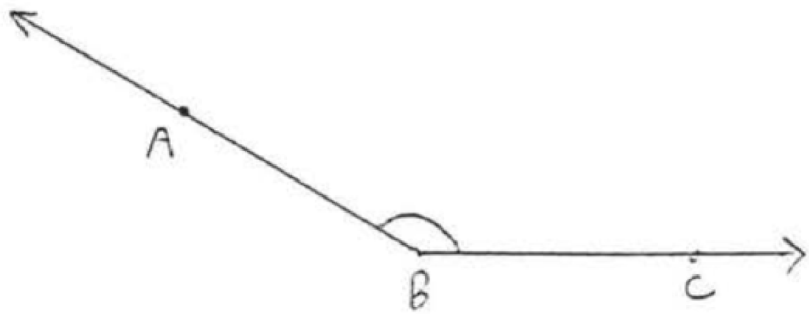


6.

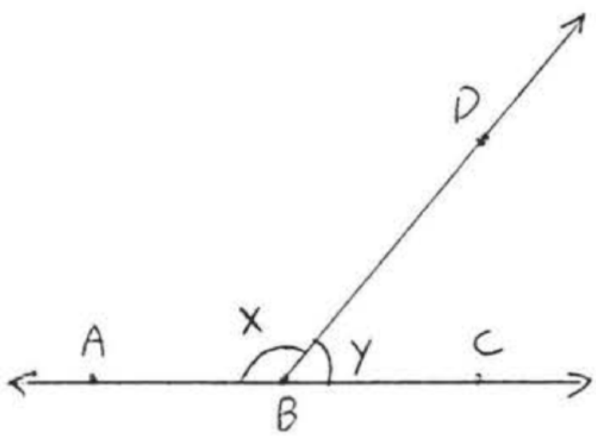
7 - 16, Measure each of the indicated angles:



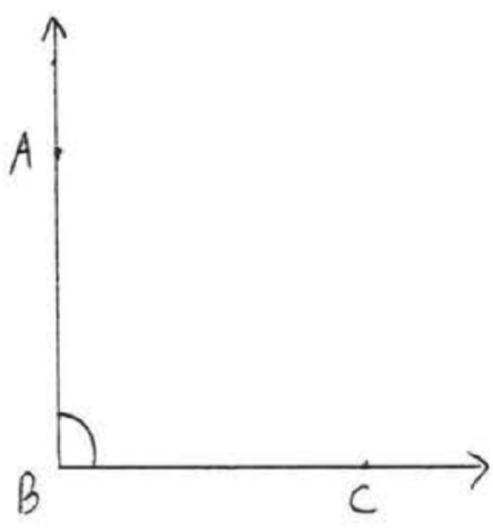
7.



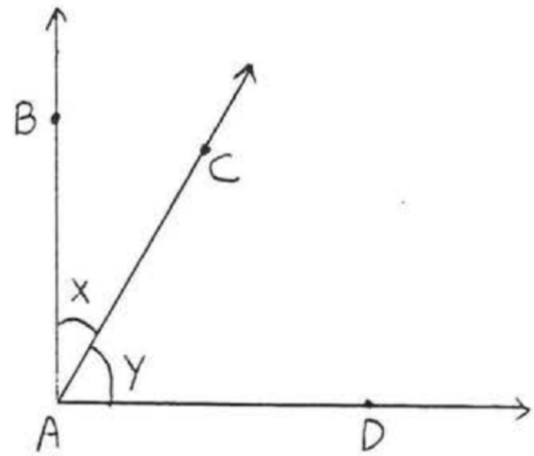
8.

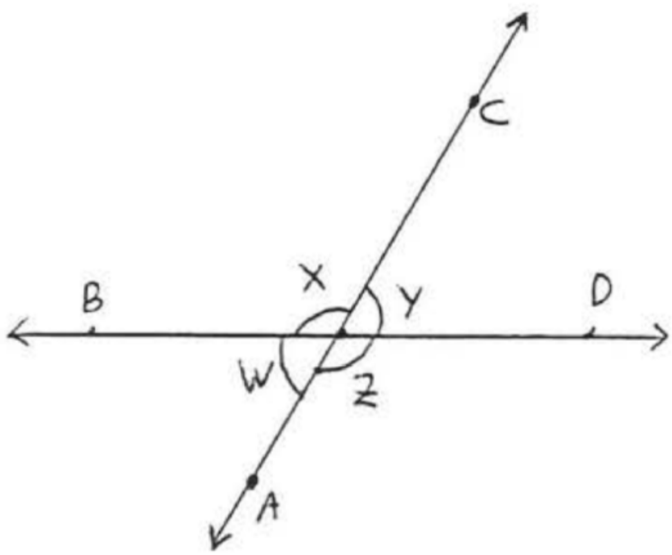


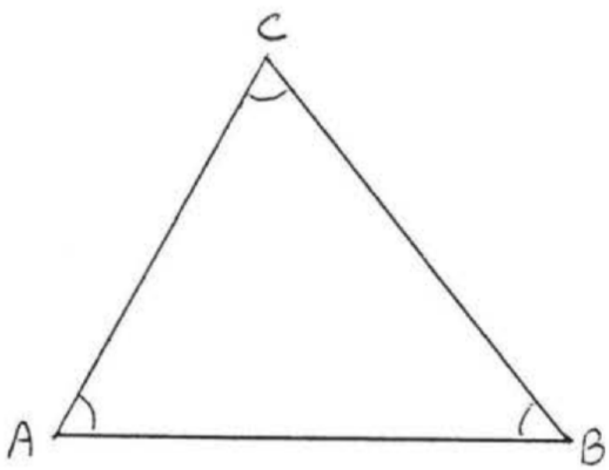
9.

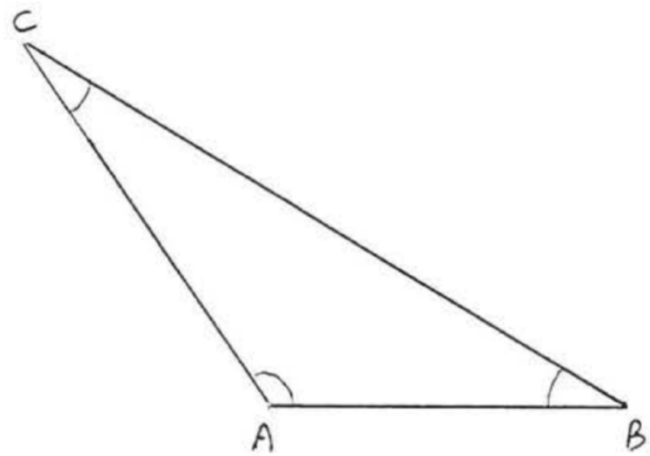


10.

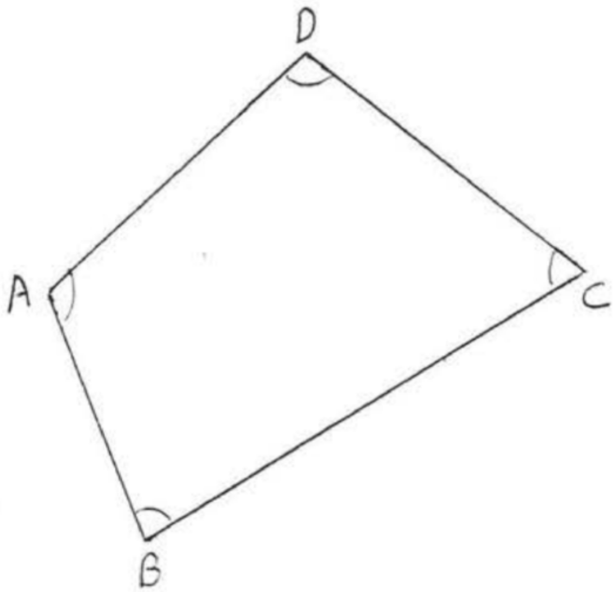
11. 

12. 

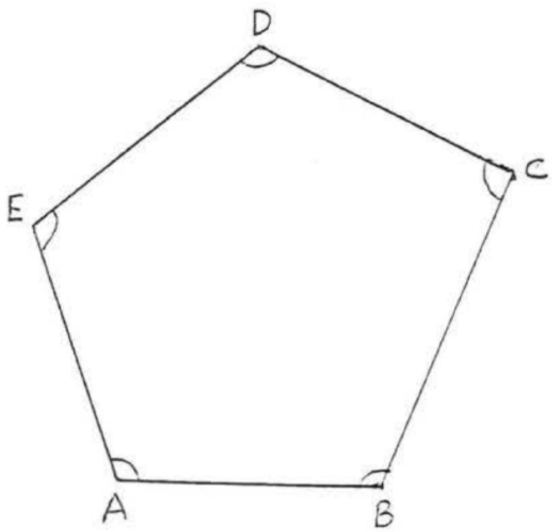
13. 



14.



15.

16. 

17 - 24. Draw and label each angle:

17. ∠BAC=30°

18. ∠BAC=40°

19. ∠ABC=45°

20. ∠EFG=60°

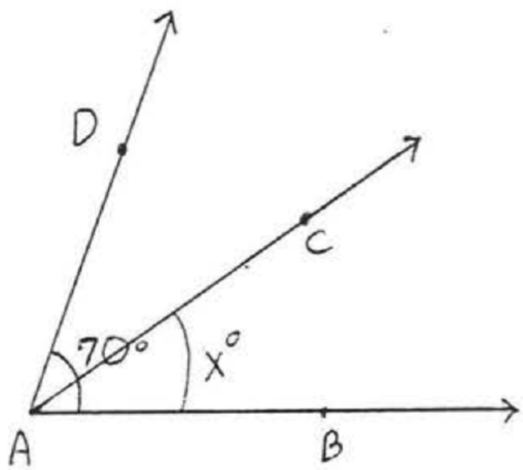
21. ∠RST=72°

22. ∠XYZ=90°

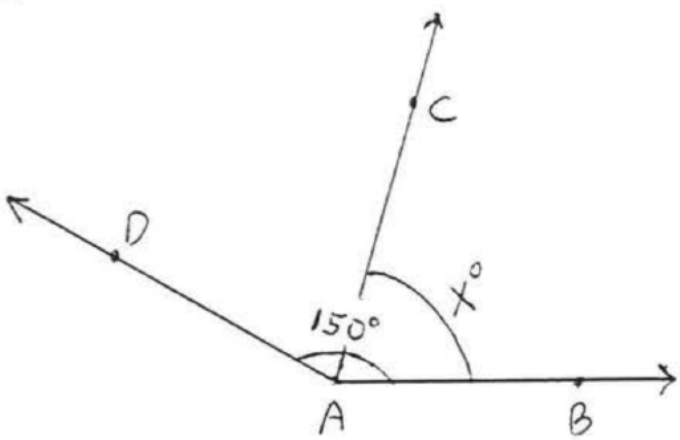
23. ∠PQR=135°

24. ∠JKL=164°

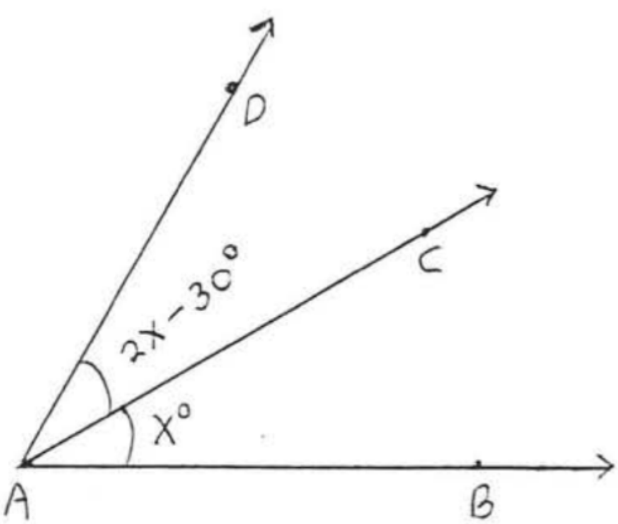
25-28. Find x if $\overrightarrow{AC}$ bisects ∠BAD:



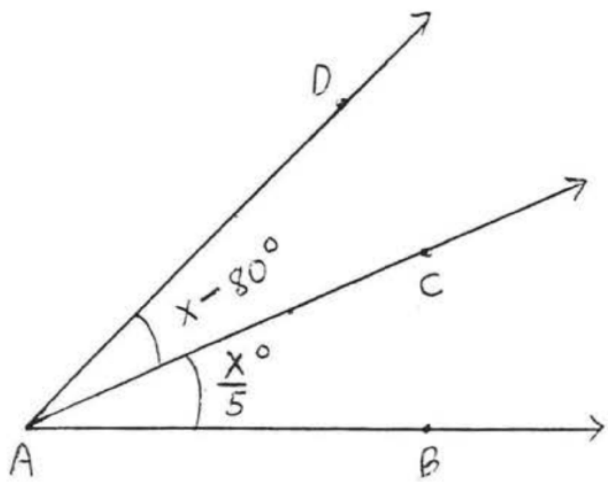
25.



26.



27.



28.

Answers to Odd Numbered Problems:

1. ∠CBD or ∠DBC

3. ∠AED or ∠DEA

5. ∠ABC or ∠CBA

7. 70°

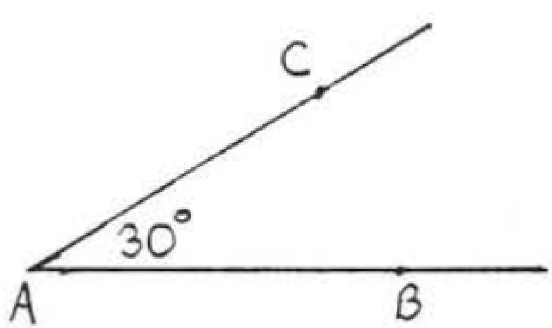
9. x=130° , y=50°

11. x=30° , y=60°

13. ∠A=60°, ∠B=50°, ∠C=70°

15. ∠A=110°, ∠B=80°, ∠C=70°, ∠D=100°

17.



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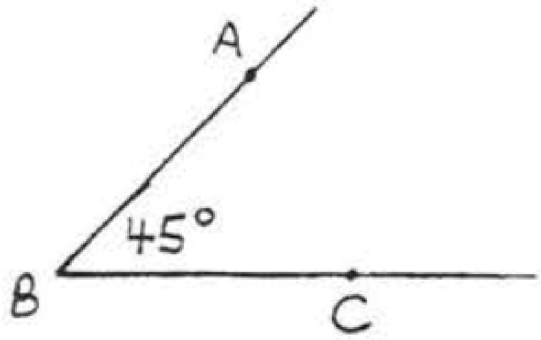

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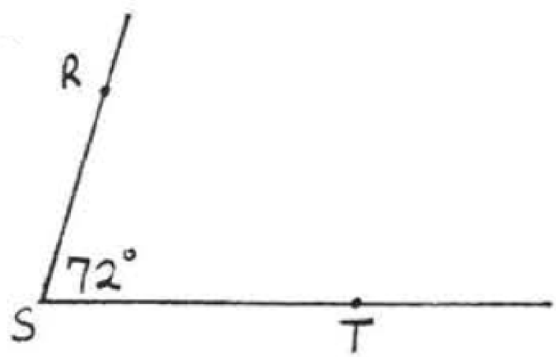
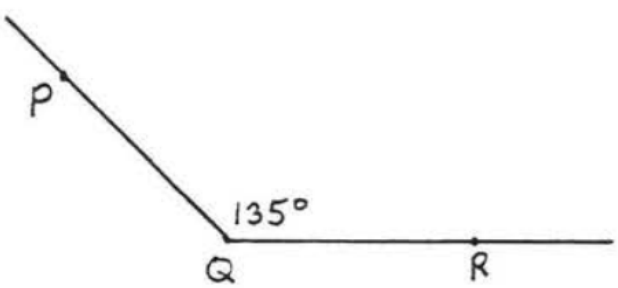


23.



25. 35°

27. 30°

1.3: Angles Classifications

Angles are classified according to their measures as follows:

- An **acute angle** is an angle whose measure is between 0° and 90°.
- A **right angle** is an angle whose measure is 90°. We often use a little square to indicate a right angle.
- An **obtuse angle** is an angle whose measure is between 90° and 180°.
- A **straight angle** is an angle whose measure is 180°. A straight angle is just a straight line with one of its points designated as the vertex.
- A **reflex angle** is an angle whose measure is greater than 180°.

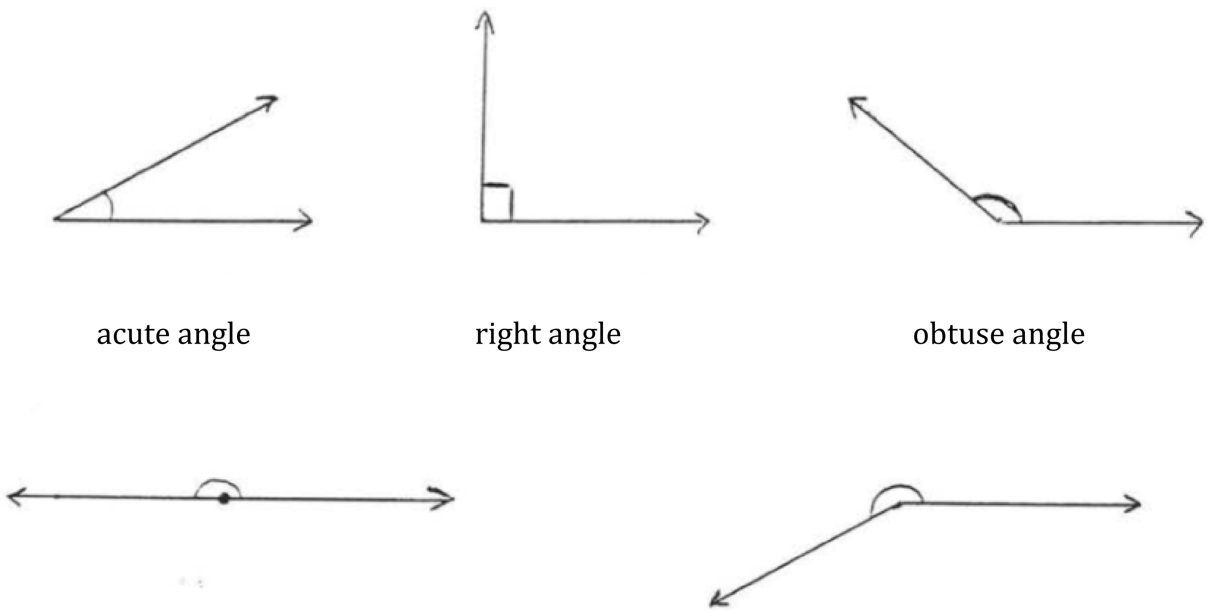


Figure 1.3.1: Angles classified according to their measures.

Notice that an angle can be measured in two ways. In Figure 1.3.2, ∠ABC is a reflex of 240° or an obtuse angle of 120° depending on how it is measured. Unless otherwise indicated, we will always assume the angle has measure less than 180°.

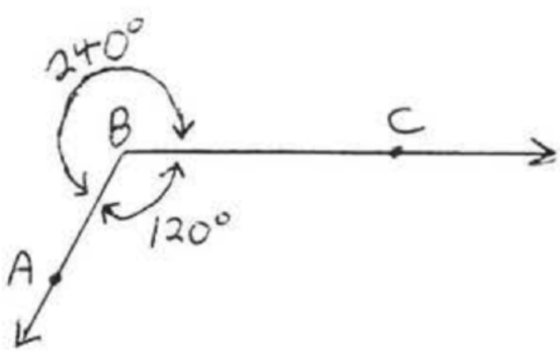


Figure 1.3.2: ∠ABC can be measured in two different ways.

The lines are perpendicular if they meet to form right angles. In Figure 1.3.3 , $\overleftrightarrow{AB}$ is perpendicular to $\overleftrightarrow{CD}$. The symbol for perpendicular is $\perp$ and we write $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$.

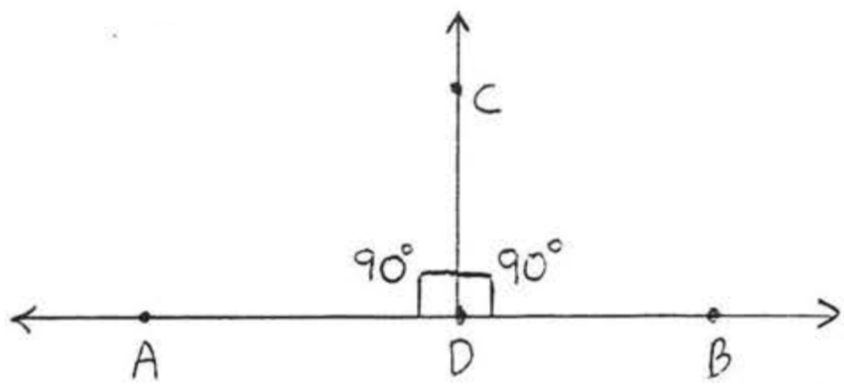


Figure 1.3.3 : $\overleftrightarrow{AB}$ is perpendicular to $\overleftrightarrow{CD}$.

The perpendicular bisector of a line segment is a line perpendicular to the line segment at its midpoint, In Figure 1.3.4 , $\overleftrightarrow{CD}$ is a perpendicular bisector of AB .

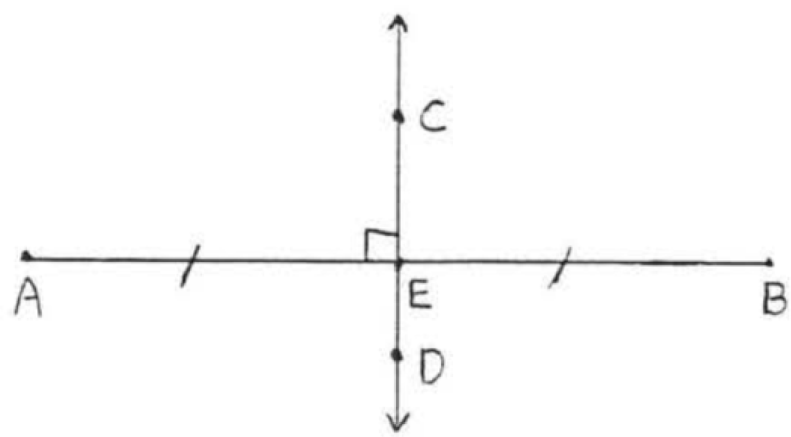


Figure 1.3.4 : $\overleftrightarrow{CD}$ is a perpendicular bisector of AB .

Two angles are called complementary if the sum of their measures is 90°. Each angle is called the complement of the other. For example, angles of 60° and 30° are complementary.

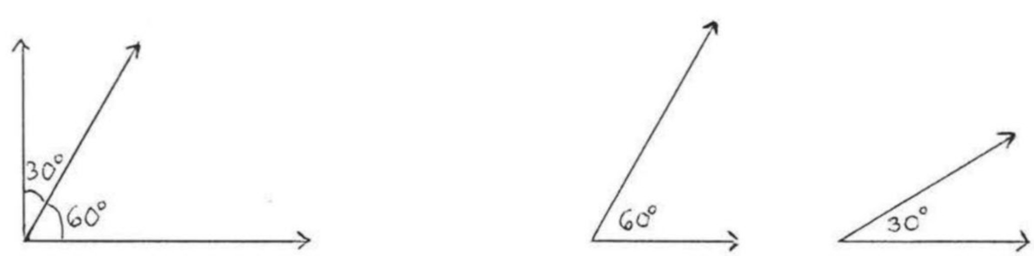


Figure 1.3.5: Complementary angles.

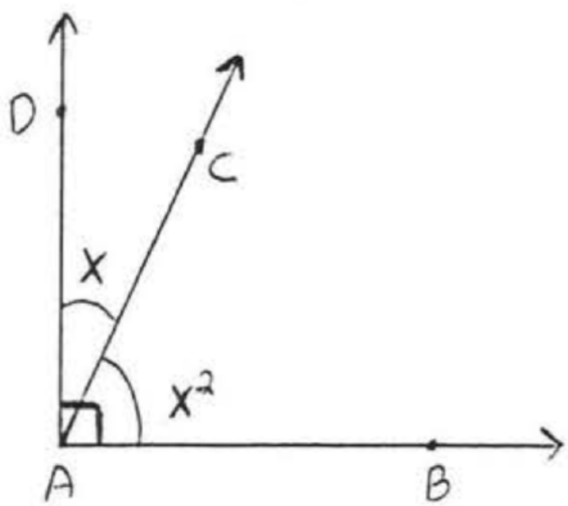
Example 1: Find the complement of a 40° angle.

Solution

$$90° - 40° = 50°$$

Answer: 50°.

Example 2: Find x and the complementary angles:



Solution:

Since ∠BAD =90°

$$\begin{aligned} x^2 + x &\quad 90^\circ \\ x^2 + x - 90 &\quad 0 \\ (x-9)(x+10) &\quad 0 \end{aligned}$$

$$\begin{aligned} x - 9 &= 0 \\ x &= 9 \end{aligned} \qquad \begin{aligned} x + 10 &= 0 \\ x &= -10 \end{aligned}$$

$\angle CAD = x = 90^\circ$. $\angle CAD = x = -10^\circ$.

$\angle BAC = x^2 = 9^2 = 81^\circ$.

$\angle BAC = \angle CAD = 81^\circ + 9^\circ = 90^\circ$.

We reject the answer x=−10 because the measure of an angle is always positive. (In trigonometry, when directed angles are introduced, angles can have negative measure. In this book, however, all angles will be thought of as having positive measure,)

Check, x=9:

$$x^2 + x = 90°$$

$$9^2 + 9$$

$$81 + 9$$

$$90°$$

Answer: x=9, ∠CAD=9°, ∠BAC=81°.

Two angles are called supplementary if the sum of their measures is 180°. Each angle is called the supplement of the other. For example, angle of 150° and 30° are supplementary.

Figure 1.3.6: Supplementary angles.

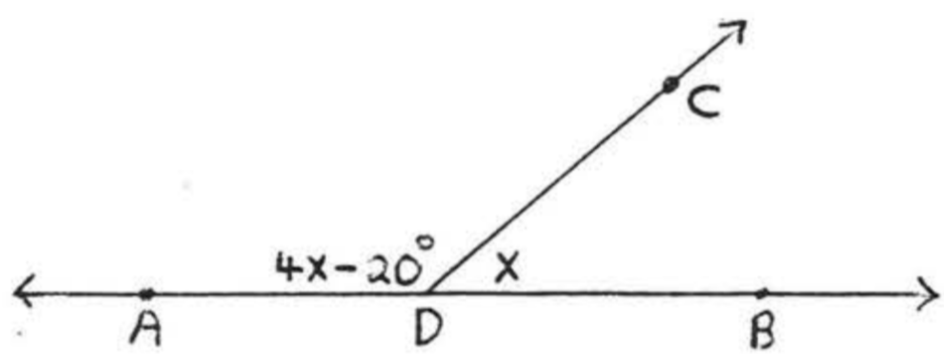
Example 3: Find the supplement of an angle of 40°.

Solution

180°−40°=140°.

Answer: 140°.

Example 4: Find x and the supplementary angles:



Solution:

Since ∠ADB= 180°

$$4x - 20 + x = 180°$$

$$5x = 180 + 20$$

$$5x = 200$$

$$x = 40$$

∠ADC= $4x - 20 = 4(40) - 20 = 160 - 20 = 140°$

$\angle$ BDC $= x = 40°$,

$\angle$ADC + $\angle$BDC=$140° + 40° = 180°$

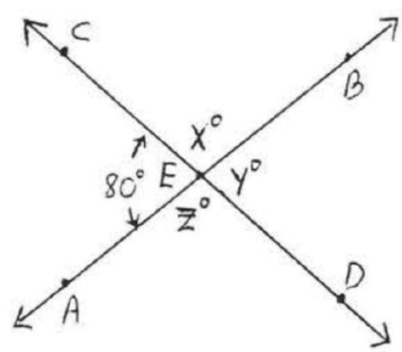
Check:

$$4x - 20 + x = 180°$$
$$4(40) - 20 + 40$$
$$160 - 20 + 40$$
$$140 + 40$$
$$180°$$

Answer

x=40, $\angle$ADC=140°, $\angle$BDC=40

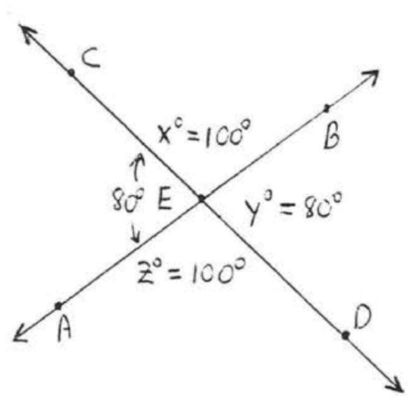
<u>Example 5:</u> Find x, y, z



Solution:

$x° = 180° - 80° = 100°$because $x°$ and 80° are the measures of supplementary angles.

$$y° = 180° - x° = 180° - 100° = 80°$$
$$z° = 180° - 80° = 100°$$



Answer: x=100, y=80, z=100.

When two lines intersect, they form two pairs of angles that are opposite to each other called vertical angles, In Figure 1.3.7, $\angle x$ and $\angle x'$ are one pair of vertical angles. $\angle y$ and $\angle y'$ are the other pair of vertical angles. As suggested by Example 1.3.5, $\angle x=\angle x'$ and $\angle y=\angle y'$. To see this in general, we can reason as follows: $\angle x$ is the supplement of $\angle y$ so $\angle x=180^\circ-\angle y$. $\angle x'$ is also the supplement of $\angle y$ so $\angle x'=180-\angle y$. Therefore $\angle x=\angle x'$. Similarly, we can show $\angle y=\angle y'$. Therefore, vertical angles are always equal.

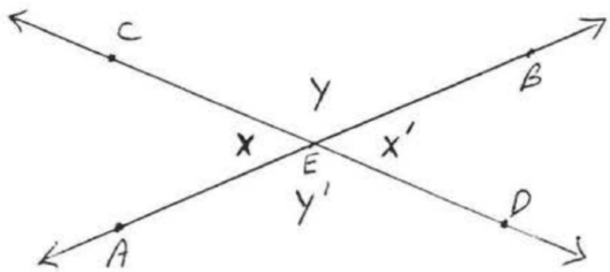
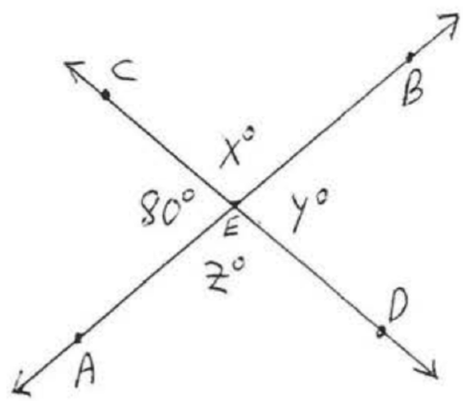


Figure 1.3.7: $\angle x$, and $\angle y$, are pairs of vertical angles.

We can now use "vertical angles are equal" in solving problems:

<u>Example 6:</u> Find x, y, and z:



Solution

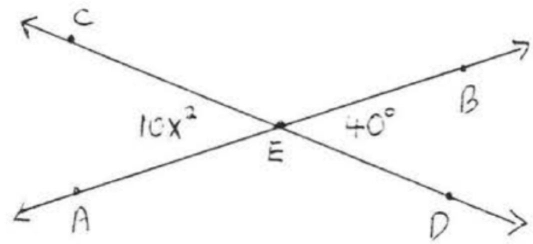
$\angle x = 180^\circ - 80^\circ = 100^\circ$ because $\angle x$ is the supplement of 80°.

$\angle y=80^\circ$because vertical angles are equal.

$\angle z=\angle x=100^\circ$ because vertical angles are equal.

Answer: x=100, y=80, z=100.

<u>Example 7:</u> Find x:



Solution:

Since vertical angles are equal, $10x^2 = 40°$

Method 1:

$$10x^2 = 40$$
$$10x^2 - 40 = 0$$
$$(10)(x^2 - 4) = 0$$
$$x^2 - 4 = 0$$
$$(x + 2)(x - 2) = 0$$

$x + 2 = 0 \quad x - 2 = 0$

$x = -2 \qquad x = 2$

Method 2:

$$10x^2 = 40$$
$$\frac{10x^2}{10} = \frac{40}{10}$$
$$x^2 = 4$$
$$x^2 = 4 \pm 2$$

If $x = 2$ then $\angle AEC = 10x^2 = 10(2)^2 = 10(4) = 40^o$.

If $x = -2$ then $\angle AEC = 10x^2 = 10(-2)^2 = 10(4) = 40^o$.

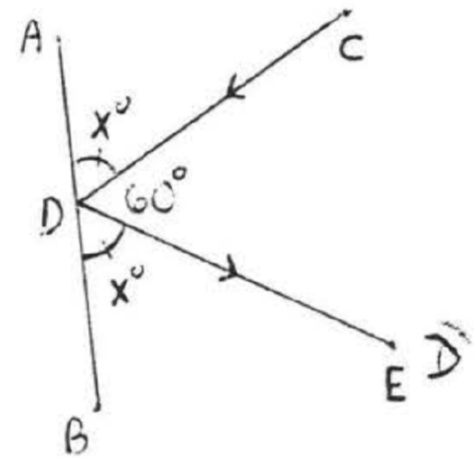
We accept the solution $x = -2$ even though x is negative because the value of the angle $10x^2$ is still positive.

Check:

$x = 2$:	$x = -2$:
$10x^2 = 40°$	$10x^2 = 40°$
$10(2)^2$	$10(-2)^2$
40	40

Answer: x=2 or x=−2.

Example 8: In the diagram, AB represents a mirror, CD represents a ray of light approaching the mirror from C, and E represents the eye of a person observing the ray as it is reflected from the mirror at D. According to a law of physics, ∠CDA, called the angle of incidence, equals ∠EDB, called the angle of reflection. If ∠CDE=60^o, how much is the angle of incidence?



Solution:

Let x^o=∠CDA=∠EDB.

$$x + x + 60 = 180$$
$$2x + 60 = 180$$
$$2x = 120$$
$$x = 60$$

Answer: 60^o

Practice Exercises

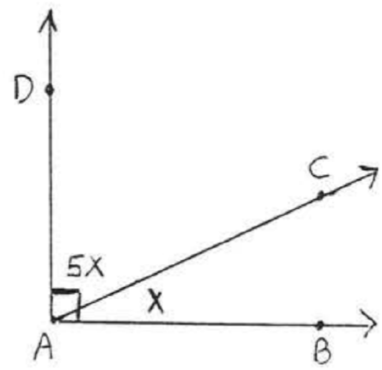
1. Find the complement of the angle:

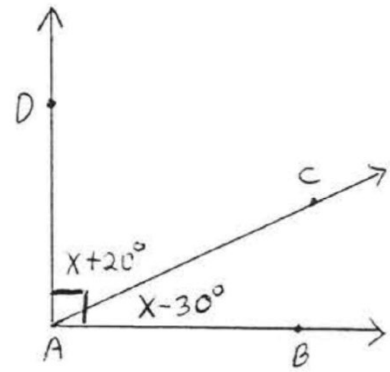
a. 37^o
b. 45^o
c. 53^o
d. 60^o

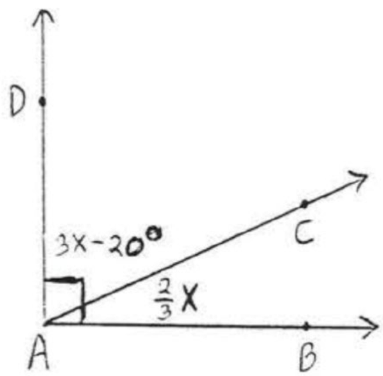
2. Find the complement of the angle:

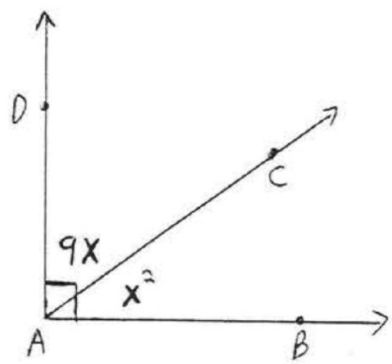
e. 30^o
f. 40^o
g. 50^o
h. 81^o

3 - 6. Find x and the complementary angles:

3. 

4. 

5. 

6. 

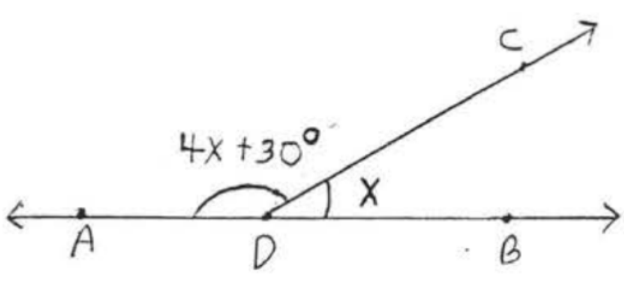
7. Find the supplement of an angle of

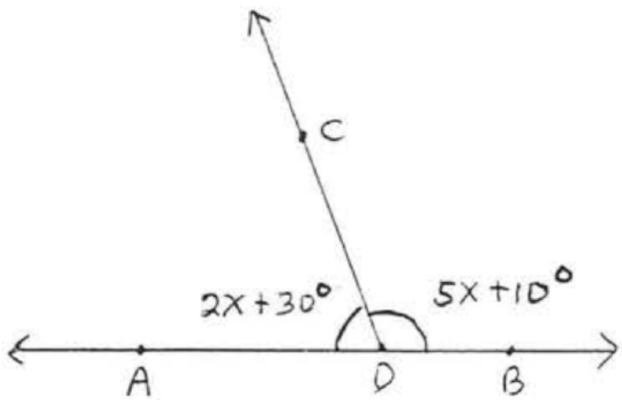
i. 30^o
j. 37^o
k. 90^o
l. 120^o

8. Find the supplement of an angle of

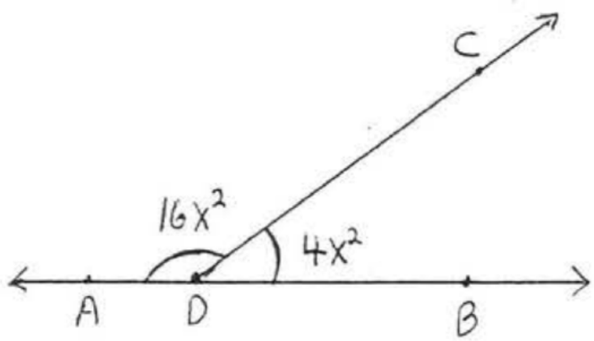
m. 45^o
n. 52^o
o. 85^o
p. 135^o

9 - 14. Find x and the supplementary angles:

9. 



10.



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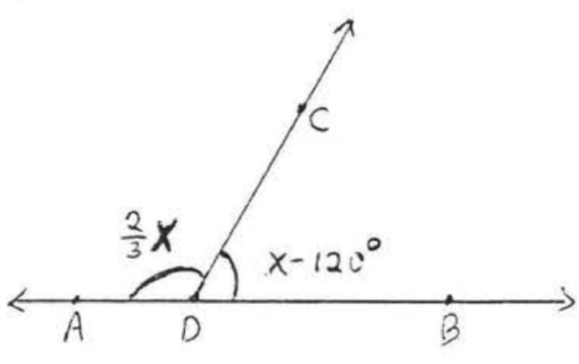

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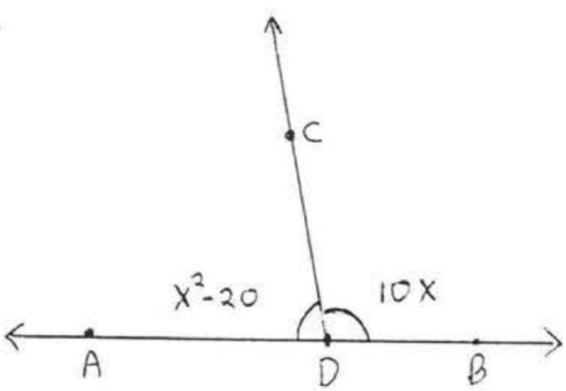

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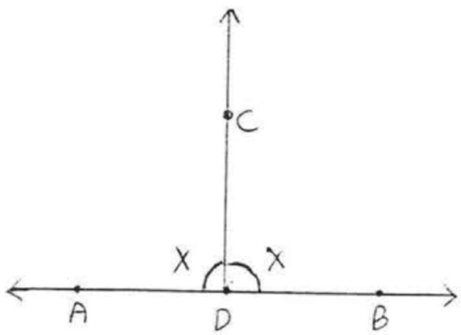
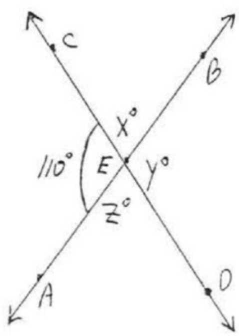


14.

15 - 22. Find x, y, and z:



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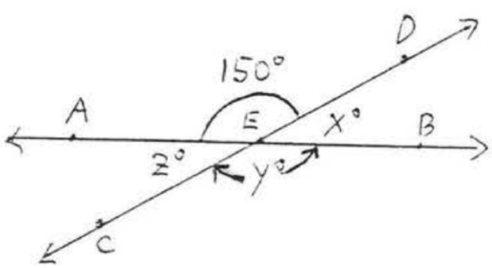

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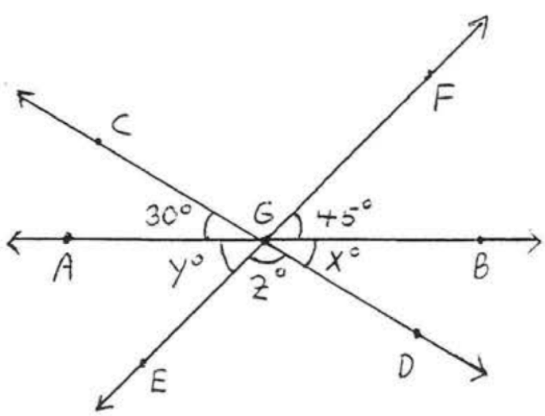

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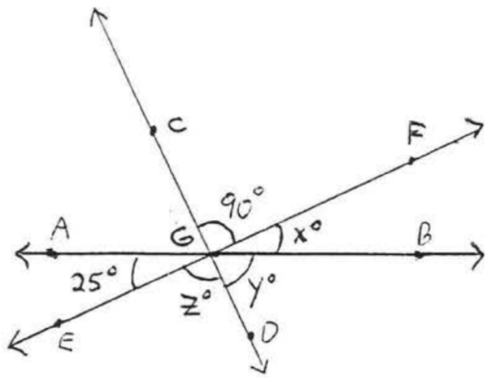

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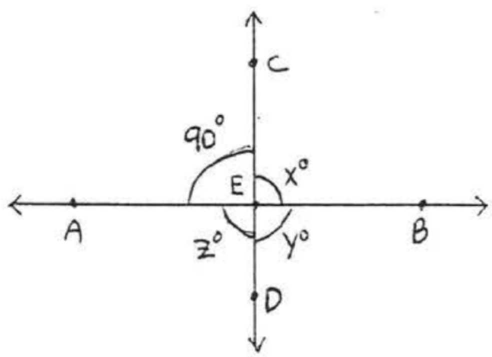

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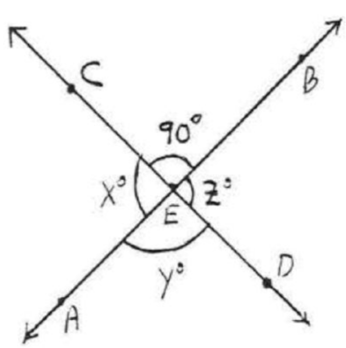

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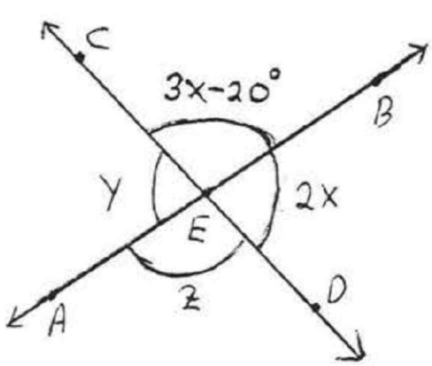

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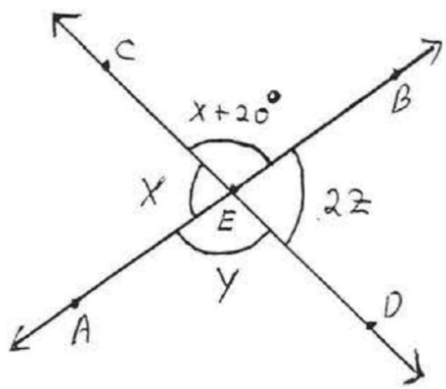
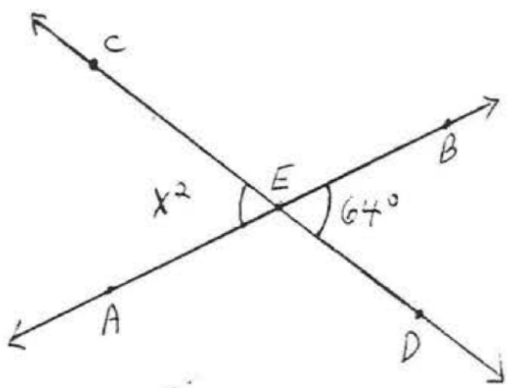


22.

23 - 26. Find x:



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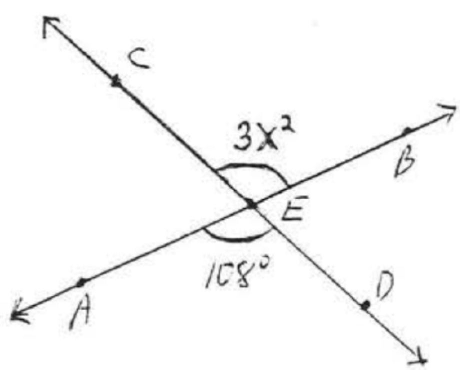

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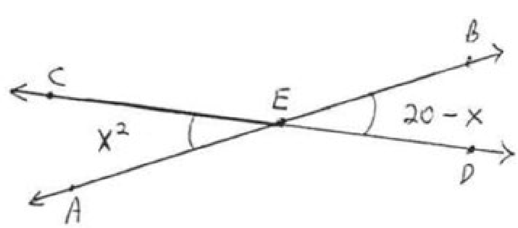

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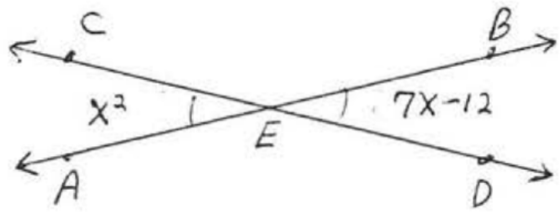
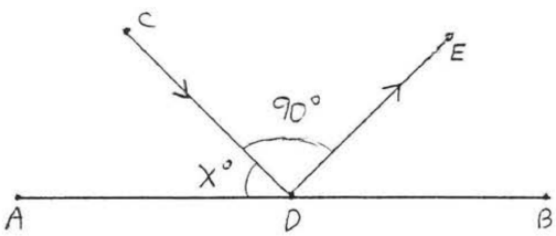
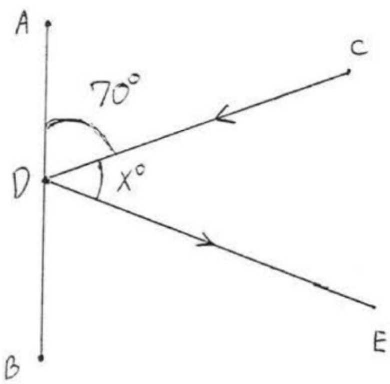


26.

27. Find the angle of incidence, ∠CDA:



28. Find x if the angle of incidence is 40^o:



Answers to Odd Number Problems

1. $(a)\ 53°\ (b)\ 45°\ (c)\ 37°\ (d)\ 30°$

3. 15°

5. 30°

7. $(a)\ 150°\ (b)\ 143°\ (c)\ 90°\ (d)\ 60°$

9. 30°

11. x=3,−3

13. 10

15. x=70, y=110, z=70

17. x=30, y=45, z=105

19. x=y=z= 90

21. x=40, y=80, z=100

23. 8, -8

25. 4, -5

27. 45°

1.4: Parallel Lines

Two lines are parallel if they do not meet, no matter how far they are extended. The symbol for parallel is $||$. In Figure 1.4.1 , $\overleftrightarrow{AB}$ $||$ $\overleftrightarrow{CD}$. The arrow marks are used to indicate the lines are parallel.

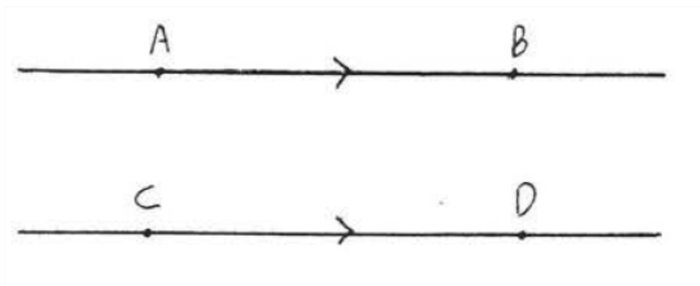


Figure 1.4.1 : $\overleftrightarrow{AB}$ and $CD^{\leftrightarrow}$

are parallel. They do not meet no matter how far they are extended.

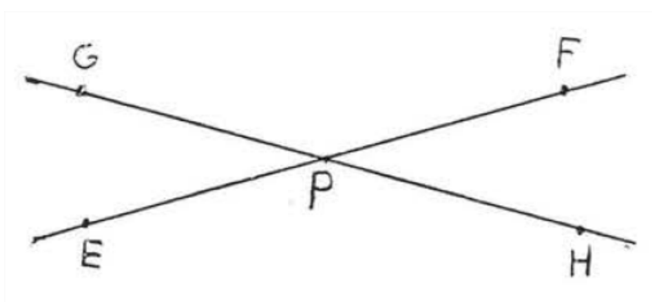


Figure 1.4.1 : $\overleftrightarrow{EF}$ and $\overleftrightarrow{GH}$ are not parallel. They meet at point P .

Through a point not on a given line one and only one line can be drawn parallel to the given line. So, in Figure 1.4.3, there is exactly one line that can be drawn through C that is parallel to $\overleftrightarrow{AB}$.

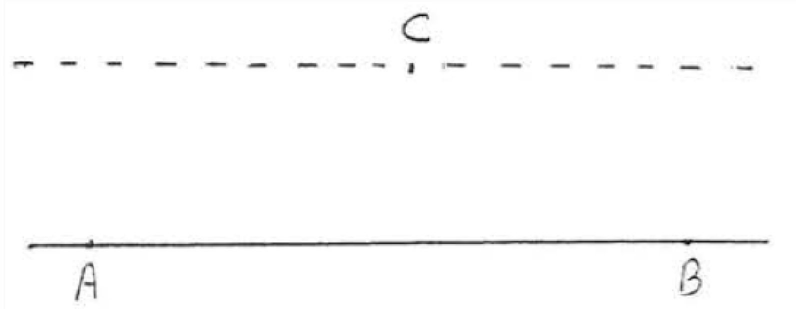


Figure 1.4.3: There is exactly one line that can be drawn through C parallel to $\overleftrightarrow{AB}$.

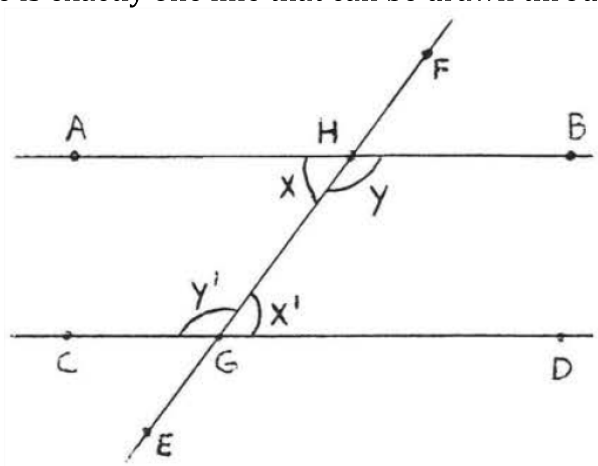


Figure 1.4.4 : $\overleftrightarrow{EF}$ is a transversal.

A **transversal** is a line that intersects two other lines at two distinct points. In Figure 1.4.4 $\overleftrightarrow{EF}$ is a transversal. ∠x and ∠x′ are called **alternate interior angles** of lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. The word "alternate," here, means that the angles are on different sides of the transversal, one angle formed with $\overleftrightarrow{AB}$ and the other formed with $\overleftrightarrow{CD}$. The word "interior" means that they are between the two lines. Notice that they form the letter "Z." (Figure 1.4.5). ∠y and ∠y′ are also alternate interior angles. They also form a "Z" though it is stretched out and backwards. Viewed from the side, the letter "Z" may also look like an "N."

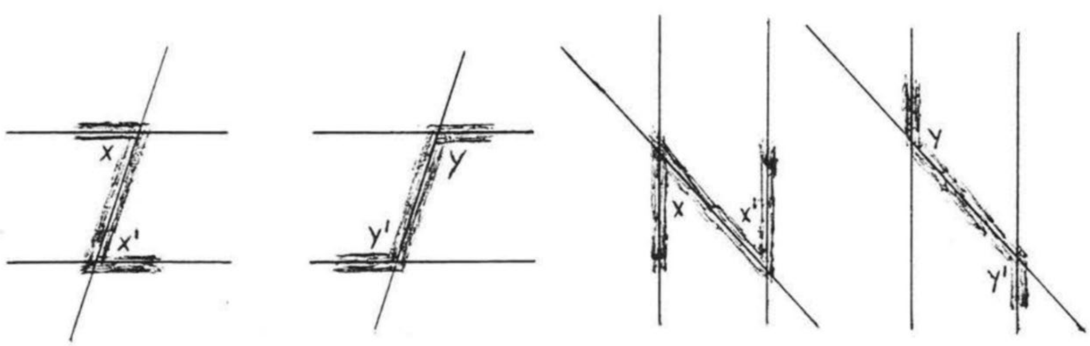


Figure 1.4.5: Alternate interior form the letters "Z" or "N". The letter may be stretched out or backwards.

Alternate interior angles are important because of the following theorem:

The "Z" Theorem

If two lines are parallel then their alternate interior angles are equal, If the alternate interior angles of two lines are equal then the lines must be parallel.

In Figure 1.4.6, $\overleftrightarrow{AB}$ must be parallel to $\overleftrightarrow{CD}$ because the alternate interior angles are both 30^o. Notice that the other pair of alternate interior angles, ∠y and ∠y′, are also equal. They are both 150^o. In Figure 1.4.7, the lines are not parallel and none of the alternate interior angles are equal.

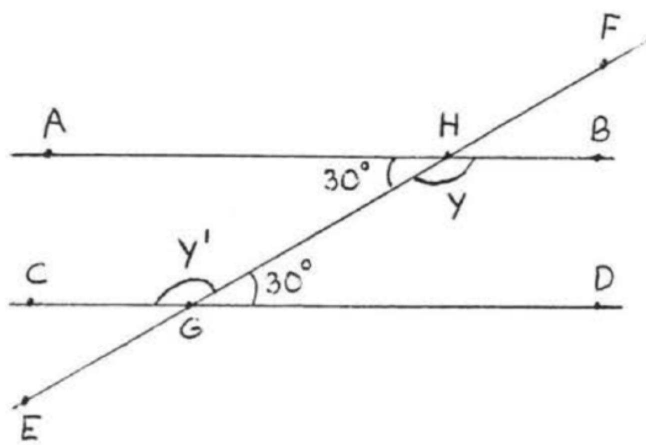


Figure 1.4.6: The lines are parallel and their alternate interior angles are equal.

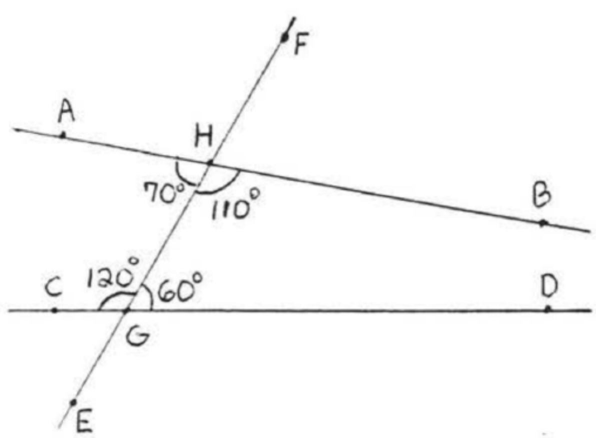
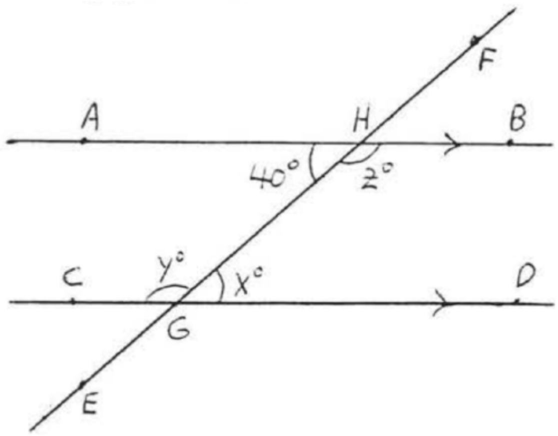


Figure 1.4.7: The lines are not parallel and their alternate interior angles are not equal.

Example 1:

Find x, y and z:



Solution

$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ since the arrows indicate parallel lines, $x° = 40°$ because alternate interior angles of parallel lines are equal. $y° = z° = 180° - 40° - 140°$.

Answer: $x = 40°,\ y = 140°\ ,\ z = 140°$

Corresponding angles of two lines are two angles which are on the same side of the two lines and the same side of the transversal, In Figure 1.4.8, ∠w and ∠w′ are corresponding angles of lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. They form the letter "F." ∠x and ∠x′, ∠y and ∠y′, and ∠z and ∠z′ are other pairs of corresponding angles of $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. They all form the letter "F", though it might be a backwards or upside down "F" (Figure 1.4.9).

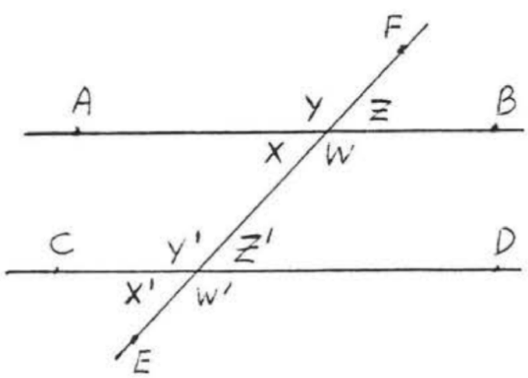


Figure 1.4.8: Four pairs of corresponding angles are illustrated.

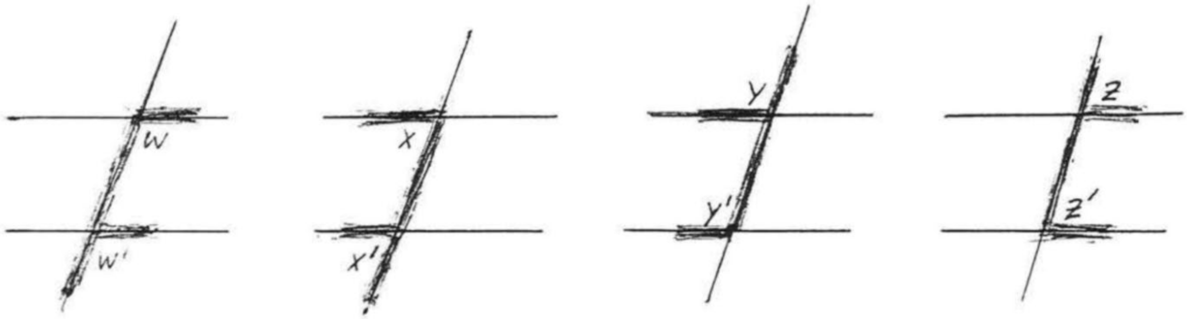


Figure 1.4.9: Corresponding angles form the letter "F," though it may be a backwards or upside down "F."

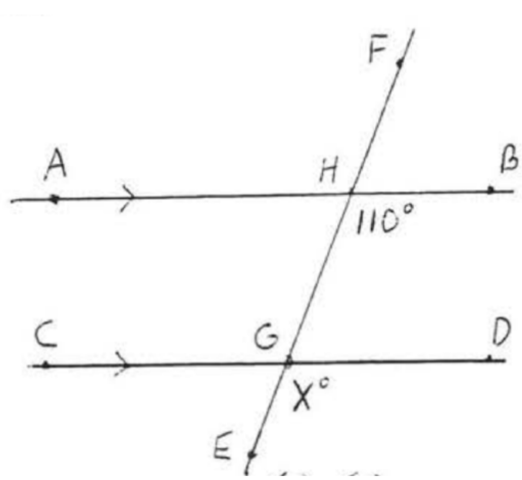
Corresponding angles are important because of the following theorem:

The "F" Theorem

If two lines are parallel, then their corresponding angles are equal. If the corresponding angles of two lines are equal, then the lines must be parallel.

Example 2:

Find x:



Solution

The arrow indicates $\overleftrightarrow{AB}$ / $\overleftrightarrow{CD}$. Therefore $x° = 110°$ because $x°$ and $110°$ are the measures of corresponding angles of the parallel lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.

Answer: $x = 110$.

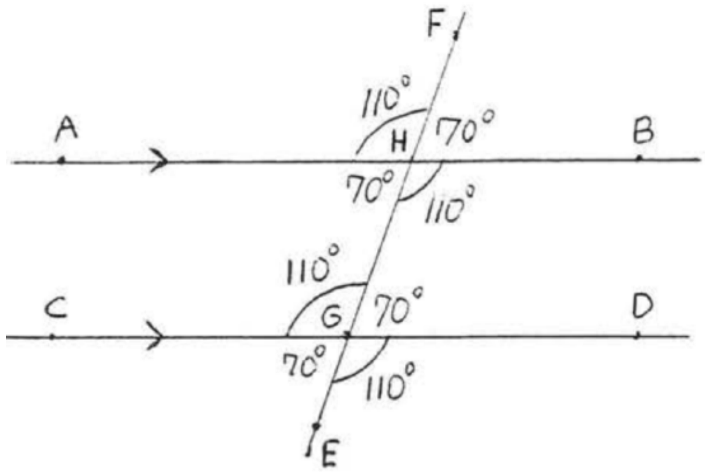


Figure 1.4.10: Each pair of corresponding angles is equal.

Each one is either supplementary to one of the 110° angles or forms equal vertical angles with one of them (Figure 1.4.10). Therefore, **all** the corresponding angles are equal, also each pair of alternate interior angles is equal. It is not hard to see that if just one pair of corresponding angles or one pair of alternate interior angles are equal then so are all other pairs of corresponding and alternate interior angles.

Proof of Theorem 1.4.2: The corresponding angles will be equal if the alternate interior angles are equal and vice versa. Therefore Theorem 1.4.2 follows directly from Theorem 1.4.1.

In Figure 1.4.11, $\angle x$ and $\angle x'$ are called **interior angles on the same side of the transversal.** $\angle y$ and $\angle y'$ are also interior angles on the same side of the transversal, notice that each pair of angles forms the letter "C." Compare Figure 1.4.11 with Figure 10 and also with Example 1.4.1, The following theorem is then apparent:

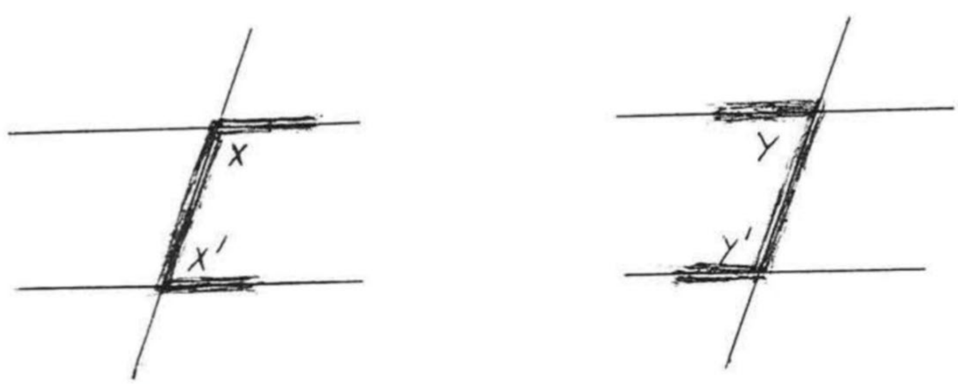


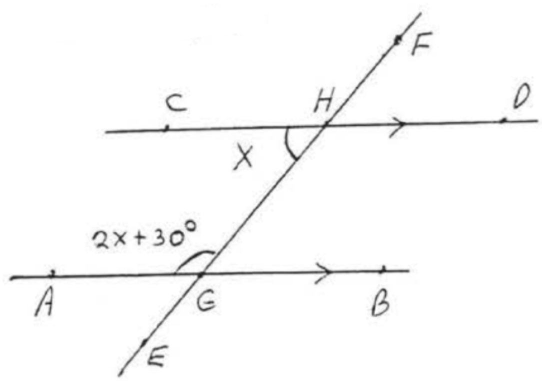
Figure 1.4.11: Interior angles on the same side of the transversal form the letter "C". It may also be a backwards "C."

Theorem 1.4.3: The "C" Theorem

If two lines are parallel, then the interior angles on the same side of the transversal are supplementary (they add up to 180∘). If the interior angles of two lines on the same side of the transversal are supplementary, then the lines must be parallel.

Example 3:

Find x and the marked angles:



Solution:

The lines are parallel so by Theorem 1.4.3 the two labelled angles must be supplementary.

$$x + 2x + 30 = 180$$

$$3x + 30 = 180$$

$$3x = 180 - 30$$

$$3x = 150$$

$$x = 50$$

$\angle CHG = x = 50°$

∠AGH=2x +30= 2(50) +30=100+30=130

Check:

$$x + 2x + 30 = 180$$

$$50 + 2(50) + 30$$

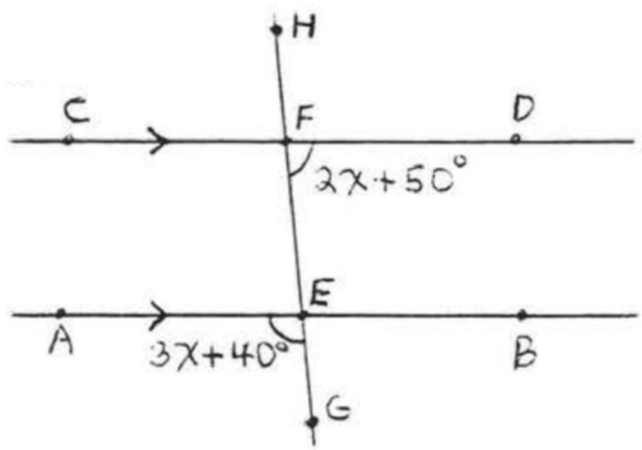
$$50 + 130$$

$$180$$

Answer: $x = 50$, $\angle CHG = 50°$ **,** $\angle AGH = 130°$

Example 4:

Find x and the marked angles:



Solution

$\angle BEF = 3x+40°$ because vertical angles are equal. $\angle BEF$ and $\angle DEF$ are interior angles on the same side of the transversal, and therefore are supplementary because the lines are parallel.

$$3x + 40 + 2x + 50 = 180$$

$$5x + 90 = 180$$

$$5x = 180 - 90 \quad (1.4.2)$$

$$5x = 90$$

$$x = 18$$

$\angle AEG = 3x + 40 = 3(18) + 40 = 54 + 40 = 94°$

$\angle DFE = 2x + 50 = 2(18) + 50 = 36 + 50 = 86°$

Check:

$$3x + 40 + 2x + 50 = 180$$

$$3(18) + 40 + 2(18) + 50$$

$$54 + 40 + 36 + 50$$

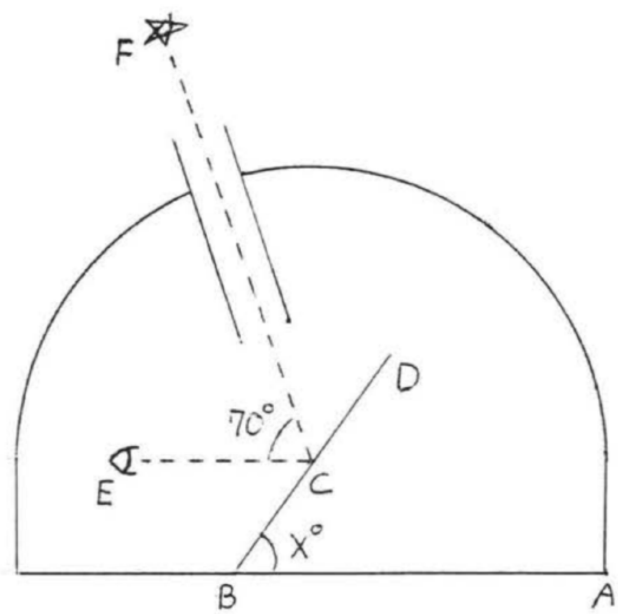
$$94 + 86$$

$$180$$

Answer: $x = 18,\ \angle AEG = 94°, \angle DFE = 86°$

Example 5:

A telescope is pointed at a star 70° above the horizon, what angle $x°$ must the mirror BD make with the horizontal so that the star can be seen in the eyepiece E?



Solution

$x° = \angle BCE$ because they are alternate interior angles of parallel lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CE}$.
$\angle DCF = \angle BCE = x°$ because the angle of incidence is equal to the angle of reflection. Therefore

$$x + 70 + x = 180$$

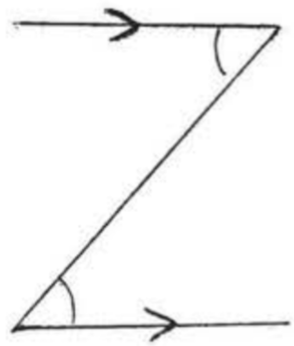
$$2x + 70 + x = 180$$

$$2x = 110$$

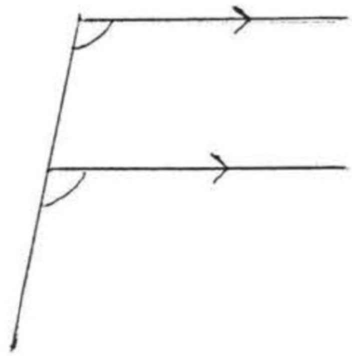
$$x = 55$$

Answer: 55°

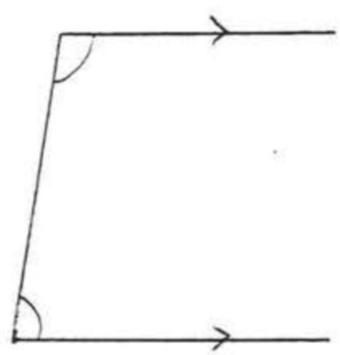
SUMMARY



Alternate interior angles of parallel lines are equal. They form the letter "Z."



Corresponding angles of parallel lines are equal. They form the letter "F."



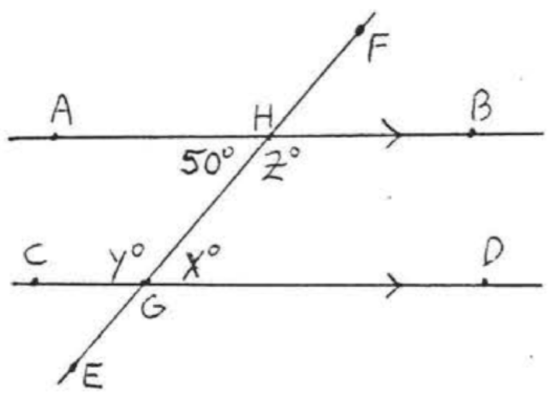
Interior angles on the same sides of the transversal of parallel lines are supplementary. They form the letter "C."

Practice Exercises

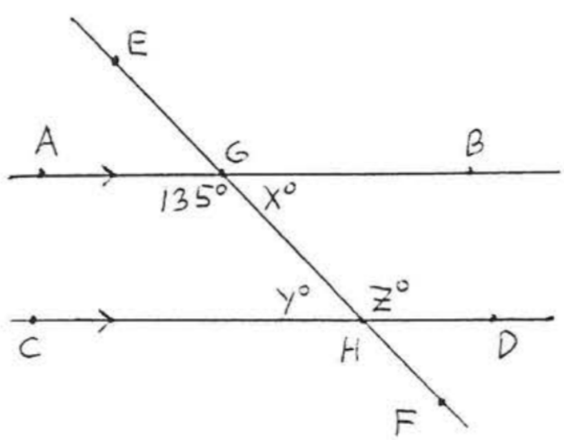
For each of the following, state the theorem(s) used in obtaining your answer (for example, "alternate interior angles of parallel lines are equal"). Lines marked with the same arrow are assumed to be parallel,

1 - 2. Find x, y, and z:

1.

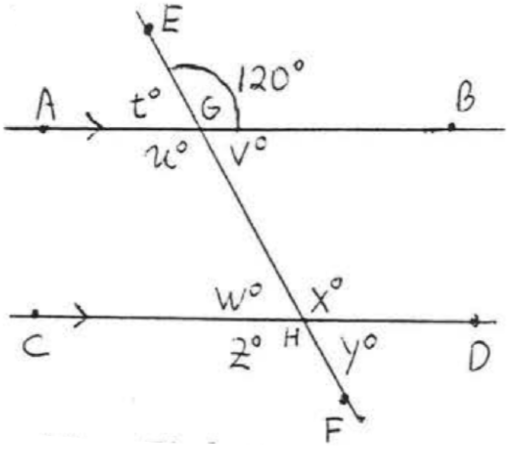


2.



3 - 4. Find t, u, v, w, x, y, and z:

3.



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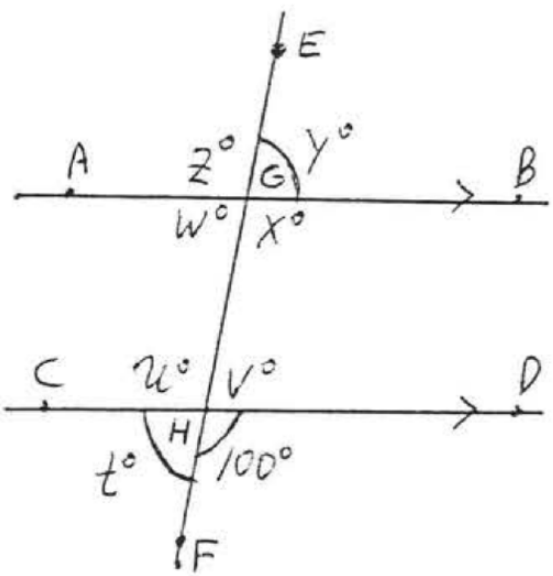
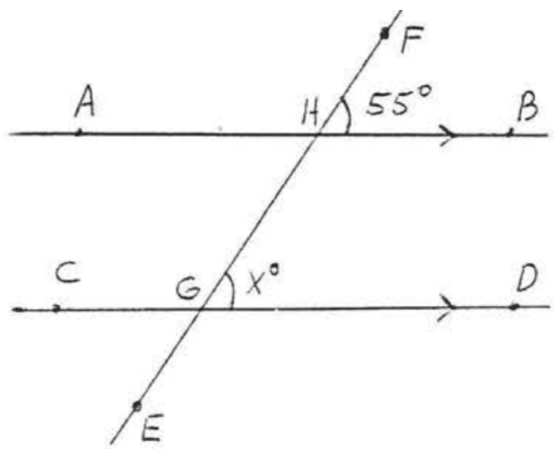


5 - 10. Find x:

5.



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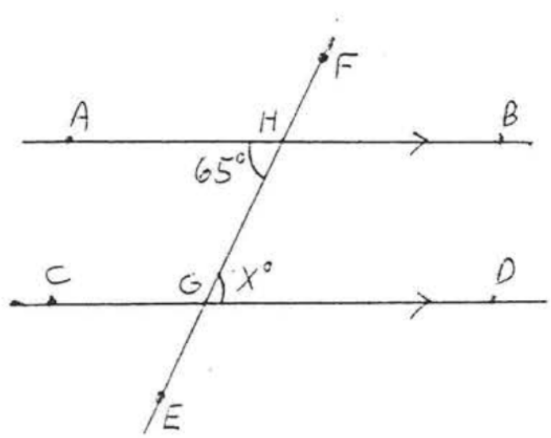

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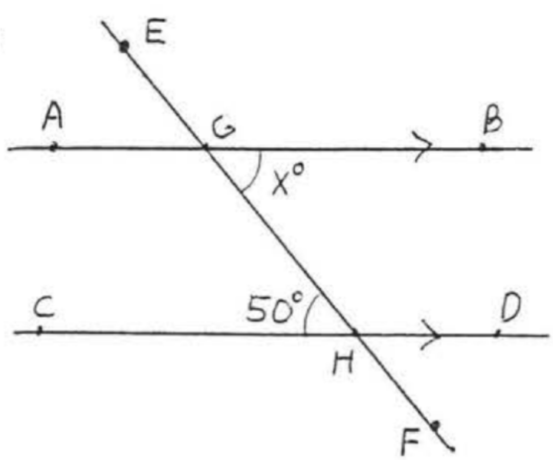

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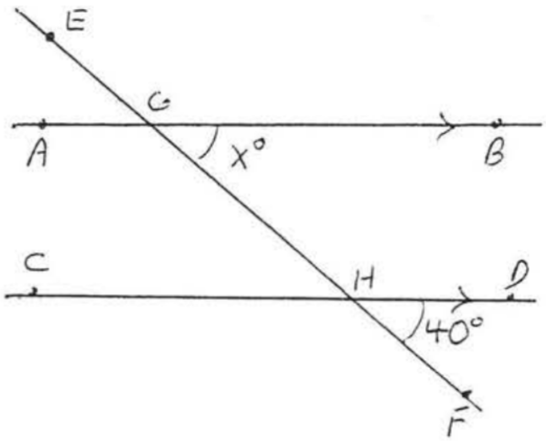

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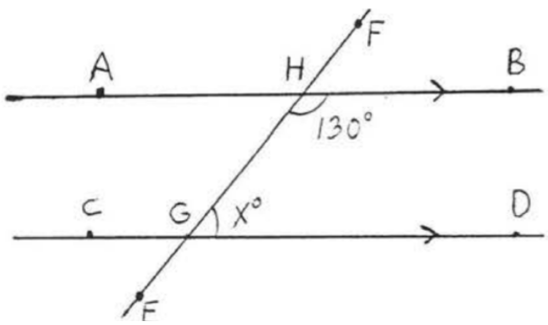

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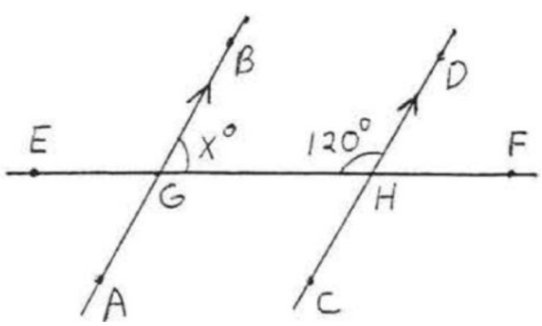
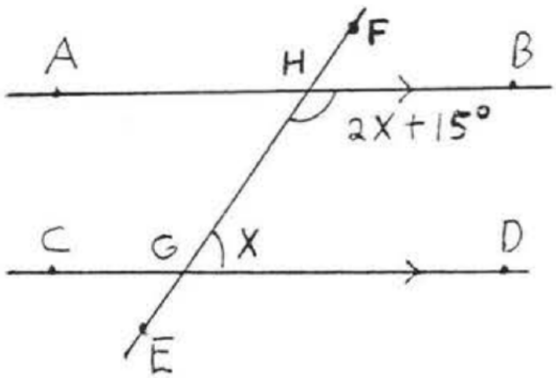


11 - 18. Find x and the marked angles:

11.



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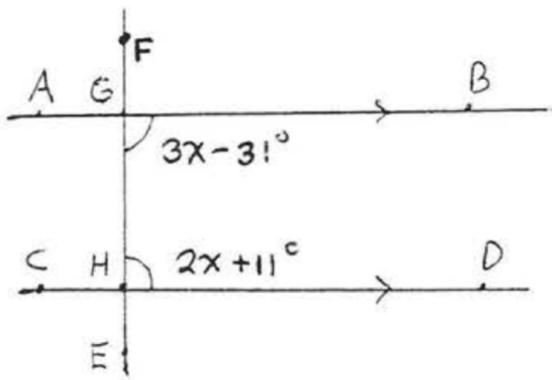

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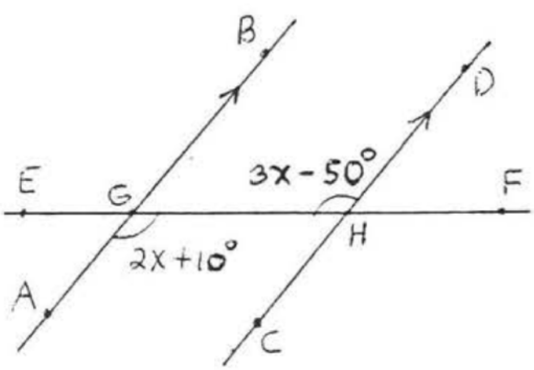

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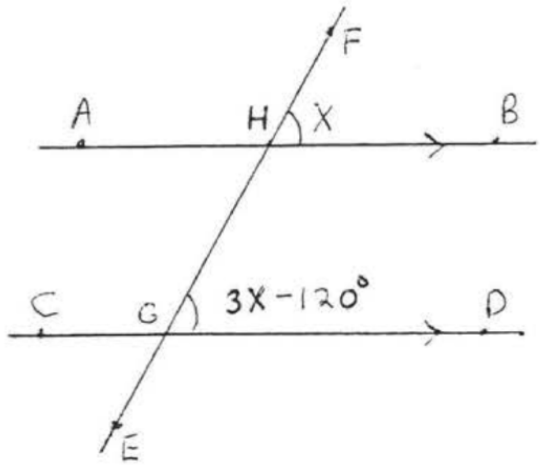

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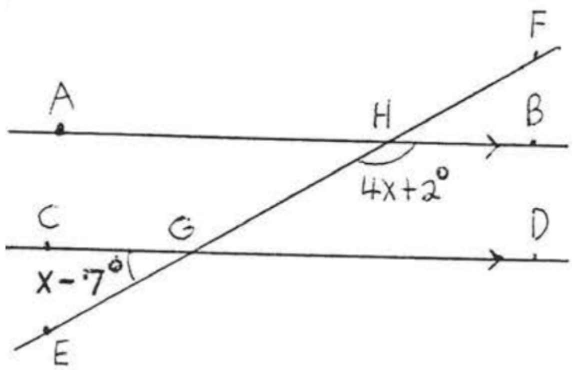

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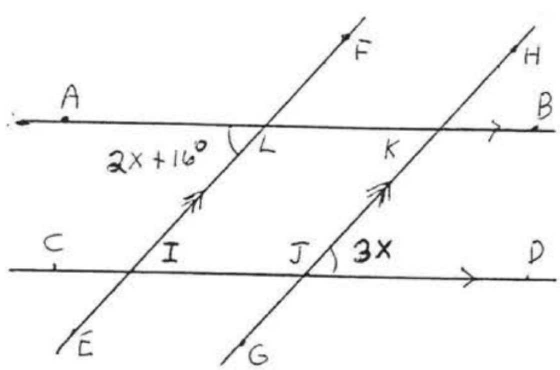

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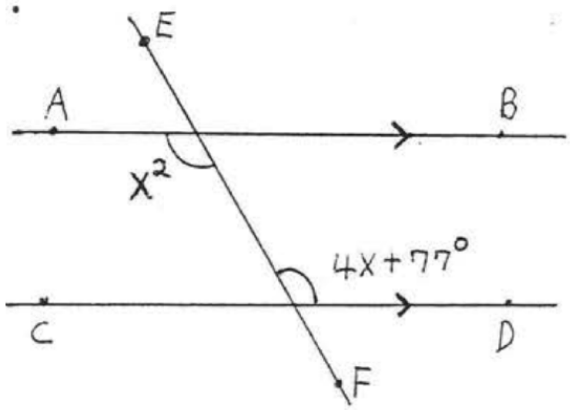
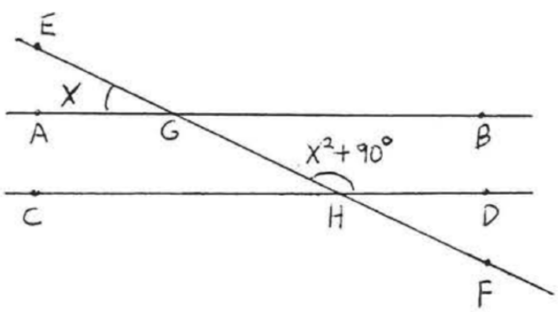
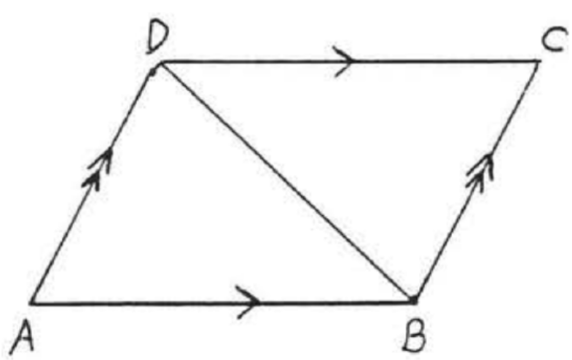
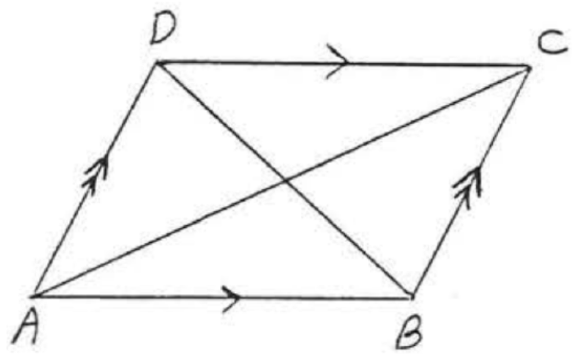


18.

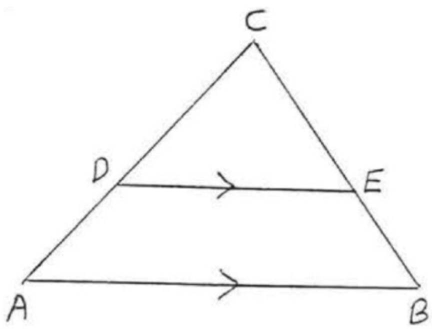


19 - 26. For each of the following, list all pairs of alternate interior angles and corresponding angles, If there are none, then list all pairs of interior angles on the same side of the transversal. Indicate the parallel lines which form each pair of angles.

19. 



20.



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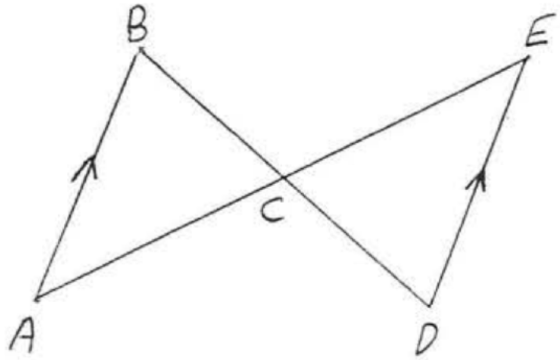

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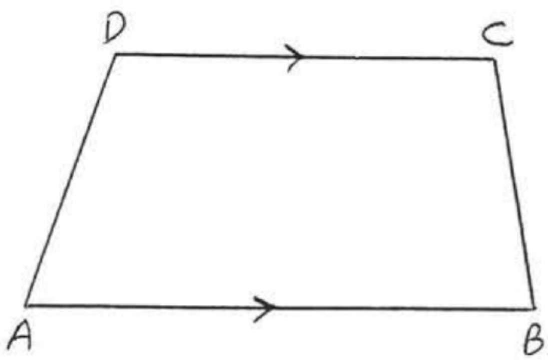

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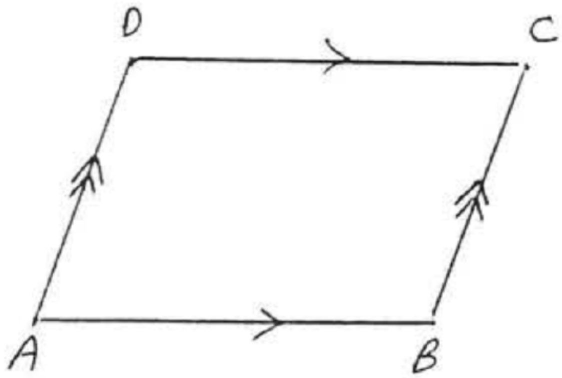

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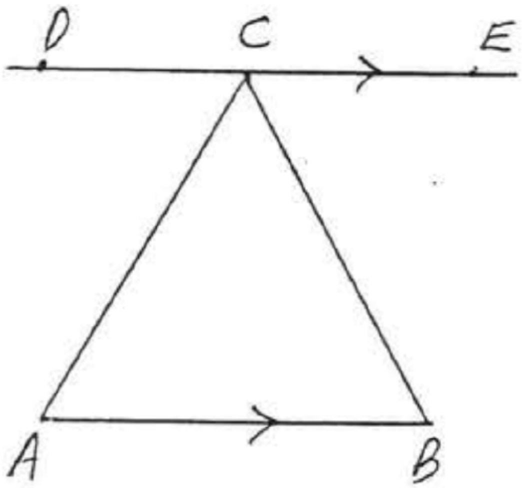
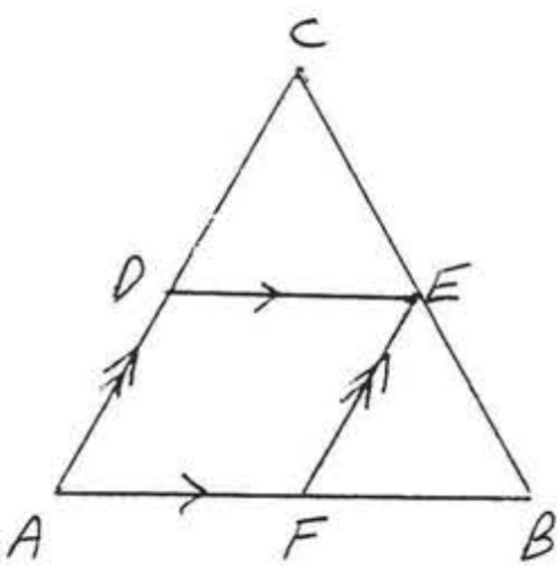
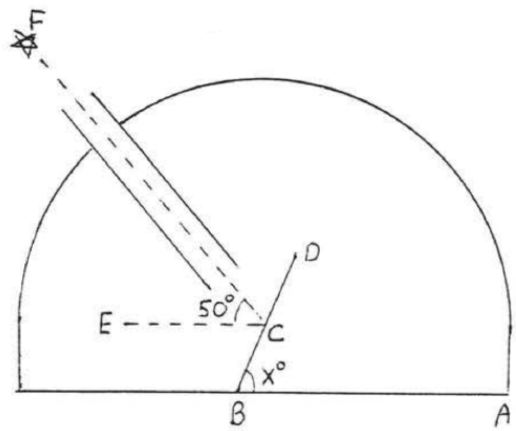


25.

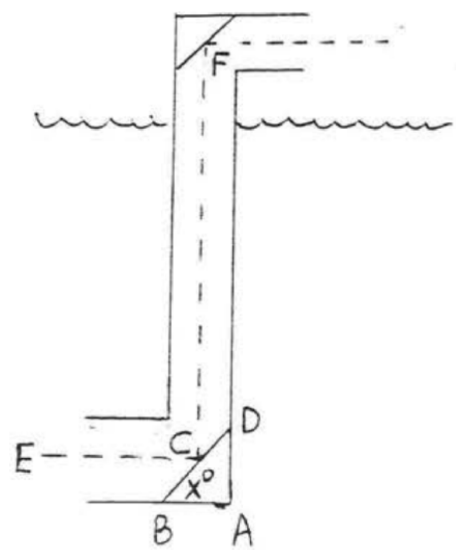


26.

27. A telescope is pointed at a star 50° above the horizon. What angle $x°$ must the mirror BD make with the horizontal so that the star can be seen in the eyepiece E?



28. A periscope is used by sailors in a submarine to see objects on the surface of the water, If ∠ECF=90°, what angle $x°$ does the mirror BD make with the horizontal?



Answers to Odd Numbered Problems

1. $x = 50, y = z = 130$

3. $u = x = z = 120, t = v = w = y = 60$

5. 55

7. 50

9. 50

11. 55

13. 60

15. 37

17. 11

19. alternate interior: $\angle ABD$ & $\angle CDB - AB||CD$; $\angle ADB$ & $\angle CBD - AD||BC$

21. corresponding: $\angle BAC$ & $\angle EDC - AB||DE$; $\angle ABC$ & $\angle DEC - AB||DE$

23. interior on same side of transversal: $\angle BAD$ & $\angle CDA - AB||CD$; $\angle ABC$ & $\angle DCB - AB||CD$

25. alternate interior: $\angle BAC$ & $\angle DCA - AB||DE$; $\angle ABC$ & $\angle ECB - AB||DE$

27. 65°

1.5: Triangle

A **triangle** is formed when three straight line segments bound a portion of the plane. The line segments are called the **sides of the triangle.** A point where two sides meet is called a **vertex of the triangle**, and the angle formed is called an **angle of the triangle.** The symbol for triangle is △.

The triangle in Figure 1.5.1 is denoted by △ABC (or △BCA or △CAB, etc.).

- Its sides are AB, AC, and BC.
- Its vertices are A, B, and C.
- Its angles are ∠A, ∠B, and ∠C.

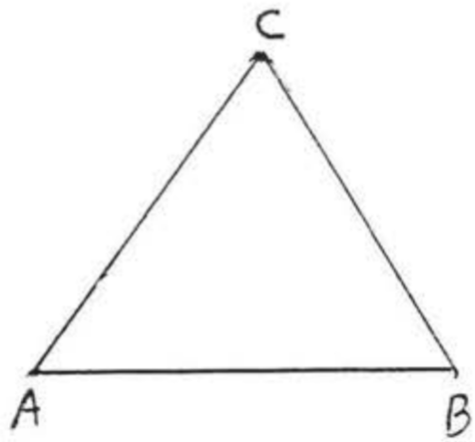


Figure 1.5.1: Triangle ABC.

The triangle is the most important figure in plane geometry, this is because figures with more than three sides can always be divided into triangles (Figure 1.5.2). If we know the properties of a triangle, we can extend this knowledge to the study of other figures as well.

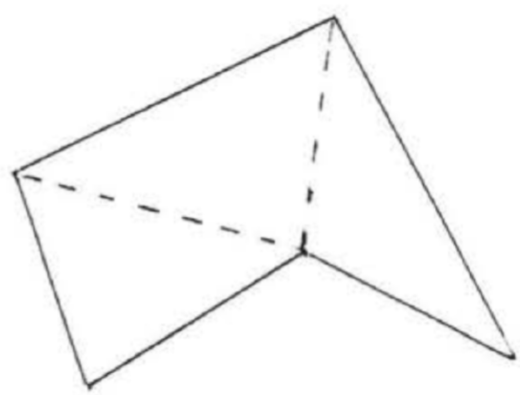


Figure 1.5.2: A closed figure formed by more than three straight lines can be divided into triangles.

A fundamental property of triangles is the following:

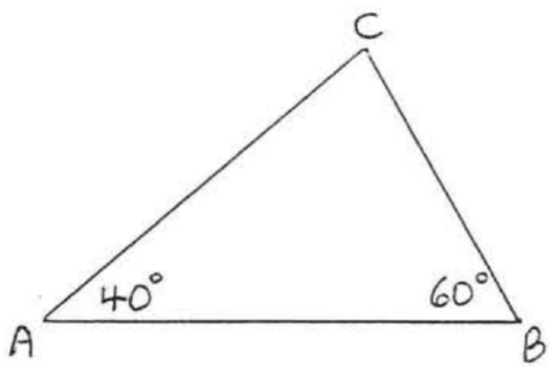
Theorem 1.5.1

The sum of the angles of a triangle is 180°.

In △ABC of Figure 1.5.1, $\angle A + \angle B + \angle C = 180°$.

Example 1:

Find ∠C:



Solution

$$\angle A + \angle B + \angle C = 180°$$

$$40° + 60° + \angle C = 180°$$

$$100° + \angle C = 180°$$

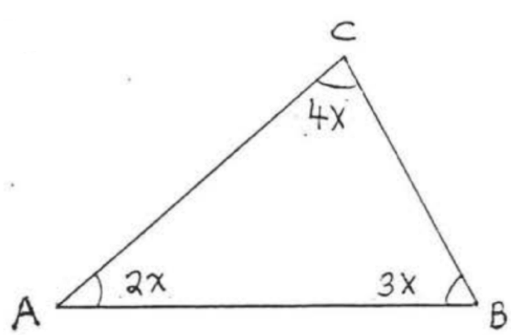
$$\angle C = 180° - 100°$$

$$\angle C = 80°$$

Answer: $\angle C = 80°$

Example 2:

Find x.



Solution

$$\angle A + \angle B + \angle C = 180°$$

$$2x + 3x + 4x = 180$$

$$9x = 180$$

$$x = 20$$

Check:

$$\angle A + \angle B + \angle C = 180°$$

$$2x + 3x + 4x$$

$$2(20) + 3(20) + 4(20)$$

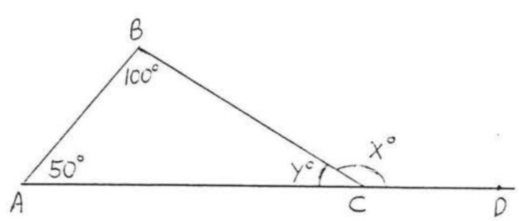
$$40° + 60° + 80°$$

$$180°$$

Answer: x=20.

Example 3:

Find y and x:



Solution

$$\mathbf{50 + 100 + y = 180}$$

$$\mathbf{150 + y = 180}$$

$$y = 180 - 150$$

$$y = 30$$

$$x = 180 - 30 = 150$$

Answer: $y = 30, x = 150$

In Figure 1.5.4, ∠x is called an **exterior angle** of △ABC, ∠A, ∠B, and ∠y is called the **interior angles** of △ABC. ∠A and ∠B are said to be the interior angles **remote** from the exterior angle ∠x.

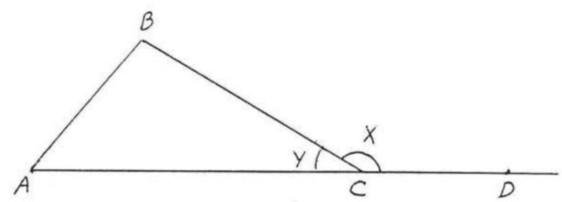


Figure 1.5.4: ∠x is an exterior angle of △ABC.

The results of Example 1.5.3 suggest the following theorem.

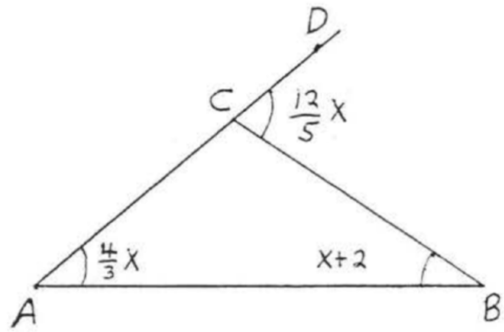
Theorem 1.5.2

An exterior angle is equal to the sum of the two remote interior angles,

In Figure 1.5.4, ∠x=∠A+∠B.

Example 4:

Find x:



Solution

∠BCD is an exterior angle with remote interior angles ∠A and ∠B . By Theorem 1.5.2 ,

$$\angle BCD = \angle A + \angle B$$

$$\frac{12}{5}x = \frac{4}{3}x + x + 2$$

The least common denominator (1, c, d) is 15.

$$\begin{aligned} \overset{3}{(\cancel{15})}\frac{12}{\cancel{5}}x &= \overset{3}{(\cancel{15})}\frac{4}{\cancel{3}}x + (15)x + (15)(2) \\ 36x &= 20x + 15x + 30 \\ 36x &= 35x + 30 \\ 36x - 35x &= 30 \\ x &= 30 \end{aligned}$$

Check:

$\angle BCD = \angle A + \angle B$

$\frac{12}{5}x = \frac{4}{3}x + x + 2$

$\frac{12}{5}(30) = \frac{4}{3}(30) + 30 + 2$

$72° = 40° + 32°$

$72°$

Answer: x=30

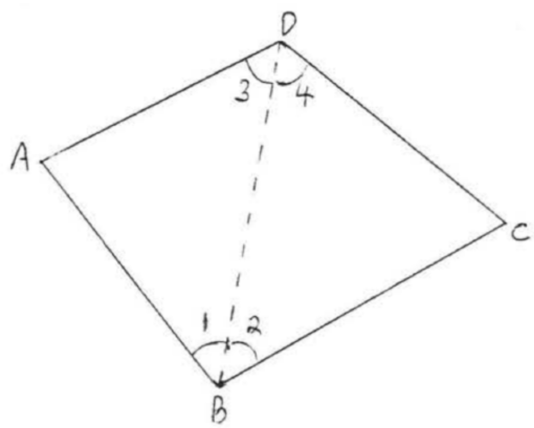
Our work on the sum of the angles of a triangle can easily be extended to other figures:

Example 5:

Find the sum of the angles of a quadrilateral (four- sided figure),

Solution

Divide the quadrilateral into two triangles as illustrated,



$$\angle A + \angle B + \angle C + \angle D = \angle A + \angle 1 + \angle 3 + \angle 2 + \angle 4 + \angle C$$

$$= 180° + 180°$$

$$360°$$

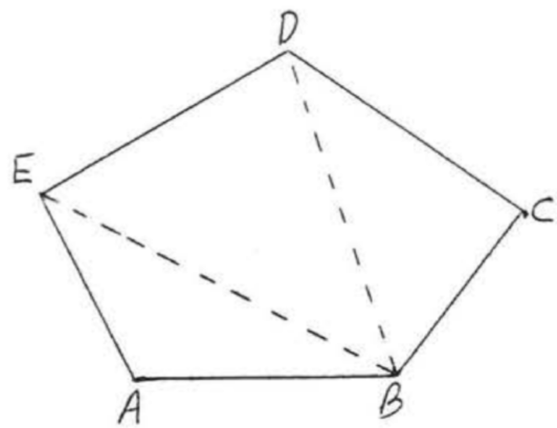
Answer: 360°

Example 6:

Find the sum of the angles of a pentagon (five-sided figure).

Solution

Divide the pentagon into three triangles as illustrated, the sum is equal to the sum of the angles of the three triangles = $(3)\ (180°)\ = 540°$.



Answer: 540°.

There is one more simple principle which we will derive from Theorem 1.5.1, Consider the two triangles in Figure 1.5.5.

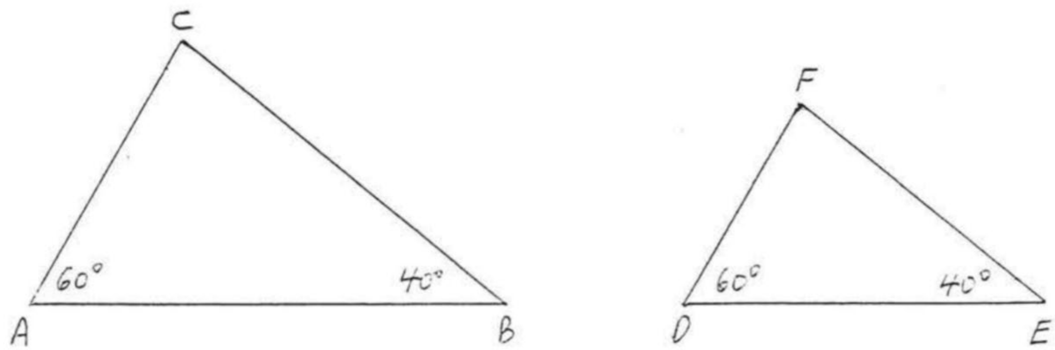


Figure 1.5.5: Each triangle has an angle of 60° and 40°.

We are given that $\angle A = \angle D = 60°$ and $\angle B = \angle E = 40°$. A short calculation shows that we must also have ∠C=∠F=80∘. This suggests the following theorem:

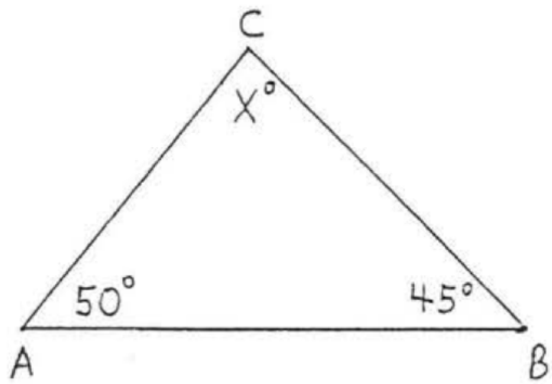
Theorem 1.5.3

If two angles of one triangle are equal respectively to two angles of another triangle, then their remaining angles are also equal.

In Figure 1.5.6, if ∠A=∠D and ∠B=∠E then ∠C=∠F.

Practice Exercises

1 - 12. Find x and all the missing angles of each triangle:



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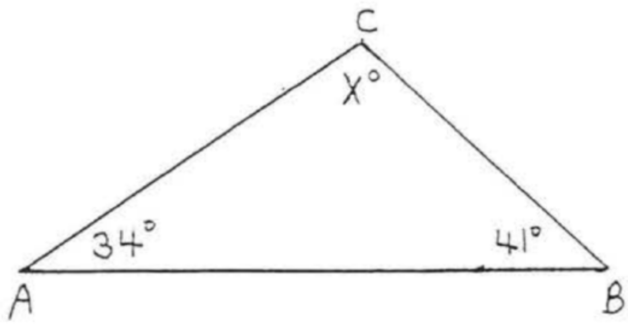

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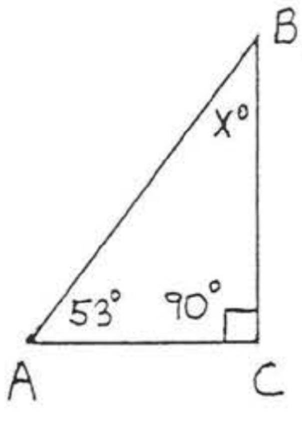

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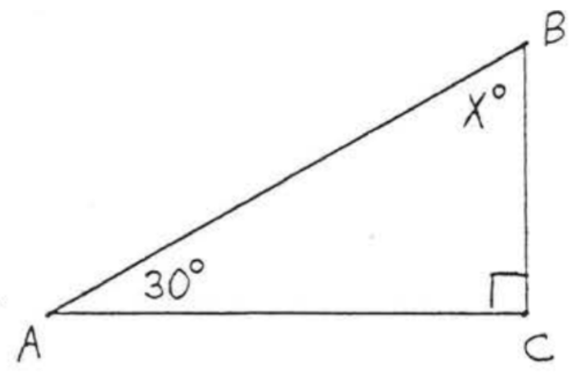
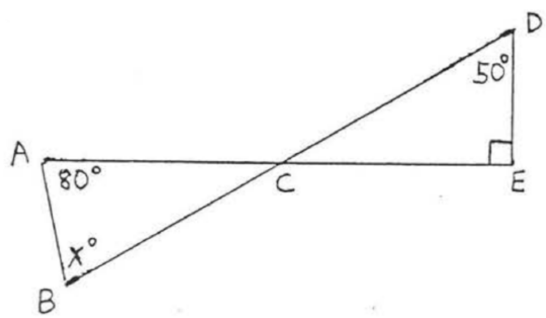
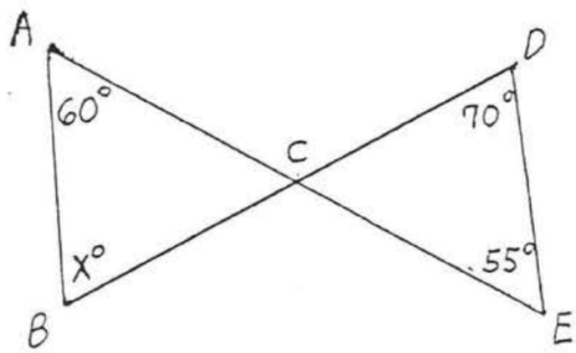
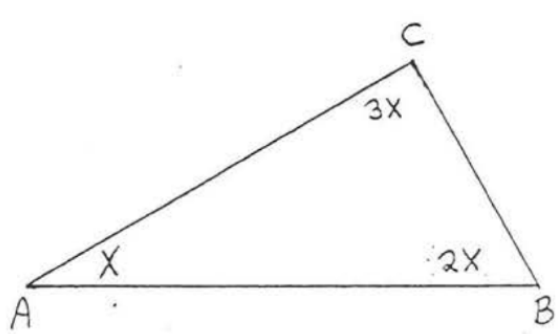
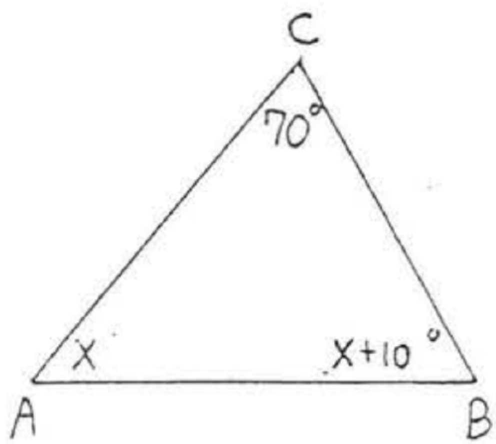
4. 

5. 

6. 

7. 



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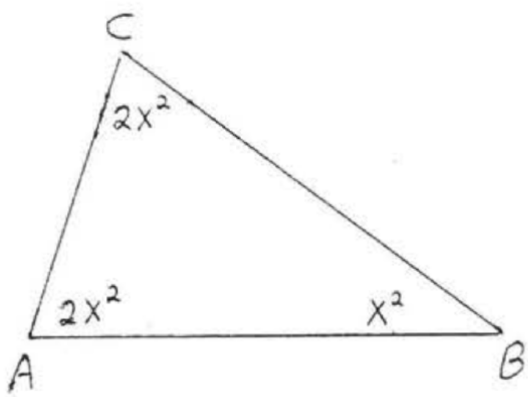

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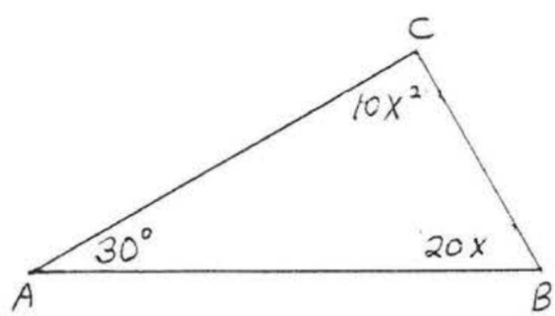
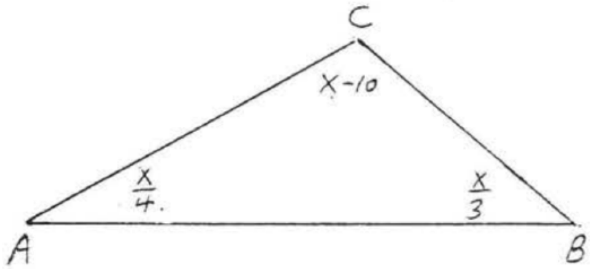
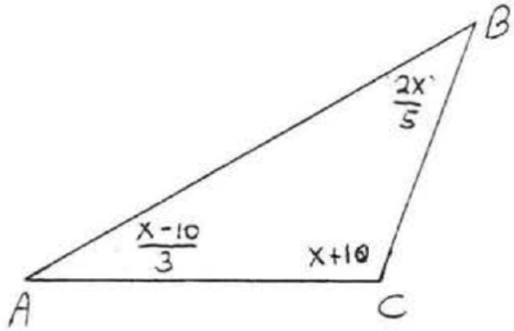


10.

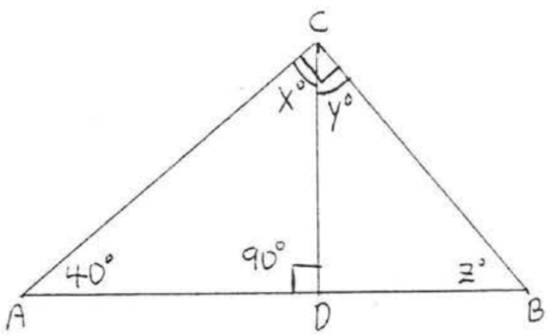


11.

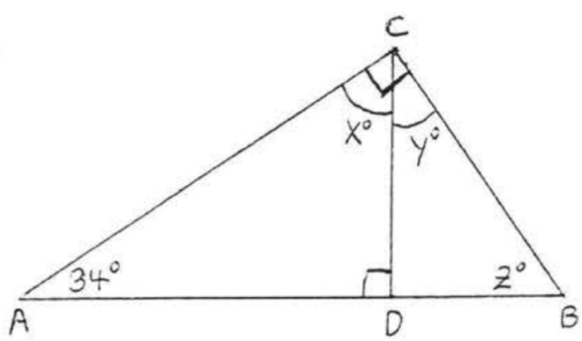


12.

13 - 14. Find x, y, and z:

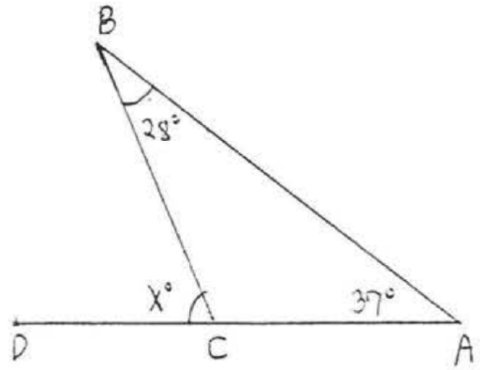


13.

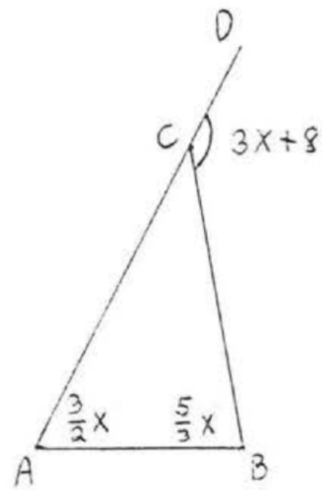


14.

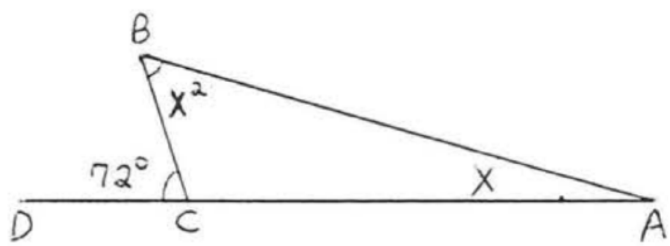
15 - 20. Find x:


B
28°
x°
37°
D
C
A

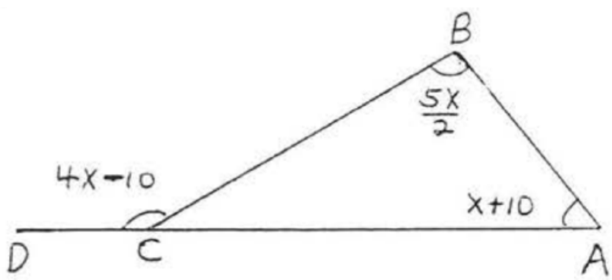
15.


D
C
3x + 8
$\frac{3}{2}$x
$\frac{5}{3}$x
A
B

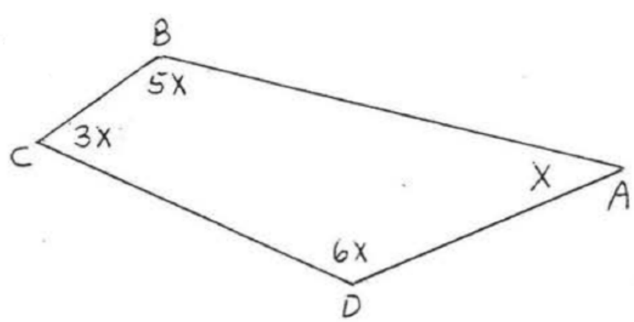
16.


B
x²
72°
x
D
C
A

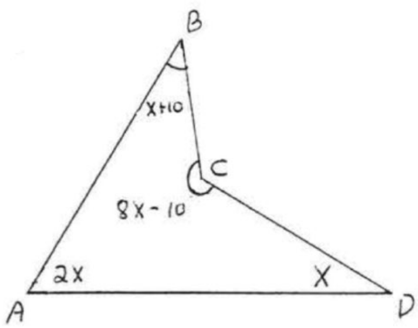
17.


B
5x/2
4x − 10
x + 10
D
C
A

18.


B
5x
C
3x
x
A
6x
D

19.

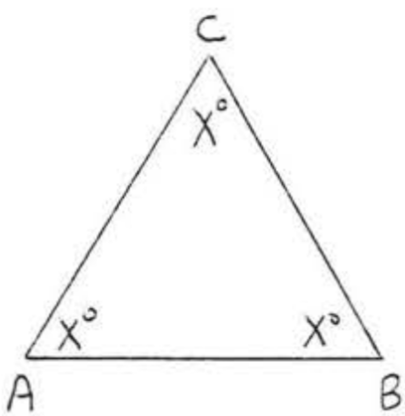


20.

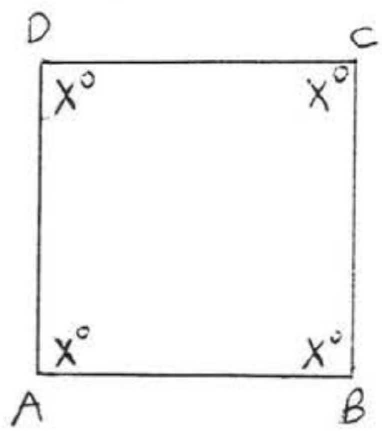
21. Find the sum of the angles of a hexagon (6-sided figure).

22. Find the sum of the angles of an octagon (8-sided figure).

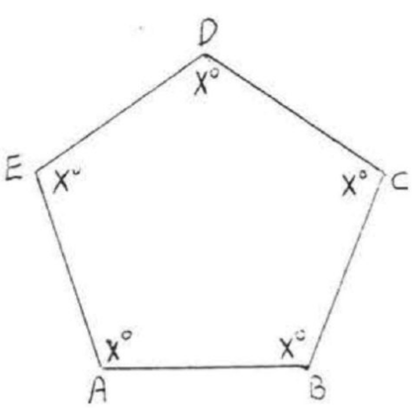
23 - 26. Find x:



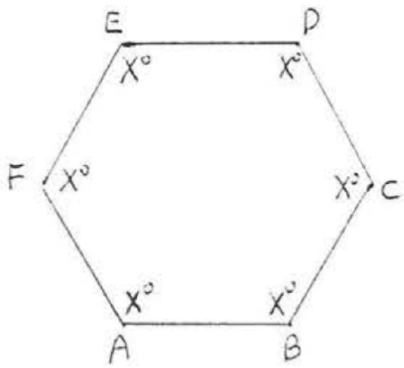
23.



24.



25.



26.

Answers to Odd Numbered Problems

1. 85°

3. 37°

5. 60°

7. 30

9. 6

11. 120

13. x=50, y=40, $\angle A = \angle D = 60^\circ$ $\angle B = \angle E = 40^\circ$∠C=∠F=80 σz=50

15. 65°

17. 8

19. 24

21. 720°

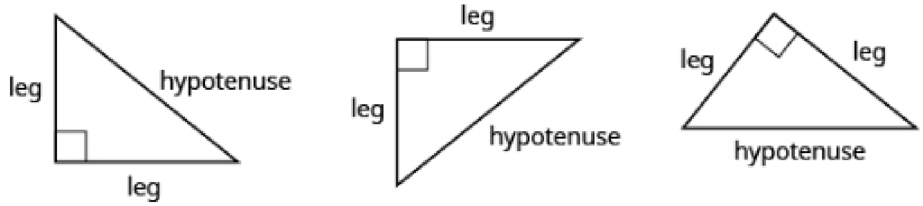
23. 60°

25. 108°

Pythagorean Theorem

The **Pythagorean Theorem** is a special property of right triangles that has been used since ancient times. It is named after the Greek philosopher and mathematician Pythagoras who lived around 500 BCE.

Remember that a right triangle has a 90° angle, which we usually mark with a small square in the corner. The side of the triangle opposite the 90 angle is called the **hypotenuse**, and the other two sides are called the **legs**.



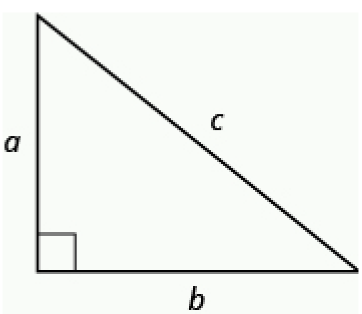
In a right triangle, the side opposite the 90° angle is called the hypotenuse and each of the other sides is called a leg.

The Pythagorean Theorem tells how the lengths of the three sides of a right triangle relate to each other. It states that in any right triangle, the sum of the squares of the two legs equals the square of the hypotenuse.

In any right triangle ΔABC,

$$a^2 + b^2 = c^2$$

where c is the length of the hypotenuse a and b are the lengths of the legs.



To solve problems that use the Pythagorean Theorem, we will need to find square roots. we introduced the notation $\sqrt{m}$ and defined it in this way:

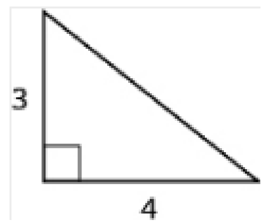
$$\text{If } m = n^2, \text{ then } \sqrt{m} = n \text{ for } n \geq 0$$

For example, we found that $\sqrt{25}$ is 5 because $5^2=25$.

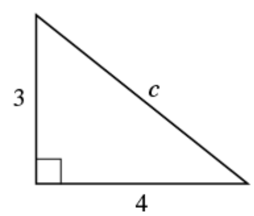
We will use this definition of square roots to solve for the length of a side in a right triangle.

Example 1:

Use the Pythagorean Theorem to find the length of the hypotenuse.

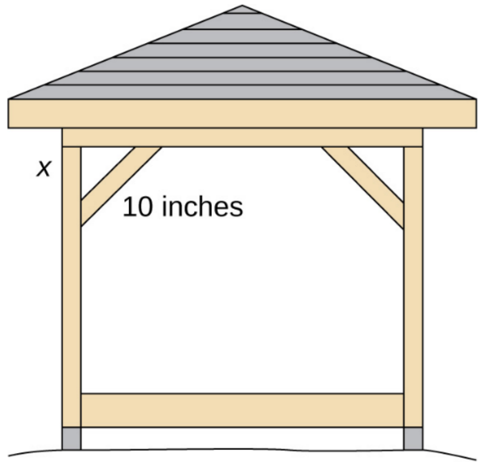


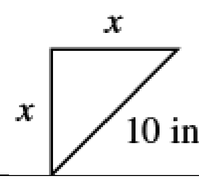
Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	the length of the hypotenuse of the triangle
Step 3. Name. Choose a variable to represent it.	Let c=the length of the hypotenuse 
Step 4. Translate. Write the appropriate formula. Substitute.	$a^2 + b^2 = c^2$ $3^2 + 4^2 = c^2$
Step 5. Solve the equation.	$9 + 16 = c^2$ $25 = c^2$ $\sqrt{25} = c^2$ $5 = c$
Step 6. Check:	$3^2 + 4^2 = 5^2$ $9 + 16 \stackrel{?}{=} 25$ $25 = 25$ ✓
Step 7. Answer the question.	The length of the hypotenuse is 5.

Example 2:

Kelvin is building a gazebo and wants to brace each corner by placing a 10-inch wooden bracket diagonally as shown. How far below the corner should he fasten the bracket if he wants the distances from the corner to each end of the bracket to be equal? Approximate to the nearest tenth of an inch.



Step 1. Read the problem.	
Step 2. Identify what you are looking for.	the distance from the corner that the bracket should be attached
Step 3. Name. Choose a variable to represent it.	Let x = the distance from the corner 
Step 4. Translate. Write the appropriate formula. Substitute.	$a^2 + b^2 = c^2$ $x^2 + x^2 = 10^2$
Step 5. Solve the equation. Isolate the variable. Use the definition of the square root. Simplify. Approximate to the nearest tenth.	$2x^2 = 100$ $x^2 = 50$ $\sqrt{x^2} = x = \sqrt{50}$ $b \approx 7.1$
Step 6. Check: $a^2 + b^2 = c^2$ $(7.1)^2 + (7.1)^2 \overset{?}{\approx} 10^2$ Yes	

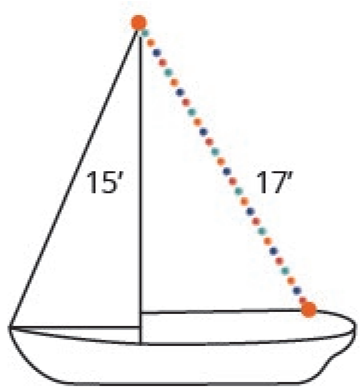
Step 7. Answer the question.	Kelvin should fasten each piece of wood approximately 7.1" from the corner.

Practice Exercises:

John puts the base of a 13-ft ladder 5 feet from the wall of his house. How far up the wall does the ladder reach?

1.

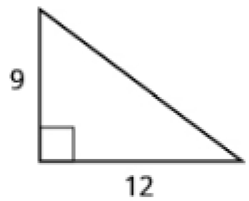
Randy wants to attach a 17-ft string of lights to the top of the 15-ft mast of his sailboat. How far from the base of the mast should he attach the end of the light string?

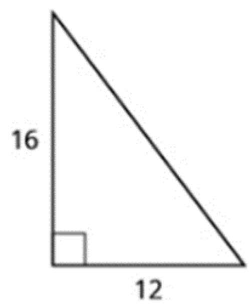


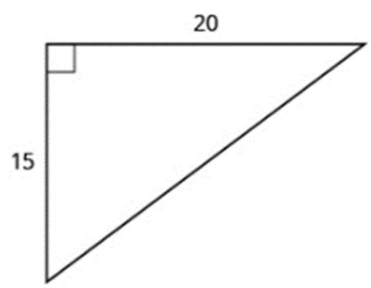
2.

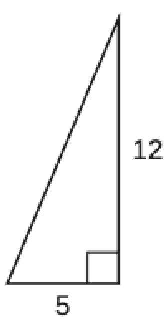
Use the Pythagorean Theorem

In the following exercises, use the Pythagorean Theorem to find the length of the hypotenuse.

3. 

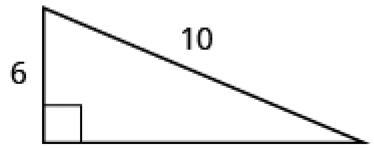
4. 

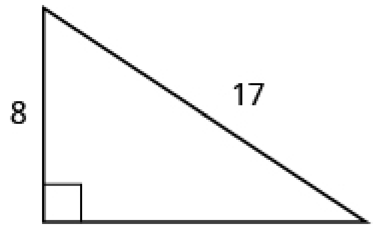
5. 

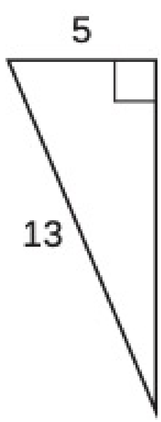
6. 

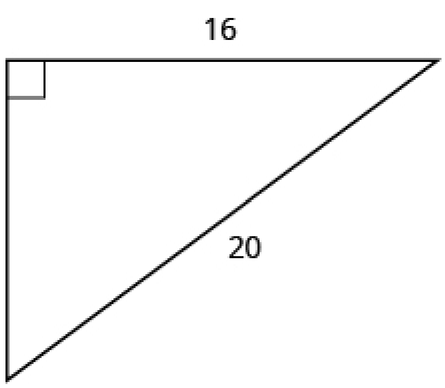
Find the Length of the Missing Side

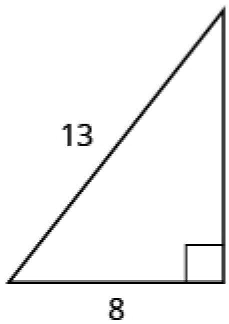
In the following exercises, use the Pythagorean Theorem to find the length of the missing side. Round to the nearest tenth, if necessary.

7. 

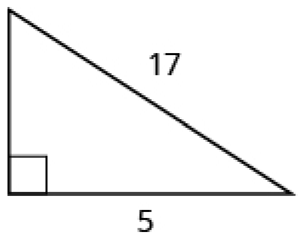
8. 

9. 

10. 

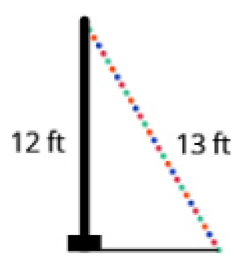


11.



12.

13. A 13-foot string of lights will be attached to the top of a 12-foot pole for a holiday display. How far from the base of the pole should the end of the string of lights be anchored?



14. Pam wants to put a banner across her garage door to congratulate her son on his college graduation. The garage door is 12 feet high and 16 feet wide. How long should the banner be to fit the garage door?

15. Chi is planning to put a path of paving stones through her flower garden. The flower garden is a square with sides of 10 feet. What will the length of the path be?

16. Brian borrowed a 20-foot extension ladder to paint his house. If he sets the base of the ladder 6 feet from the house, how far up will the top of the ladder reach?

ANSWERS

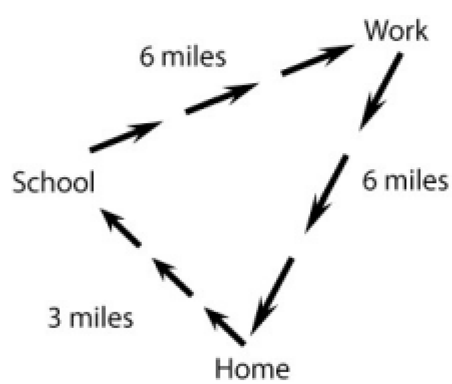
1. 12 feet

3. 15 feet
5. 25
7. 8
9. 12
11. 10.2
13. 5 feet
15. 14.1

Pythagorean Theorem Video https://youtu.be/6ge9tJzIibc

Perimeter

Perimeter is a one-dimensional measurement that is taken around the outside of a closed geometric shape. Let's start our discussion of the concept of perimeter with an example. Joseph does not own a car so he must ride the bus or walk everywhere he goes. On Mondays, he must go to school, to work, and back home again. His route is pictured below.



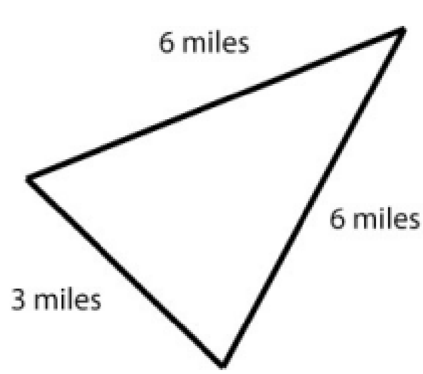
The obvious question to ask in this situation is, "how many miles does Joseph travel on Mondays"?

To compute, we each distance:

$$3 + 6 + 6 = 15$$

Joseph travels 15 miles on Mondays.

Another way to work with this situation is to draw a shape that represents Joseph's travel route and is labeled with the distance from one spot to another.



Notice that the shape made by Joseph's route is that of a closed geometric figure with three sides (a triangle). What we can ask about this shape is, "what is the perimeter of the triangle"?

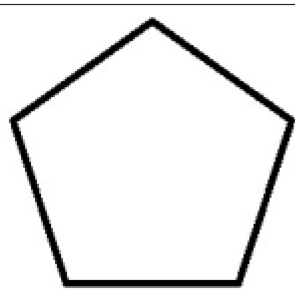
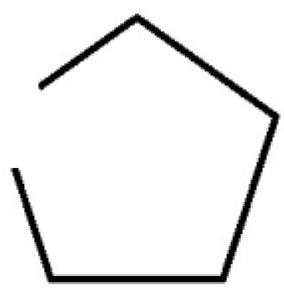
Perimeter means "distance around a closed

	figure or shape" and to compute we add each length: $3 + 6 + 6 = 15$ Our conclusion is the same as above. Joseph travels 15 miles on Mondays. However, what we did was model the situation with a geometric shape and then apply a specific geometric concept (perimeter) to computer how far Joseph traveled.

Notes on *Perimeter:*

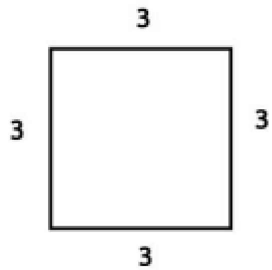
- *Perimeter* is a one-dimensional measurement that represents the distance around a closed geographic figure or shape (no gaps)
- To find *perimeter*, add the lengths of each side of the shape.
- If there are units, include units in your final result. Units will always be of single dimension (i.e., feet, inches, yards, centimeters, etc...)

To compute perimeter, our shapes must be closed. The images below show the difference between a closed figure and an open figure.

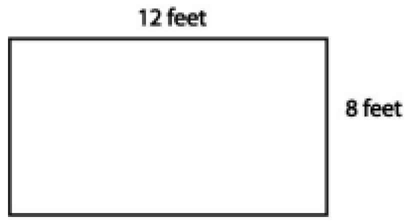
	
Close figure: There is an indide and an outside to the shape. To get from inside to outside, you must cross the boundary of the shape.	Open figure Not closed: There isn't an inside or outside. Even a portion that seems enclosed can be reached without crossing the boundary of the shape.

Example 1: Find the perimeter for each of the shapes below.

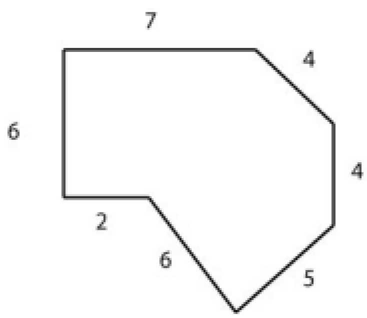
a. Add the lengths of each side.

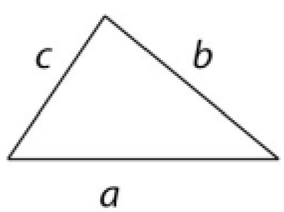
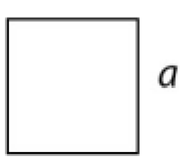


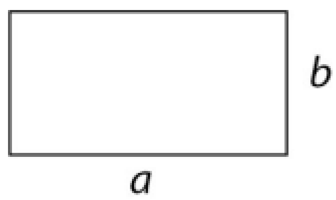
b. Sometimes you have to make assumptions if lengths are not labeled.



Example 2: How do we find the perimeter of this more complicated shape? Just keep adding those side lengths.



<table>
<tr><th>Shape</th><th>Perimeter</th></tr>
<tr><td>Triangle with side lengths a, b, c
</td><td>$P = a + b + c$</td></tr>
<tr><td>Square with side length a
</td><td>$P = a + a + a + a$
$P = 4a$</td></tr>
<tr><td>Rectangle with side lengths a, b</td><td>$P = a + b + a + b$</td></tr>
</table>

	$P = a + a + b + b$ $P = 2a + 2b$

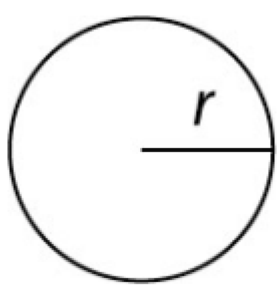
Circumference

You may realize that we have not yet discussed the distance around a very important geometric shape: a circle! The distance around a circle has a special name called the circumference. To find the circumference of a circle, we use the formula below:

$$C = 2\pi r$$

In this formula, π is pronounced “pi”, and is defined as the circumference of a circle divided by its diameter, $\pi = \frac{C}{d}$. We usually replace π with the approximation 3.14. The letter *r* represents the *radius* of the circle.

Let’s see where the formula for circumference comes from. Below is a generic circle with radius r.

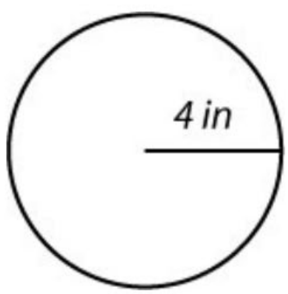
 Notes about $C = 2\pi r$ Note: Remember that in the formula, when computing the circumference $C = 2\pi r$, we multiply as follows USUALLY substituting 3.14 in place of π $C = 2 \cdot 3.14 \cdot r$	Origins of $C = 2\pi r$ As mentioned earlier, the special number π is defined as the ratio of a circle’s circumference to its diameter. We can write this in equation form as: $\frac{C}{d} = \pi$ We know from our previous work that to identify the unknown, C, we can move d to the other side of the equation by writing: $C = \pi d$

Often, the use of ( ) will help make the different parts of the formula easier to see: $C = (2)\cdot(3.14)\cdot(r)$	The diameter is all the way across the circle's middle so the diameter is twice the radius. We can update C in terms of the radius as: $C = \pi(2r)$ With a little final rearranging of the order our parts are written in, we can say that: $C = 2\pi r$

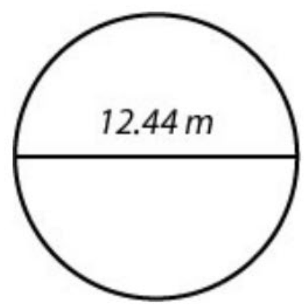
Let's use the formula to find the circumference of a few circles.

Example 3: Find the circumference of each of the following circles. Leave your answers first in exact form and then in rounded form (to the hundredths place). [Note that when a radius is given, its value is centered above a radius segment. When a diameter is given, its value is centered above a diameter segment.]

a.



b.



<u>Practice Problems:</u>

Find the circumference or perimeter given in each described situation below. Include a drawing of the shape with the included information. Use examples to help determine what shapes to draw. Show all work. As in the examples, if units are included then units should be present in your final result. Round to tenths unless indicated otherwise.

a. Find the perimeter of a square with side length 2.17 feet.

b. Find the perimeter of a rectangle with sides of length 4.2 and 3.8.

c. Find the perimeter of a triangle with sides of length 2, 5, 7.

d. Find the circumference of a circle with a radius of 6 inches. Present answer in exact form and also compute using 3.14 for π. Present rounded form to the nearest tenth.

e. Find the circumference of a circle with a diameter of 14.8 inches. Present answer in exact form and also compute using 3.14 for π. Present rounded form to the nearest tenth.

Answers:

a: 8.7 feet

b: 16

c: 14

d: Exact 12π in, Rounded 37.7 in

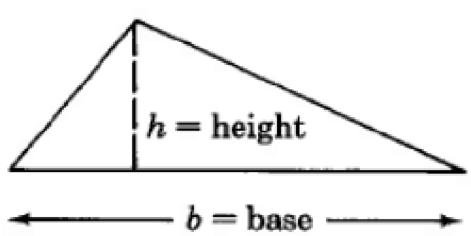
e: Exact 14.8π in, Rounded 46.5 in

Area

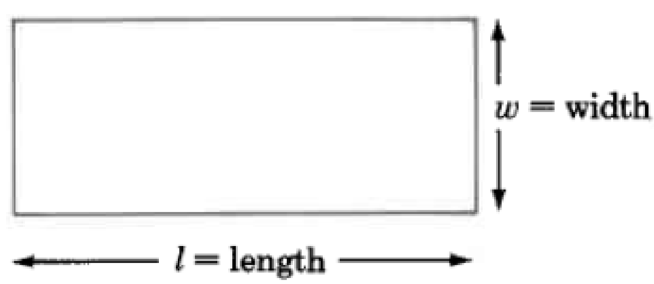
The area of a surface is the amount of square length units contained in the surface.

For example, 3 sq in. means that 3 squares, 1 inch on each side, can be placed precisely on some surface. (The squares may have to be cut and rearranged so they match the shape of the surface.)

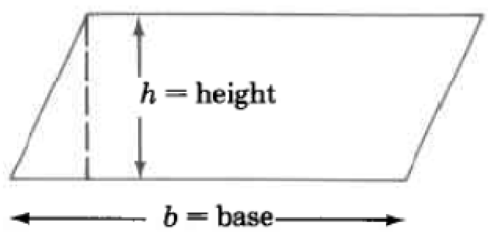
We will examine the area of the following geometric figures.



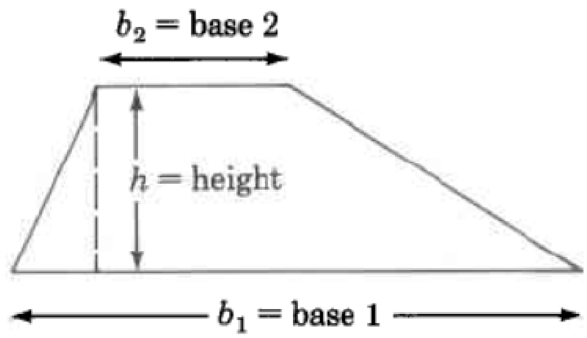
Triangles



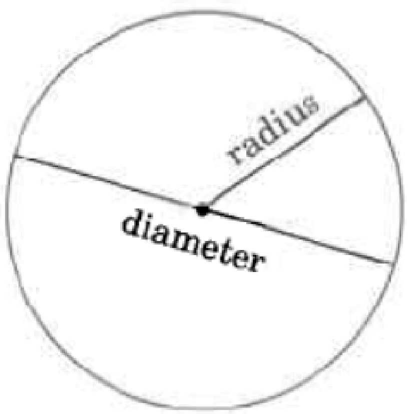
Rectangles



Parallelograms



Trapezoids

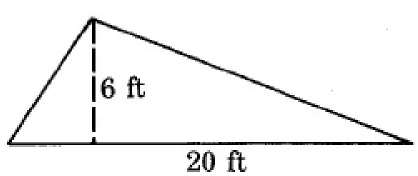


Circles

Finding Areas of Some Common Geometric Figures

Examples

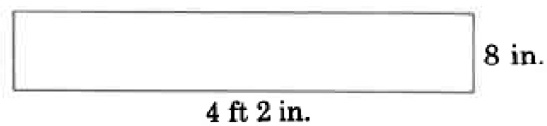
Find the area of the triangle.



$$\begin{aligned} A_T &= \tfrac{1}{2} \cdot b \cdot h \\ &= \tfrac{1}{2} \cdot 20 \cdot 6 \ \text{ sq ft} \\ &= 10 \cdot 6 \ \text{ sq ft} \\ &= 60 \text{ sq ft} \\ &= 60 \text{ ft}^2 \end{aligned}$$

The area of this triangle is 60 sq ft, which is often written as $60ft^2$.

Find the area of the rectangle.



Let's first convert 4 ft 2 in. to inches. Since we wish to convert to inches, we'll use the unit fraction $\frac{12\ in}{1ft}$ since it has inches in the numerator. Then,

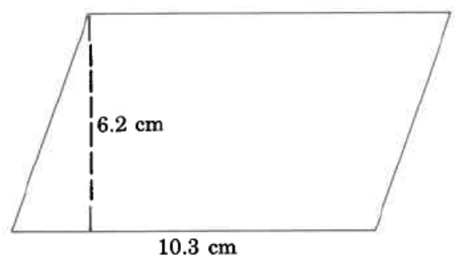
$$\begin{aligned} 4\text{ft} &= \frac{4 \text{ ft}}{1} \cdot \frac{12\text{in.}}{1 \text{ ft}} \\ &= \frac{4 \cancel{\text{ft}}}{1} \cdot \frac{12\text{in.}}{1 \cancel{\text{ft}}} \\ &= 48\text{in.} \end{aligned}$$

Thus, $4\ ft\ 2\ in. = 48\ in. + 2\ in. = 50\ i$

$$\begin{aligned} A_R &= l \cdot w \\ &= \ 50 \ \text{ in.} \cdot 8 \text{ in.} \\ &= 400 \text{ sq in.} \end{aligned}$$

The area of this rectangle is 400 sq in.

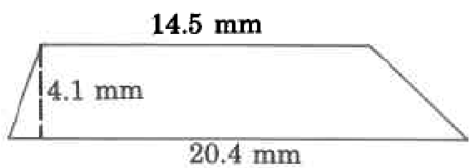
Find the area of the parallelogram.



$$\begin{aligned} A_P &= b \cdot h \\ &= 10.3 \text{ cm} \cdot 6.2 \text{ cm} \\ &= 63.86 \text{ sq cm} \end{aligned}$$

The area of this parallelogram is 63.86 sq cm.

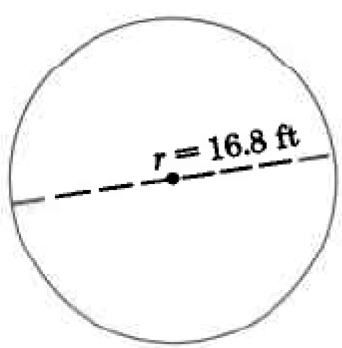
Find the area of the trapezoid.



$$\begin{aligned} A_{Trap} &= \tfrac{1}{2} \cdot (b_1 + b_2) \cdot h \\ &= \tfrac{1}{2} \cdot (14.5 \text{ mm}, +, 20.4 \text{ mm}) \cdot (4.1 \text{ mm}) \\ &= \tfrac{1}{2} \cdot (34.9 \text{ mm}) \cdot (4.1 \text{ mm}) \\ &= \tfrac{1}{2} \cdot (143.09 \text{ sq mm}) \\ &= 71.545 \text{ sq mm} \end{aligned}$$

The area of this trapezoid is 71.545 sq mm.

Find the approximate area of the circle.

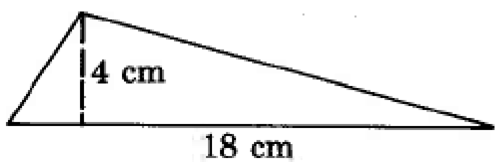


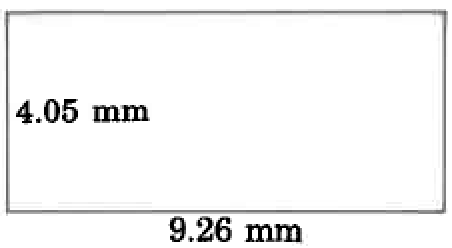
$$
\begin{aligned}
A_c &= \pi \cdot r^2 \\
&\approx (3.14) \cdot (16.8 \text{ ft})^2 \\
&\approx (3.14) \cdot (282.24 \text{ sq ft}) \\
&\approx 888.23 \text{ sq ft}
\end{aligned}
$$

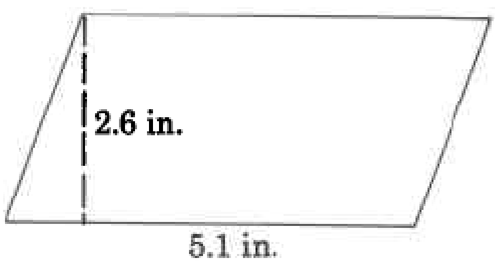
The area of this circle is approximately 886.23 sq ft.

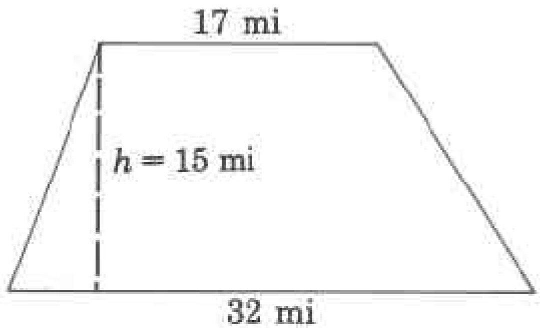
Practice Exercises

Find the area of each of the following geometric figures

1. 

2. 

3. 



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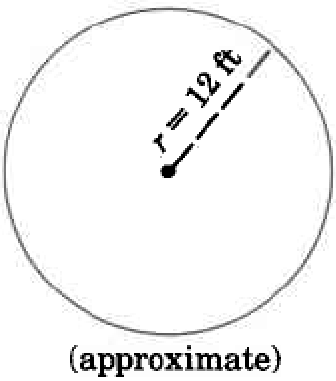
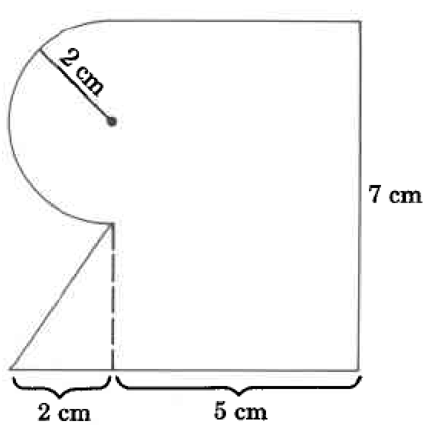


5.

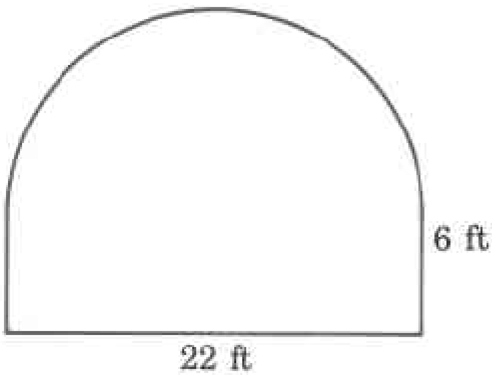
Answers

1. 36 sq cm
2. 37.503 sq mm
3. 13.26 sq in
4. 367.5 sq mi
5. 452.16 sq ft

Combined Area Exercises



1.



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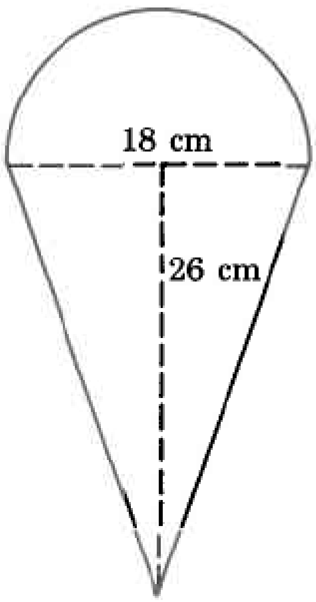

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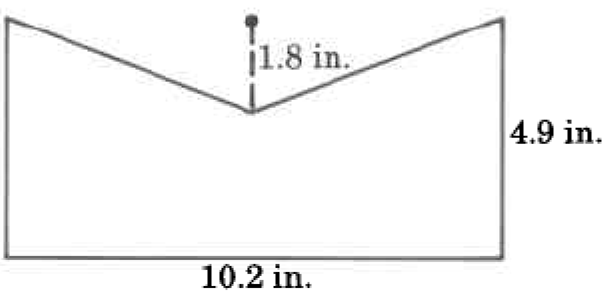

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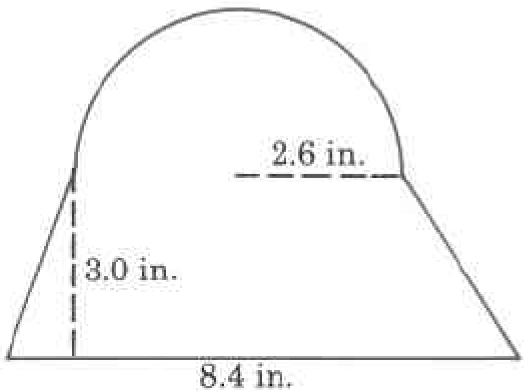
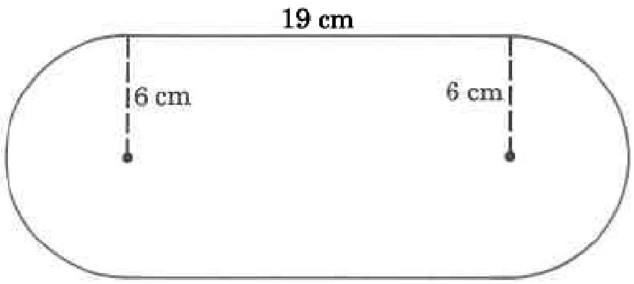
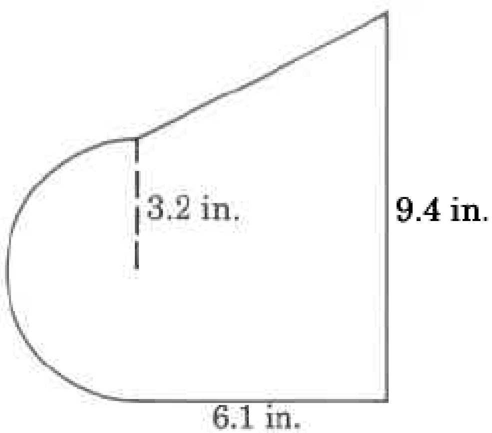
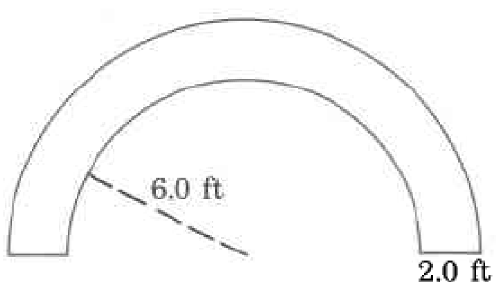
5. 

6. 

7. 

8. 

Answers

1. 44.28 sq cm
2. (60.5π +132) sq ft
3. 361.17 sq cm

4. 40.8 sq in
5. 31.0132 sq in
6. 341.04 sq cm
7. 64.2668 sq in
8. 43.96 sq ft

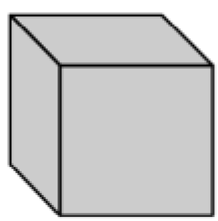
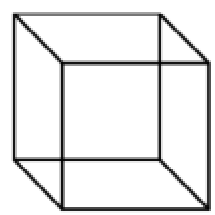
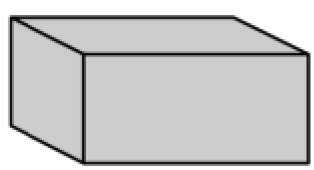
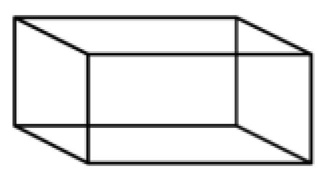
Volume of Solids

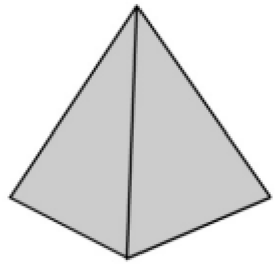
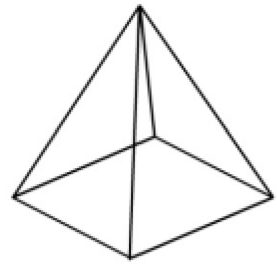
Living in a two-dimensional world would be pretty boring. Thankfully, all of the physical objects that you see and use every day—computers, phones, cars, shoes—exist in three dimensions. They all have length, width, and height. (Even very thin objects like a piece of paper are three-dimensional. The thickness of a piece of paper may be a fraction of a millimeter, but it does exist.)

In the world of geometry, it is common to see three-dimensional figures. In mathematics, a flat side of a three-dimensional figure is called a face. Polyhedrons are shapes that have four or more faces, each one being a polygon. These include cubes, prisms, and pyramids. Sometimes you may even see single figures that are composites of two of these figures. Let's take a look at some common polyhedrons.

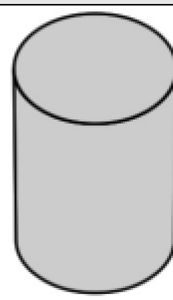
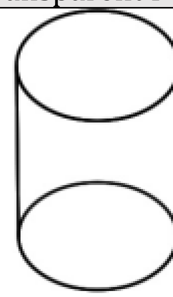
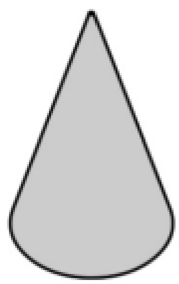
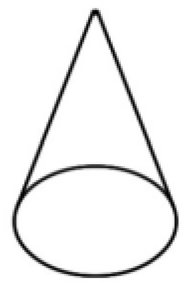
Identifying Solids

The first set of solids contains rectangular bases. Have a look at the table below, which shows each figure in both solid and transparent form:

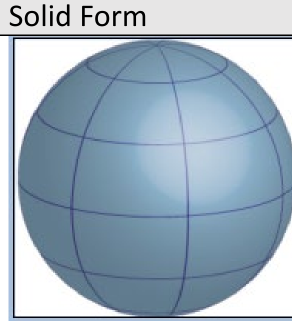
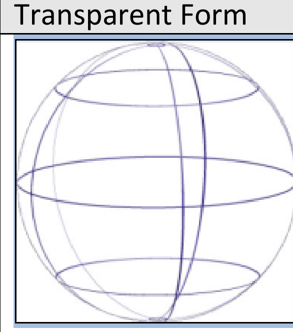
Name	Definition	Solid Form	Transparent Form
Cube	A six-sided polyhedron that has congruent squares as faces.		
Rectangular prism	A polyhedron that has three pairs of congruent, rectangular, parallel faces.		

Pyramid	A polyhedron with a polygonal base and a collection of triangular faces that meet at a point.		

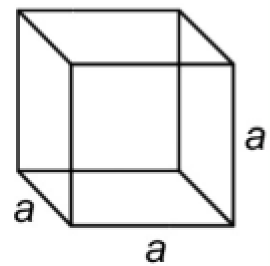
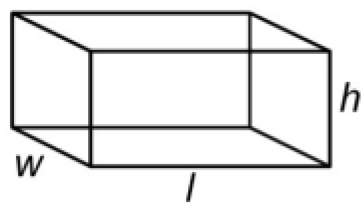
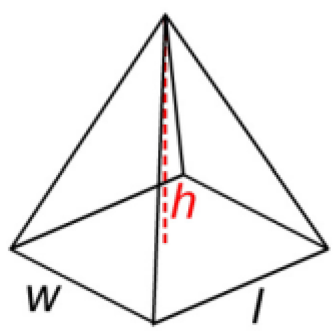
In this next set of solids, each figure has a circular base.

Name	Definition	Solid Form	Transparent Form
Cylinder	A solid figure with a pair of circular, parallel bases and a round, smooth face between them.		
Cone	A solid figure with a single circular base and a round, smooth face that diminishes to a single point.		

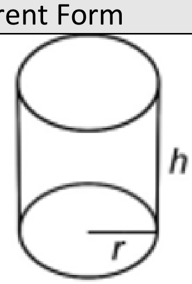
Finally, let's look at a shape that is unique: a sphere.

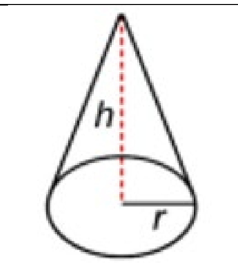
Name	Definition	Solid Form	Transparent Form
Sphere	A solid, round figure where every point on the surface is the same distance from the center.		

As you look through the list below, you may notice that some of the volume formulas look similar to their area formulas. To find the volume of a rectangular prism, you find the area of the base and then multiply that by the height.

Name	Transparent Form	Volume Formula
Cube		$V = a \cdot a \cdot a = a^3$ a = the length of side
Rectangular prism		$V = l \cdot w \cdot h$ l = Length w=width h=height
Pyramid		$V = \frac{l \cdot w \cdot h}{3}$ l = Length w=width h=height

Now let's look at solids that have a circular base.

Name	Transparent Form	Volume Formula
Cylinder		$V = \pi \cdot r^2 \cdot h$ r=radius h=height

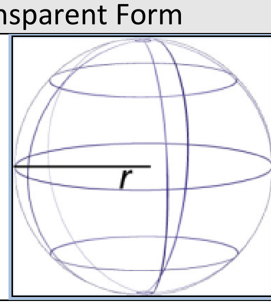
Cone		$V = \frac{\pi \cdot r^2 \cdot h}{3}$ r=radius h=height

Here you see the number π again.

The volume of a cylinder is the area of its base, πr^2 , times its height, h.

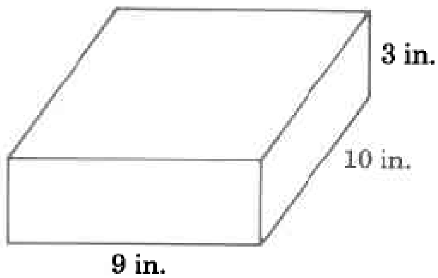
Compare the formula for the volume of a cone ($V = \frac{\pi \cdot r^2 \cdot h}{3}$) with the formula for the volume of a pyramid ($V = \frac{l \cdot w \cdot h}{3}$). The numerator of the cone formula is the volume formula for a cylinder, and the numerator of the pyramid formula is the volume formula for a rectangular prism. Then divide each by 3 to find the volume of the cone and the pyramid. Looking for patterns and similarities in the formulas can help you remember which formula refers to a given solid.

Finally, the formula for a sphere is provided below. Notice that the radius is cubed, not squared and that the quantity πr^3 is multiplied by $\frac{4}{3}$.

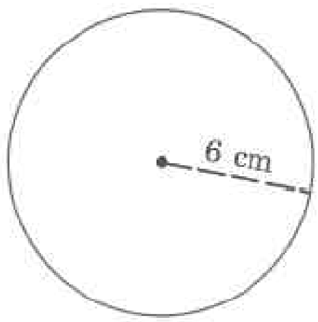
Name	Transparent Form	Volume Formula
Sphere		$V = \frac{4}{3}\pi r^2$ r=radius

Practice Exercises

Find the volume of each geometric object. If ππ is required, approximate it with 3.14 and find the approximate volume.



1.



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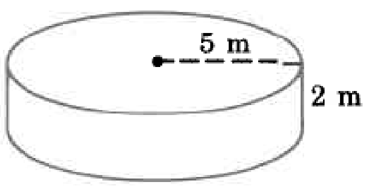

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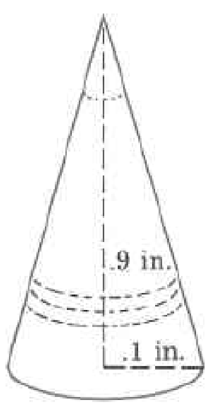

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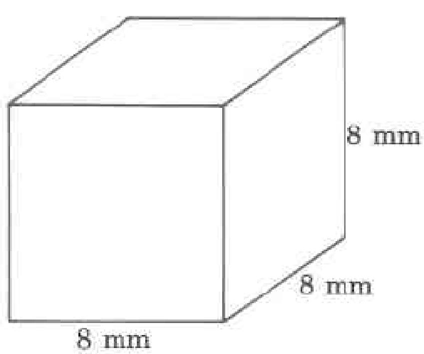

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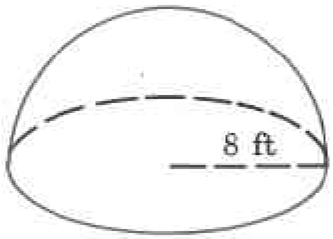
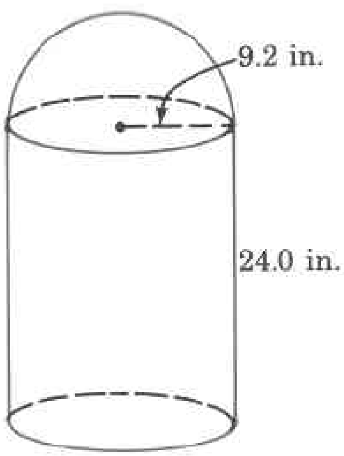


6.

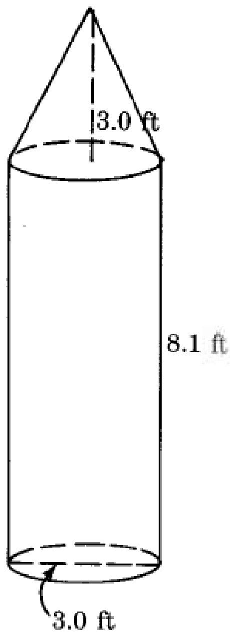
Answers

1. 21 in^3
2. 904.32 ft^3
3. 157 m^3
4. 0.00942 in^3
5. 512 cm
6. $\frac{1024}{3}\pi$ ft^3

Combined Volume Exercises



1.

2. 

Answers

1. 9,638.58 cu in
2. 63.585 sq ft

Surface Area

In this section, we will finish our study of geometry applications. We find the **surface area** of some three-dimensional figures. Since we will be solving applications, we will show our Problem-Solving Strategy for Geometry Applications.

Problem Solving Strategy for Geometry Applications

Step 1. **Read** the problem and make sure you understand all the words and ideas. Draw the figure and label it with the information given.

Step 2. **Identify** what you are looking for.

Step 3. **Name** what you are looking for. Choose a variable to represent that quantity.

Step 4. **Translate** into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.

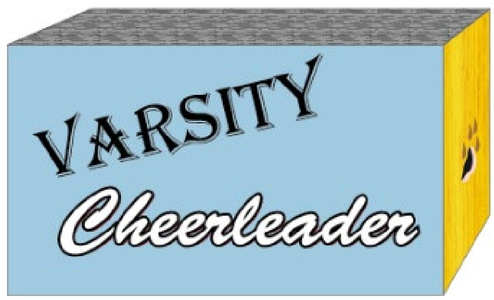
Step 5. **Solve** the equation using good algebra techniques.

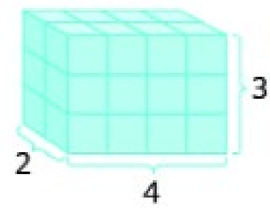
Step 6. **Check** the answer to the problem and make sure it makes sense.

Step 7. **Answer** the question with a complete sentence.

Finding Surface Area of Rectangular Solids

A cheerleading coach is having the squad paint wooden crates with the school colors to stand on at the games. The amount of paint needed to cover the outside of each box is the **surface area**, a square measure of the total area of all the sides. The amount of space inside the crate is the volume, a cubic measure.





To find the *surface area* of a rectangular solid, think about finding the area of each of its faces. How many faces does the rectangular solid above have? You can see three of them.

$Afront = L \times W$ $\quad$ $Aside = L \times W$ $\quad$ $Atop = L \times W$

$Afront = 4 \cdot 3$ $\quad$ $Aside = 2 \cdot 3$ $\quad$ $Atop = 4 \cdot 2$

$Afront = 12$ $\quad$ $Aside = 6$ $\quad$ $Atop = 8$

Notice for each of the three faces you see, there is an identical opposite face that does not show:

Solution:

$S =$ **(front + back) + (left side + right side) + (top + bottom)**

$S =$ **(2· front) + (2· left side) + (2· top)**

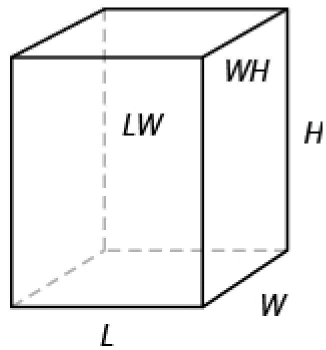
$S = 2 \cdot 12 + 2 \cdot 6 + 2 \cdot 8$

$S = 24 + 12 + 16$

$S =$ **52 sq. units**

In general, to find the surface area of a rectangular solid, remember that each face is a rectangle, so its area is the product of its length and its width. Find the area of each face that you see and then multiply each area by two to account for the face on the opposite side.

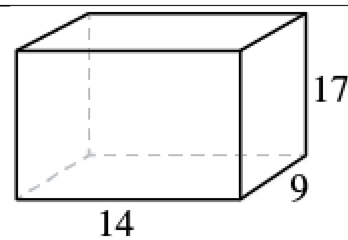
S=2LH+2LW+2WH



<u>Example 1:</u>

For a rectangular solid with length 14cm, height 17cm, and width 9cm, find the surface area.

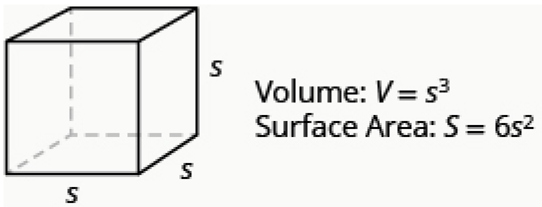
Solution:

Step 1. **Read** the problem. Draw the figure and label it with the given information.	
Step 2. **Identify** what you are looking for.	The surface area of the solid
Step 3. **Name**. Choose a variable to represent it.	Let S = surface area
Step 4. **Translate.** Write the appropriate formula. Substitute	$S = 2LH + 2LW + 2WH$ $S = 2(14 \cdot 17) + 2(14 \cdot 9) + 2(9 \cdot 17)$
Step 5. **Solve the equation.**	S **= 1,034**
Step 6. **Check:** Double-check with a calculator	
Step 7. **Answer** the question.	The surface area is **1,034** square centimeters.

Surface Area of a Cube

A **cube** is a rectangular solid whose length, width, and height are equal. Substituting, *s* for the length, width and height into the formulas for surface area of a rectangular solid, we get:

For any cube with sides of length s,



Solution:

$S= 2LH + 2LW + 2WH$

$S= 2s \cdot s + 2s \cdot s + 2s \cdot s$

$S = 2s^2 + 2s^2 + 2s^2$

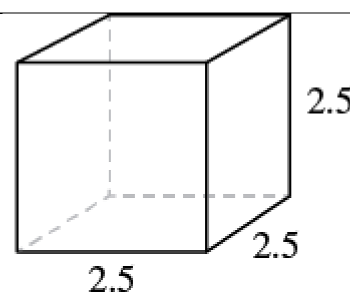
$S= 6s^2$

So, for a cube, the formulas for surface area S**$= 6s^2$**

Example 2:

A cube is 2.5 inches on each side. Find its surface area.

Solution:

Step 1. **Read** the problem. Draw the figure and label it with the given information.	
Step 2. **Identify** what you are looking for.	The surface area of the cube
Step 3. **Name.** Choose a variable to represent it.	Let S= surface area
Step 4. **Translate.**	**$S=6s^2$**

Write the appropriate formula.	
Step 5. **Solve.** Substitute and solve.	**$S= 6\bullet (2.5)^2$** **$S= 37.5$**
Step 6. **Check:** The check is left to you.	
Step 7. **Answer** the question	The surface area is **37.5** square inches.

Surface Area of a Sphere

A **sphere** is the shape of a basketball, like a three-dimensional circle. Just like a circle, the size of a sphere is determined by its radius, which is the distance from the center of the sphere to any point on its surface. The formula for surface area of a sphere is given below.

Showing where these formulas come from, like we did for a rectangular solid, is beyond the scope of this course. We will approximate π with 3.14.

VOLUME AND SURFACE AREA OF A SPHERE

For a sphere with radius r:

Example 3:

A sphere has a radius of 6 inches. Find its surface area.

Solution:

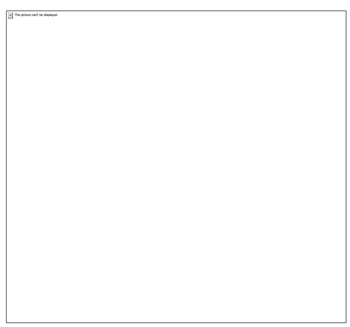
Step 1. **Read** the problem. Draw the figure and label it with the given information.	
Step 2. **Identify** what you are looking for.	The surface area of a cube
Step 3. **Name**. Choose a variable to represent it.	Let S= surface area

Step 4. **Translate.** Write the appropriate formula.	$S = 4\pi r^2$
Step 5. **Solve**	$S = 4(3.14)6^2$ $S = 452.16$
Step 6. **Check**: Double-check your math on a calculator	
Step 7. **Answer** the question	The surface area is approximately 452.16 square inches.

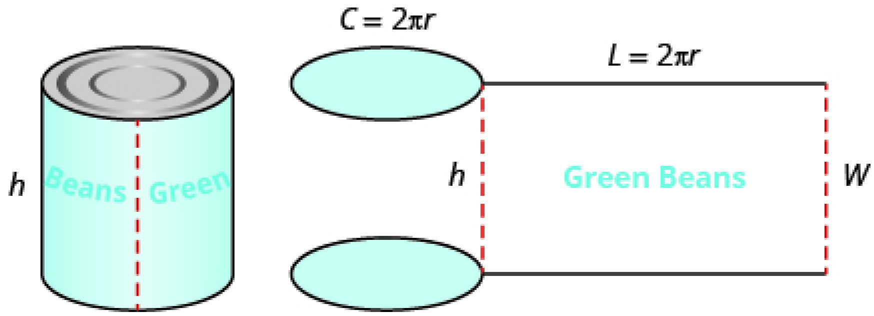
Surface Area of a Cylinder

If you have ever seen a can of soda, you know what a cylinder looks like. A **cylinder** is a solid figure with two parallel circles of the same size at the top and bottom. The top and bottom of a cylinder are called the bases. The height of a cylinder is the distance between the two bases. For all the cylinders we will work with here, the sides and the height, h, will be perpendicular to the bases.

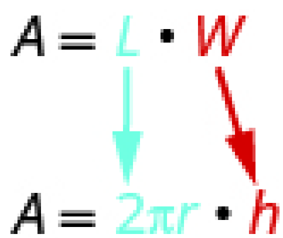
A right circular cylinder with radius *r* and height *h*



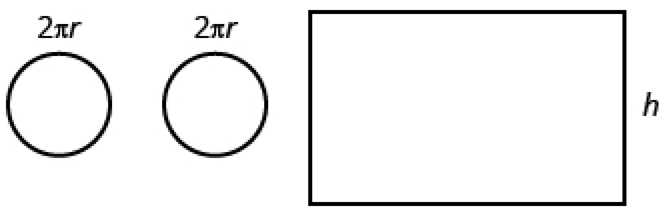
To understand the formula for the surface area of a cylinder, think of a can of vegetables. It has three surfaces: the top, the bottom, and the piece that forms the sides of the can. If you carefully cut the label off the side of the can and unroll it, you will see that it is a rectangle.



The distance around the edge of the can is the **circumference** of the cylinder's base it is also the length Lof the rectangular label. The height of the cylinder is the width W of the rectangular label. So, the area of the label can be represented as



To find the total surface area of the cylinder, we add the areas of the two circles to the area of the rectangle.



$$S = A_{\text{top circle}} + A_{\text{bottom circle}} + A_{\text{rectangle}}$$

$$S = \underbrace{\pi r^2 + \pi r^2} + 2\pi r \cdot h$$

$$S = 2 \cdot \pi r^2 + 2\pi rh$$

$$S = 2\pi r^2 + 2\pi rh$$

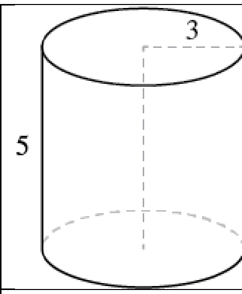
The surface area of a cylinder with radius r and height h, is

$$\boldsymbol{S = 2\pi r^2 + 2\pi rh}$$

Example 4:

A cylinder has a height of 5 centimeters and radius 3 centimeters. Find the surface area.

Solution

Step 1. **Read** the problem. Draw the figure and label it with the given information.	
Step 2. **Identify** what you are looking for	The surface area of the cylinder
Step 3. **Name**. Choose a variable to represent it	Let S= surface area
Step 4. **Translate** Write the appropriate formula. Substitute. (Use 3.14 for π.	$S = 2\pi r^2 + 2\pi rh$ $S = 2(3.14)3^2 + 2(3.14)(3)5$
Step 5. **Solve**	$S = 150.72$
Step 6. **Check**: We leave it to you to check your calculations.	
Step 7. **Answer** the question.	The surface area is approximately 150.72 square inches.

Practice Exercises:

Find Surface area of Rectangular Solids

In the following exercises, find the surface area of the rectangular solid with the given dimensions.

1. length 2 meters, width 1.5 meters, height 3 meters

2. length 5 feet, width 8 feet, height 2.5 feet

3. length 3.5 yards, width 2.1 yards, height 2.4 yards

4. length 8.8 centimeters, width 6.5 centimeters, height 4.2 centimeters

In the following exercises, solve.

5. A rectangular moving van has length 16 feet, width 8 feet, and height 8 feet. Find its surface area.

6. A rectangular gift box has length 26 inches, width 16 inches, and height 4 inches. Find its surface area.

7. A rectangular carton has length 21.3 cm, width 24.2 cm, and height 6.5 cm. Find its surface area.

8. A rectangular shipping container has length 22.8 feet, width 8.5 feet, and height 8.2 feet. Find its surface area.

In the following exercises, find ⓐ the volume and ⓑ the surface area of the cube with the given side length.

9. 5 centimeters

10. 6 inches

11. 10.4 feet

12. 12.5 meters

In the following exercises, solve.

13. Each side of the cube at the Discovery Science Center in Santa Ana is 64 feet long. Find its surface area.

14. A cube-shaped museum has sides 45 meters long. Find its surface area.

15. The base of a statue is a cube with sides 2.8 meters long. Find its surface area.

16. A box of tissues is a cube with sides 4.5 inches long. Find its surface area.

Find Surface Area of Spheres

In the following exercises, find the surface area of the sphere with the given radius. Round answers to the nearest hundredth.

17. 3 centimeters

18. 9 inches

19. 7.5 feet

20. 2.1 yards

In the following exercises, solve. Round answers to the nearest hundredth.

21. An exercise ball has a radius of 15 inches. Find its surface area.

22. The Great Park Balloon is a big orange sphere with a radius of 36 feet. Find its surface area.

23. A golf ball has a radius of 4.5 centimeters. Find its surface area.

24. A baseball has a radius of 2.9 inches. Find its surface area.

Find the Surface Area of a Cylinder

In the following exercises, find the surface area of the cylinder with the given radius and height. Round answers to the nearest hundredth.

25. radius 3 feet, height 9 feet

26. radius 5 centimeters, height 15 centimeters

27. radius 1.5 meters, height 4.2 meters

28. radius 1.3 yards, height 2.8 yards

In the following exercises, solve. Round answers to the nearest hundredth.

29. A can of coffee has a radius of 5 cm and a height of 13 cm. Find its surface area.

30. A snack pack of cookies is shaped like a cylinder with radius 4 cm and height 3 cm. Find its surface area.

31. A cylindrical barber shop pole has a diameter of 6 inches and height of 24 inches. Find its surface area.

32. A cylindrical column has a diameter of 8 feet and a height of 28 feet. Find its surface area.

<u>Exercise Answers</u>

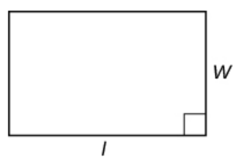
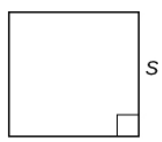
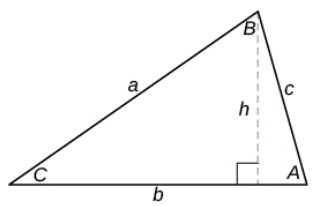
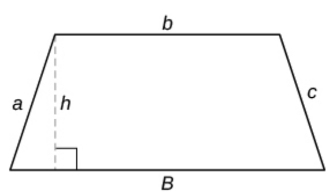
1) 27 sq. m
3) 41.58 sq. yd.
5) 640 sq. ft
7) 1,622.42 sq. cm
9) 150 sq. cm
11) 648.96 sq. ft
13) 24,576 sq. ft
15) 47.04 sq. m
17) 113.04 sq. cm
19) 706.5 sq. ft
21) 2,826 sq. in.

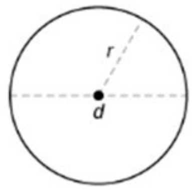
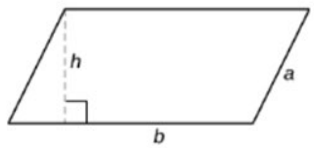
23) 254.34 sq. cm
25) 226.08 sq. ft
27) 53.694 sq. m
29) 565.2 sq. cm
31) 508.68 sq. in

Additional Geometry Word Problems https://youtu.be/GGw7cS7Csy8

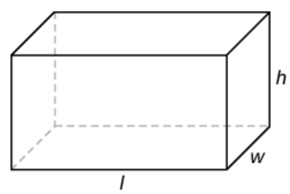
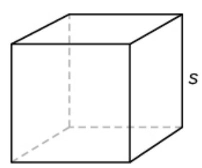
GEOMETRY FORMULAS

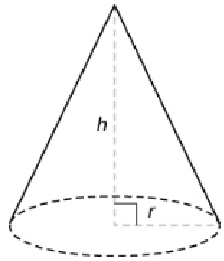
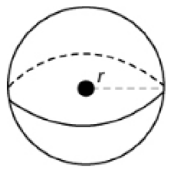
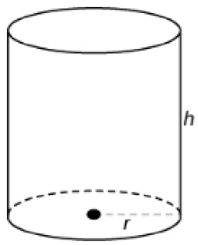
Area Formulas:

Name	Shape	Formulas
Rectangle		Perimeter: $P = 2l + 2w$ Area: $A = lw$
Square		Perimeter: $P = 4s$ Area: $A = s^2$
Triangle		Perimeter: $P = a + b + c$ Area: $A = \frac{1}{2}bh$ Sum of Angles: $A + B + C = 180^\circ$
Trapezoid		Perimeter: $P = a + b + c + B$ Area: $A = \frac{1}{2}(B + b)h$

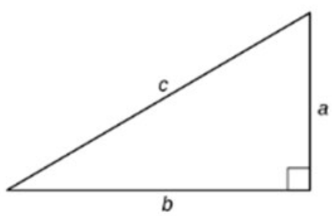
Circle		Circumference: $C = 2\pi r$ or $C = \pi d$ Area: $A = \pi r^2$
Parellelogram		Perimeter: $P = 2a + 2b$ Area: $A = bh$

Volume Formulas:

Name	Shape	Formulas
Rectangular Solid		Volume: $V = lwh$ Surface Area: $SA = 2lw + 2wh + 2hl$
Cube		Volume: $V = s^3$ Surface Area: $SA = 6s^2$

Cone		Volume: $V=\frac{1}{3}\pi r^2 h$ Surface Area: $SA=\pi r^2+\pi r\sqrt{h^2+r^2}$
Sphere		Volume: $V=\frac{4}{3}\pi r^3$ Surface Area: $SA=4\pi r^2$
Right Circular Cylinder		Volume: $V=\pi r^2 h$ Surface Area: $SA=2\pi r^2+2\pi rh$

Pythagorean Theorem

Right Triangle		Pythagorean Theorem: $a^2+b^2=c^2$ Area: $A=\frac{1}{2}ab$

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Chapter 9: Data Analysis

Introduction

In this lesson, we will learn the basic language and concepts related to a branch of mathematics that deals with collecting, organizing, and interpreting data. This branch of mathematics is called statistics. In addition, the word statistics is often used to denote the data and information that are being collected and interpreted.

KEY TERMS

The key terms listed below will help you keep track of important mathematical words and phrases that are part of this lesson.

- Average
- Measures of Central Tendency
- Mean
- Median
- Mode
- Weighted Average
- Outlier
- Range
- Variation

Measures of central tendency

When we are given a set of data points, particularly if that set is very large, we want to get a feel for the data by getting a sense of what single number most accurately represents that data. To do that, we compute one or more of the following Measures of Central Tendency or Averages.

- ***Mean*** is the sum of a set of values divided by the number of values.

$$mean = \frac{sum\ of\ all\ values}{total\ number\ of\ values}$$

- ***Median*** is the number in the middle of a set of numbers arranged in numerical order. If there are two numbers in the middle (i.e., an even number in the set) then find the mean of just the two numbers in the middle.

• ***Mode*** is the number (or numbers) that occurs most frequently in the set. If no number or numbers occur more than once, there is no mode.

Note that all of the above are numerical definitions of "average" for a given data set. However, each is computed differently and will often give different results. When the word "average" is utilized within our daily lives it is most often associated with the mean. Do not assume that the mean is the only average of a set of values.

Example 1: Find the mean, median, and mode of the following data sets. Begin by writing the set in increasing order.

a. 5, 1, 4, 5, 3, 1, 5

b. 6 0 6 3 2 2 6 2

Example 2: Find the mean, median, and mode of the data set 5 2 7 11 6 0 3 3. Begin by writing the data set in increasing order.

Mean Median Mode

Weighted average

A weighted average (which is another kind of mean) is used when some values in the number set count more heavily than others. The following examples illustrate this idea.

Example 3: A given Biology class contains 20 students. The 8 female students in the class are enrolled in an average of 14 semester credits. The 12 male students are enrolled in an average of 8 semester credits. Compute the average number of semester credits for the class as a whole. [To begin, circle the GIVENS and underline the GOAL].

Example 4: Grade point average is a classic example of a weighted average. Last term, a student's grades were as indicated in the table below. Compute the student's GPA for the term.

Course	Credits	Grade	Grade Pts	Grade Pt. Totals
Philosophy	3	C		
English	3	B		
P.E.	1	A		
Biology	5	B		
Total				

Example 5: Compute the student's GPA for the term.

Course	Credits	Grade
MATH082	3	A

ENG071	4	B
PSY100	3	C
RDG061	3	A

Variation, range, & outliers

Measures of Central Tendency are concerned with finding the most accurate center point or representative point for a given data set. If we want to understand how spread out the data are, then we need to look at the **variation** in the given data.

Example 6: Order the following from least to most variation.

a. The weights of all adults

b. The weights of all adult women

c. The weights of all 20-year-olds

d. The weights of all 20-year-old women

- **Range** is the difference between the largest and smallest value in the set and provides the most information about how spread out the data is. Be sure to write the data set in order before computing the range.

$$\text{Range} = \text{Highest Value} - \text{Lowest Value}$$

Example 7: Determine the range of the following data set: 24, 32, 12, 14, 3, 7, 12, 43, 1, 5

Example 8: Find the range of the following data set 5 2 7 11 6 0 3 3. Start by writing the data set in increasing order.

Example 9: Find the mean, median, mode, and range for the following data sets.

a. 2, 2, 3, 5, 6

b. 2, 2, 3, 5, 20

- **Outliers** are values that are far removed from the other values in a data set. In the above example, data set b has an outlier of 20. Notice how the measures of central tendency and variability are impacted.

Practice Activity

1. Determine the mean, median, mode, and range of the following data sets. Show all of your work. Round to two decimals places as needed.

	Data	Mean	Median	Mode	Range
a.	4, 15, 3, 8, 3, 6, 15, 5, 17				
b.	4, 3, 4, 5, 2, 4, 25				
c.	5, 9,7, 2, 3, 32, 8, 6				
d.	5, 2, 1, 12, 10, 8, 9, 7				
e.	1, 3, 1, 4, 1, 5, 8, 6, 7, 9				

2. Determine the mean, median, mode, and range of the following data sets. Show all your work. Round to two decimals places as needed.

	Data	Mean	Median	Mode	Range
a.	8,7,6,7,5,3,9				
b.	4,2,5,7,2,3,6,6				

3. Answer True or False for each of the following. If your answer is False, provide an example that proves your point. If your answer is true, explain. Given a typical set of numerical data with an odd number of values:

a. T or F: The mean is always one of the data values.

b. T or F: The median is always one of the data values.

c. T or F: The mode is always one of the data values.

d. T or F: The range measures the variability of the given data set.

e. T or F: The mean is always the best measure of central tendency to use.

4. Compute the following weighted average. You may need to add information to the given table to help you make the correct computations.

Over a given time period, a convenience store had visits from delivery trucks in the following categories with the indicated charge per delivery. What is the average delivery charging the store pays each week?

Category	Deliveries per week	Per Delivery Charge
Snacks	3	$25.00
Alcohol	2	$100.00
Dairy	4	$75.00

5. Compute the following weighted average. You may need to add information to the given table to help you make the correct computations.

Danielle has started her own exercise company. She charges varying amounts for different classes (charged per month) as shown in the table below. Determine the average charge per person using the weighted average.

Category	Number of People	Dollars per person
Yoga	13	$17.00
Aerobics	24	$19.00
Weight Training	7	$26.00

Attributions:

Chapter 10: Tables and Graphs

Tables and Graphs are often used to display and organize data as illustrated in the examples below. Look for a legend or headers to understand what the different parts of the table or graph represent.

Example 1: A table presents information in rows and columns as shown in this example.

Country	Birth Rate (Per 1000 population per year)	Population
French	15.53	294,935
Brazil	17.79	203,429,800
Austria	12.33	21,766,710
Sudan	36.12	45,047,500
Russia	11.05	138,739,900
India	20.97	1,189,173,000
Bulgaria	9.32	7,093,635

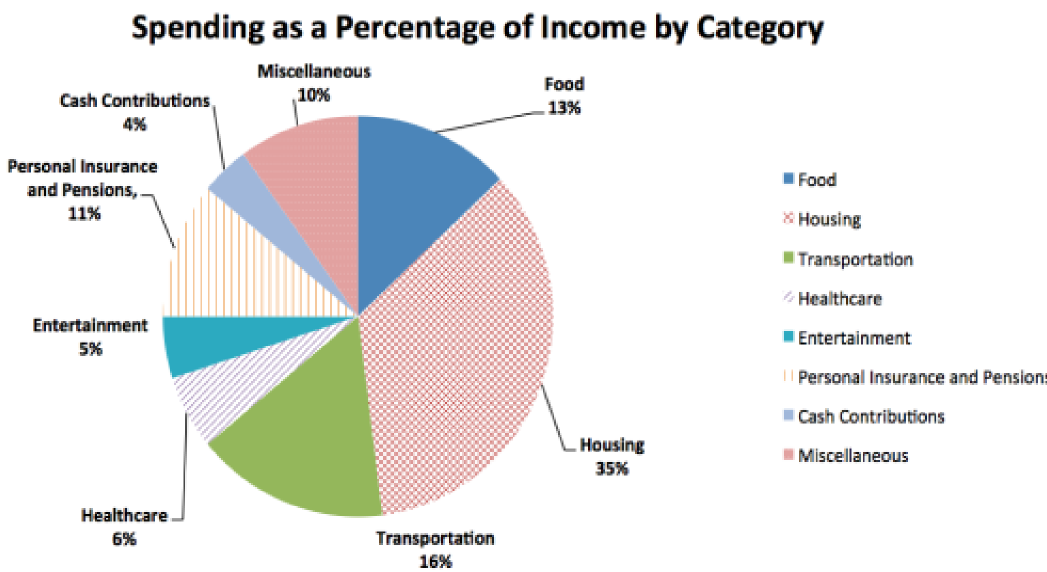
Which country has a birth rate of 17.79? ___________________________

Which country has the smallest birth rate? ___________________________

Which country has the largest population? __________________________

Circle Graphs

Example 2: A Circle Graph (also called a Pie Graph) is used to show how the whole amount is broken up into parts. *(Source: Consumer Expenditure Survey, U.S. Bureau of Labor Statistics, October, 2010)*



Legend

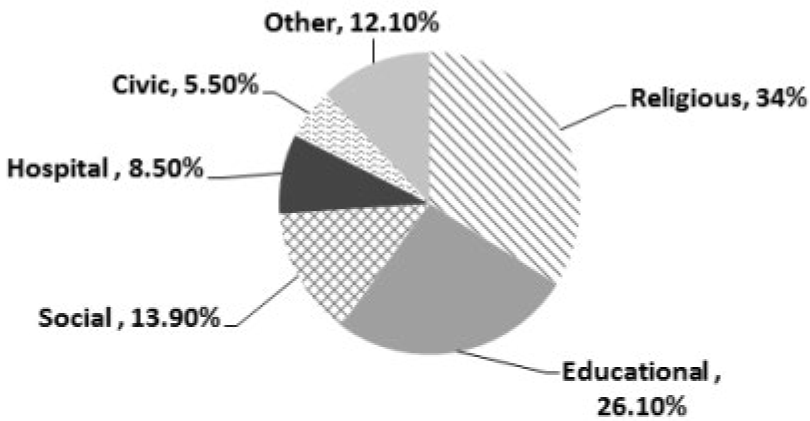
- Food 13%
- Housing 35%
- Transportation 16%
- Healthcare 6%
- Entertainment 5%
- Personal Insurance and Pensions 11%
- Cash Contributions 4%
- Miscellaneous 10%

a. How much of their income does the average American spend on healthcare? ______

b. For the average person, what is the single biggest category of expense? __________

c. Suppose your monthly salary is $2200. How much should you be spending on Food?

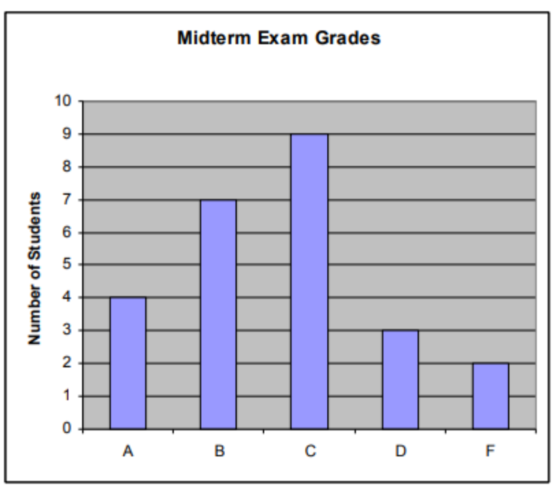
Example 3: In 2009, the Bureau of Labor Statistics reported a surge in volunteerism. At this time, there were a reported 63,361volunteers in the U.S. The pie chart below shows the different categories in which these people volunteered.



Find the number of people who volunteered in an educational capacity. Round your answer to the nearest whole number.

Bar Graph

Example 4:



a. How many students made a B on the Midterm?

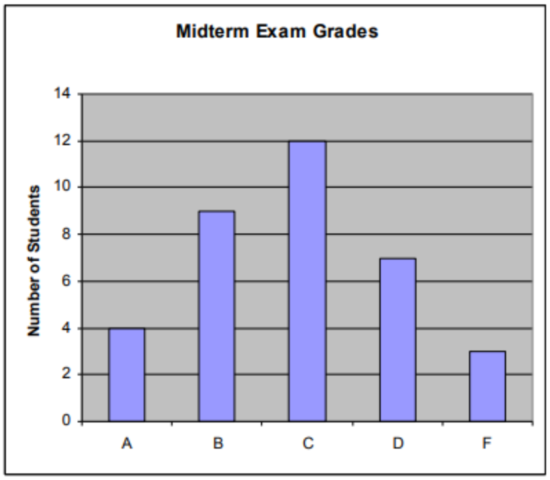
b. How many students were in the class?

c. What percentage of the class made a B on the midterm? Round to hundredths.

d. What percentage of students made a passing grade (A, B, or C) on the Midterm?

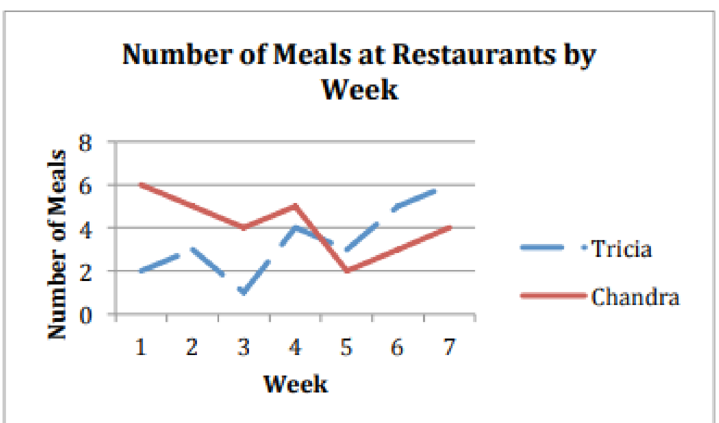
	Round to hundredths.

Example 5:

	a. How many students made a B on the Midterm? b. How many students were in the class? c. What percentage of the class made a B on the midterm? Round to hundredths. d. What percentage of students made a passing grade (A, B, or C) on the Midterm? Round to hundredths.

Practice Activity

1. Tricia and Chandra love to go to restaurants but want to save money by eating at home. The double line graph below shows how many meals they ate at restaurants per week for a 7-week time period.



a. How many meals did Tricia eat at restaurants during this 7-week time period?

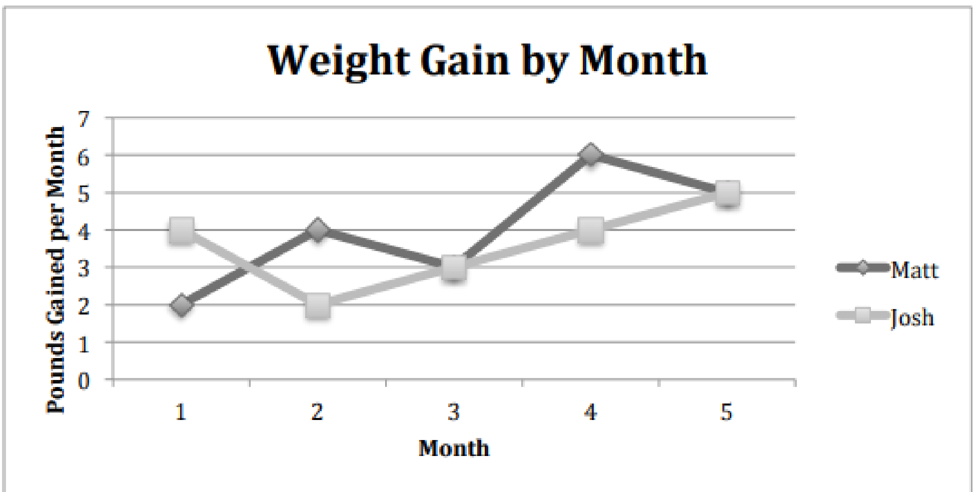
b. What was Tricia's rate of meals at restaurants per week for the given seven weeks (i.e., compute a unit rate from the given data). Round to two decimals as needed.

c. How many meals did Chandra eat at restaurants during this 7-week time period?

d. What was Chandra's rate of meals at restaurants per week for the given seven weeks (i.e., compute a unit rate from the given data). Round two decimals as needed.

e. During which week(s) did Tricia eat more meals at restaurants than Chandra?

2. Matt and Josh are wrestlers and begin a diet to gain weight for the season. The line graph below shows the pounds they gained per month over a 5-month period.



a. How much weight did Matt gain over the 5-month time period?

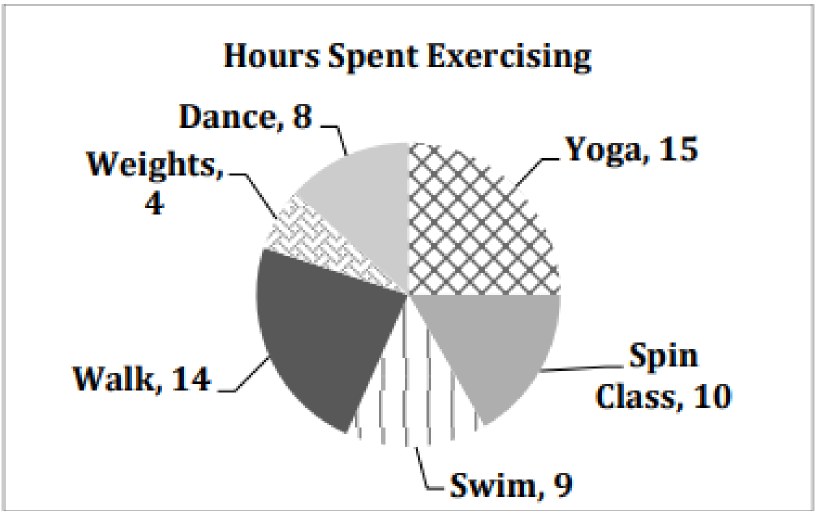
b. What was Matt's rate of weight gain for the given five months (i.e., compute a unit rate from the given data). Round two decimals as needed.

c. How much weight did Josh gain over the 5-month time period?

d. What was Josh's rate of weight gain for the given five months (i.e., compute a unit rate from the given data). Round two decimals as needed.

e. During which month(s) did Josh gain more weight than Matt?

8. The graph below displays the number of hours per month that Amber spends in varying exercise activities. Complete each item in the table below including the information in the Total row. DO NOT reduce your fraction answers to lowest terms other than in the Total Fraction of Budget cell. When you are finished, answer the questions below the table.



Category	Amount	Fraction of Exercise	Percent of Exercise
Dance			
Weights			
Walk			
Spin Class			
Swim			
Yoga			
Total			

a. Given the number of hours spent in one month doing Yoga, how many hours would Amber spend on yoga in 12 months?

b. What do you notice about the totals in the Fraction of Budget column and the Percent of Budget column?

c. Reduce all your items in the Fraction of Budget column and then add them together. Do you get the same final, simplified result that you did in the table?

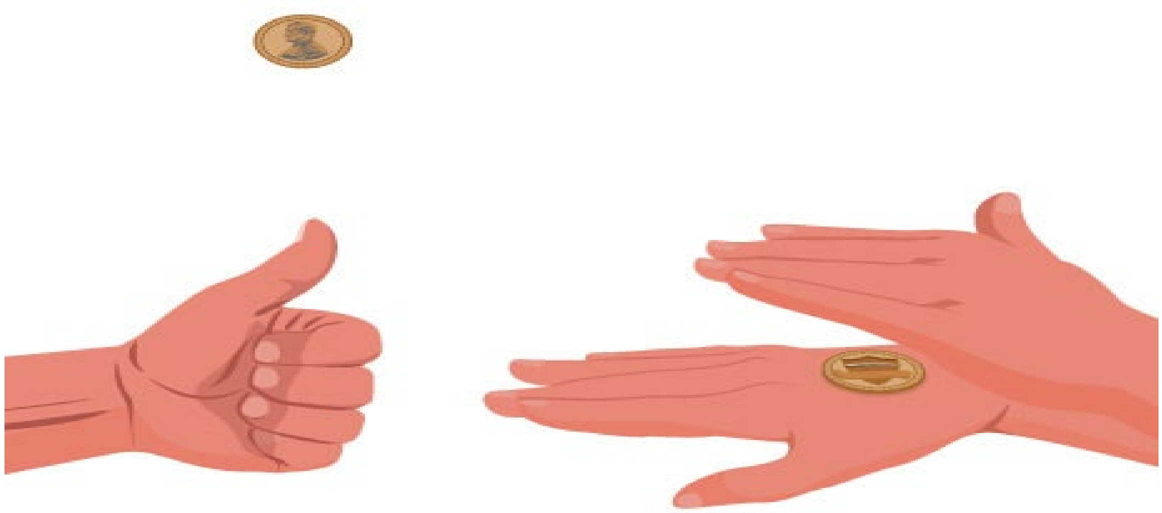
Reading Graphs Video https://youtu.be/C0-eJfOg3nc

Attribution:

Chapter 11: Probability

Probability

Probability is the likelihood, or chance, that a certain event will occur. The easiest way to understand probability is with coin tosses (see the **Figure** below). When you toss a coin, the chance of a head turning up is 50 percent. This is because a coin has only two sides, so there is an equal chance of a head or tail turning up on any given toss.



Tossing a Coin. Competitions often begin with the toss of a coin. Why is this a fair way to decide who goes first? If you choose heads, what is the chance that the toss will go your way?

If you toss a coin twice, you might expect to get one head and one tail. But each time you toss the coin, the chance of a head is still 50 percent. Therefore, it's quite likely that you will get two or even several heads (or tails) in a row. What if you tossed a coin ten times? You would probably get more or less than the expected five heads. For example, you might get seven heads (70 percent) and three tails (30 percent). The more times you toss the coin, however, the closer you will get to 50 percent heads. For example, if you toss a coin 1000 times, you might get 510 heads and 490 tails.

Exercise:

Choose the best answer.

1)If you toss a coin twice, what is the probability that you will get heads the second time?

a) 50%
b) 25%
c) 75%
d) None of the above

2)Which of the following factors can increase the likelihood of getting the expected outcome when tossing a coin?

a) Increasing the amount of times, you toss the coin
b) Starting with the same side of the coin each time
c) Starting with opposite sides of the coin each time
d) None of the above

Answers

1)a
2)a

Basic Concepts

If you roll a die, pick a card from deck of playing cards, or randomly select a person and observe their hair color, we are executing an experiment or procedure. In probability, we look at the likelihood of different outcomes. We begin with some terminology.

Events and Outcomes

- The result of an experiment is called an **outcome.**
- An **event** is any particular outcome of a group of outcomes.
- A **simple event** is an event that cannot be broken down further.
- The **sample space** is the set of all possible simple events.

Example 1:

If we roll a standard 6-sided die, describe the sample space and some simple events. The sample space is the set of all possible simple events: {1, 2, 3, 4, 5, 6}

Some examples of simple events:

We roll a 1

We roll a 5

Some compound events:

We roll a number bigger than 4

We roll an even number

Basic Probability

Given that all outcomes are equally likely, we can compute the probability of an event E using this formula:

$$P(E) = \frac{Number\ of\ outcomes\ corresponding\ to\ the\ event\ E}{Total\ number\ of\ equally - likely\ outcomes}$$

Example 2:

If we roll a 6-sided die, calculate

a) P (rolling a 1)
b) P (rolling a number bigger than 4)

Recall that the sample space is {1, 2, 3, 4, 5, 6}

a) There is one outcome corresponding to "rolling a 1", so the probability is $\frac{1}{6}$
b) There are two outcom]es bigger than a 4, so the probability is $\frac{2}{6} = \frac{1}{3}$

Probabilities are essentially fractions and can be reduced to lower terms like fractions.

Example 3:

Let's say you have a bag with 20 cherries, 14 sweet and 6 sour. If you pick a cherry at random, what is the probability that it will be sweet?

There are 20 cherries that could be picked, so the number of possible outcomes is 20. Of these 20 possible outcomes, 14 are favorable (sweet), so the probability that the cherry will be sweet is $\frac{14}{20} = \frac{7}{10}$

There is one potential complication to this example, however. It must be assumed that the probability of picking any of the cherries is the same as the probability of picking any other. This would not be true if (let us imagine) the sweet cherries were smaller than the sour ones. (The sour cherries would come to hand more readily when you sampled from the bag.) Let us keep in mind, therefore, that when we assess probabilities in terms of the ratio of favorable to all potential cases, we rely heavily on the assumption of equal probability for all outcomes.

Cards
A standard deck of 52 playing cards consists of four **suits** (hearts, spades, diamonds, and clubs). Spades and clubs are black while hearts and diamonds are red. Each suit contains 13 cards, each of a different **rank**: an Ace (which in many games functions as both a low card and a high card), cards numbered 2 through 10, a Jack, a Queen, and a King.

Example 4:

Compute the probability of randomly drawing one card from a deck and getting an Ace.

There are 52 cards in the deck and 4 Aces so $P(Ace) = \frac{4}{52} = \frac{1}{3} \approx 0.0769$

We also think of probabilities as percents: There is 7.69% chance that a randomly selected card will be an Ace.

Notice that the smallest possible probability is 0- if there are no outcomes that correspond with the event. The largest possible probability is 1- if all possible outcomes correspond with the event.

Certain and Impossible Events
An impossible event has a probability of 0. A certain event has a probability of 1. The probability of any event must be $0 \leq P(E) \leq 1$

Working with Events

Complementary events

Now let us examine the probability that an event does not happen. As in the previous section, consider the situation of rolling a six-sided die and first compute the probability of rolling a six: the answer is P(six) =1/6. Now consider the probability that we do not roll a six: there are 5 outcomes that are not a six, so the answer is P (not a six) $= \frac{5}{6}$. Notice that

$$P(\text{not } E) = \frac{n-m}{n} = \frac{n}{n} - \frac{m}{n} = 1 - \frac{m}{n} = 1 - P(E)$$

Complement of an Event

The **complement** of an event is the event "E doesn't happen"

The notation $\tilde{E}$ is used for the complement of event *E*.

We can compute the probability of the complement using $P(\tilde{E}) = 1 - P(E)$

Notice also that $P(E) = 1 - P(\tilde{E})$

Example 5:

If you pull a random card from a deck of playing cards, what is the probability it is not a heart?

There are 13 hearts in the deck, so *P (heart)*= $\frac{13}{32} = \frac{1}{4}$

The probability of *not* drawing a heart is the complement:

P (not heart) = 1- *P (heart)*= 1-$\frac{1}{4} = \frac{3}{4}$

Probability of two Independent Events

Example 6:

Suppose we flipped a coin, rolled a die, and wanted to know the probability of getting a head on the coin and a 6 on the die.

We could list all possible outcomes: {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}.

Notice there are $2 \cdot 6 = 12$ total outcomes. Out of these, only 1 is the desired outcome, so the probability is $\frac{1}{12}$.

The prior example was looking at two independent events.

Independent Events

Events A and B are **independent events** if the probability of Event B occurring is the same as whether Event A occurs.

Example 7:

Are these events independent?

a) A fair coin is tossed two times. The two events are (1) the first toss is a head and (2) second toss is a head.

b) The two events (1) “It will rain tomorrow in Houston” and (2) “It will rain tomorrow in Galveston” (a city near Houston)

c)You draw a card from a deck, then draw a second card without replacing the first.

Answers:

a) The probability that a head comes up on the second toss is ½ regardless of whether a head came up on the first toss, so these events are independent.

b) These events are not independent because it is more likely that it will rain in Galveston on days it rains in Houston than on days it does not.

c)The probability of the second card being red depends on whether the first card is red or not, so these events are not independent.

When two events are independent, the probability of both occurring is the product of the probabilities of the individual events.

P (A and B) for independent events If events *A* and *B* are independent, then the probability of both *A* and *B* occurring is $P(A \text{ and } B) = P(A) \bullet P(B)$ Where *P (A* and *B*) is the probability of events *A* and *B* both occurring. *P(A)* is the probability of event *A* occurring, and *P(B*) is the probability of event *B* occurring.

If you look back at the coin and die example from earlier, you can see how the number of outcomes of the first event multiplied by the number of outcomes in the second event multiplied to equal the total number of possible outcomes in the combined event.

Example 8:

In your drawer you have 10 pairs of socks, 6 of which are white, and 7 tee shirts, 3 of which are white. If you randomly reach in and pull out a pair of socks and a tee shirt, what is the probability both are white?

The probability of choosing a white pair of socks is $\frac{6}{10}$

The probability of choosing a white tee shirt is $\frac{3}{7}$

The probability of both being white is $\frac{6}{10} \bullet \frac{3}{7} = \frac{18}{70} = \frac{9}{35}$

The previous examples looked at the probability of both events occurring. Now we will look at the probability of either event occurring.

Example 9:

Suppose we flipped a coin, rolled a die, and wanted to know the probability of getting a head on the coin or a 6 on the die.

Here, there are still 12 possible outcomes: {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}

By simply counting, we can see that 7 of the outcomes have a head on the coin or a 6 on the die or both – we use or inclusively here (these 7 outcomes are H1, H2, H3, H4, H5, H6, T6).

So, the probability is $\frac{7}{12}$. How could we have found this from the individual probabilities?

As we would expect, $\frac{1}{2}$ of these outcomes have a head, and $\frac{1}{6}$ of these outcomes have a 6 on the die. If we add these, ½ $+\frac{1}{6}=\frac{6}{12}+\frac{2}{12}=\frac{8}{12}$, which is not the correct probability. Looking at the outcomes we can see why: the outcome H6 would have been counted twice, since it contains both a head and a 6; the probability of both a head and rolling a 6 is $\frac{1}{2}$. If we subtract this double count, we have the correct probability: $\frac{8}{12}-\ \frac{1}{12}=\frac{7}{12}$.

P (A or B)
The probability of either *A* or *B* occurring (or both) is
P (A** or **B)** = **P(A)** + **P(B)**— **P (A** and **B)

Example 10:

Suppose we draw one card from a standard deck. What is the probability that we will get a Queen or King?

There are 4 Queens and 4 kings in the deck, hence 8 outcomes corresponding to a Queen or King out of 52 possible outcomes. Thus, the probability of drawing a Queen or a King is:

$P\ (King\ or\ Queen) = \frac{8}{52}$

Note that in this case, there are no cards that are both a Queen and a King, so $P\ (King\ and\ Queen) = 0$. Using our probability rule, we could have said:

$P\ (King\ and\ Queen) = P(King) + P(Queen)\ \text{-}\ P\ (king\ and\ Queen) = \frac{4}{52}+\frac{4}{52}-0=\frac{8}{52}$

In the example, the events were **manually exclusive**, so *P (A or B) = P(A)+ P(B*).

Example 11:

Suppose we draw one card from a standard deck. What is the probability that we will get a red card or a king?

Half the cards are read, so *P(red)*= $\frac{26}{52}$

There are four kings, so *P(King)*=$\frac{4}{52}$

There are two red kings, so P (Red and King) =$\frac{2}{52}$

We can then calculate

P (Red or King) = *P(Red) + P(King)- P9Red and King)* =$\frac{26}{52}+\frac{4}{52}-\frac{2}{52}=\frac{28}{52}$

Example 12:

The table below shows the number of survey subjects who have received and not received a speeding ticket in the last year, and the color of their car. Find the probability that a randomly chosen person:

a) Has a red car and got a speeding ticket.

b) Has a red car or got a speeding ticket.

	Speeding ticket	Not speeding ticket	Total
Red car	15	135	150
Not red car	45	470	515
Total	60	605	665

We can see that 15 people of the 665 surveyed had both a red car and got a speeding ticket, so the probability is $\frac{15}{665}\approx 0.0226$.

Notice that having a red car and getting a speeding ticket are not independent events, so the probability of both occurring is not simply the product of probabilities of each one occurring.

We could answer this question by simply adding up the numbers. 15 people with red cars and speeding tickets +135 with red cars but no ticket +45 with a ticket but no red car=195 people. So, the probability is $\frac{195}{665}\approx 0.2932$

We also could have found this probability by: P (had a red car) + P (got a speeding ticket) - P (had a red car and got a speeding ticket)

$$= \frac{150}{665} + \frac{60}{665} - \frac{15}{665} = \frac{195}{665}$$

Conditional Probability

Often it is required to compute the probability of an event given that another event has occurred.

Example 13:

What is the probability that two cards drawn at random from a deck of playing cards will both be aces?

It might seem that you could use the formula for the probability of two independent events and simply multiply $\frac{4}{52} + \frac{4}{52} - \frac{1}{169}$.
]This would be incorrect, however, because the two events are not independent. If the first card drawn is an ace, then the probability that the second card is also an ace would be lower because there would only be three left in the deck.
Once the first card chosen is an ace, the probability that the card chosen is also an ace is called the **conditional probability** of drawing an ace. In this case the "condition" is that the first card is an ace. Symbolically, we write this as:

P (ace on second draw I an ace on the first draw).

The vertical bar "I" is red as "given," so the above expression is short for "The probability that an ace is drawn on the second draw given that an ace was drawn on the first draw." What is the probability? After an ace is drawn on the first draw, there are 3 aces out of 51 total cards left. This means that the conditional probability of drawing an ace after one ace has already been drawn is $\frac{3}{51} = \frac{1}{17}$.

Thus, the probability of both cards being aces is $\frac{4}{52} \bullet \frac{3}{51} = \frac{12}{2652} = \frac{1}{221}$.

Conditional Probability
The probability the event B occurs, given that event A has happened, is represented as $P\,(B\,/\,A)$
This is read as "The probability of B given A."

Example 14:

Find the probability that a die rolled shows a 6, given that a flipped coin shows a head. There are two independent events, so the probability of the die rolling a 6 is $\frac{1}{6}$, regardless of the result of the coin flip.

Example 15:

The table below shows the number of survey subjects who have received and not received a speeding ticket in the last year, and the color of their car. Find the probability that a randomly chosen person:

a) Has a speeding ticket *given* they have a red car

b) Has a red car *given* they have a speeding ticket

	Speeding ticket	No speeding tickets	Total
Red car	15	135	150
Not red car	45	470	515
Total	60	605	665

a) Since we know the person has a red car, we are only considering the 150 people in the first row of the table. of those, 15 have a speeding ticket, so

P(ticket/ red car)=$\frac{15}{150} = \frac{1}{10} = 0.1$

b) Since we know the person has a speeding ticket, we are only considering the 60 people in the first column of the table. Of those, 15 have a red car, so

P(red car/ ticket)=$\frac{15}{60} = \frac{1}{4} = 0.25$

Notice from the last example that P (B | A) is not equal to P (A | B).

These kinds of conditional probabilities are what insurance companies use to determine your insurance rates. They look at the conditional probability of you having an accident, given your age, your car, your car color, your driving history, etc., and price your policy based on that likelihood.

Conditional Probability Formula

If Events *A* and *B* are not independent, then

P (A and *B) = P(A) • P(B/A)*

Example 16:

If you pull 2 cards out of a deck, what is the probability that both are spades?

The probability that the first card is a spade is $\frac{13}{52}$.

The probability that the second card is a spade, given the first was a spade, is $\frac{12}{51}$, since there is one less spade in the deck, and one less total card.

The probability that both cards are spades is $\frac{13}{52} \cdot \frac{12}{51} = \frac{156}{2652} \approx 0.0588$

Example 17:

If you draw two cards from a deck, what is the probability that you will get the Ace of Diamonds and a black card?

You can satisfy this condition by having Case A or case B as follows:

Case A) you can get the Ace of Diamonds first and then a black card or

Case B) you can get a black card first and then the Ace of Diamonds.

Let's calculate the probability of Case A. The probability that the first card is the Ace of Diamonds is $\frac{1}{52}$. The probability that the second card is black given that the first card is the Ace of Diamonds is $\frac{26}{51}$ because 26 of the remaining 51 cards are black. The probability is therefore $\frac{1}{52} \bullet \frac{26}{51} = \frac{1}{102}$

Note for Case B: the probability that the first card is black is $\frac{26}{52} = \frac{1}{2}$. The probability that the second card is the Ace of Diamonds given that the first card is black is $\frac{1}{51}$.

The probability of Case B is therefore $\frac{1}{2} \cdot \frac{1}{51} = \frac{1}{102}$, the same as the probability of Case 1.

Recall that the probability of A or B is P(A)+P(B)-P (A and B). In this problem. P (A and B)=0 since the first card cannot be the Ace of Diamonds and be a black card. Therefore, the probability of Case A or Case B is $\frac{1}{102} + \frac{1}{102} = \frac{2}{102} = \frac{1}{51}$. The probability that you will get the Ace of Diamonds and a black card when drawing two cards from a deck is $\frac{1}{51}$.

Example 18:

A home pregnancy test was given to woman, then pregnancy was verified through blood tests. The following table shows the home pregnancy test results. Find

a) P(not pregnant / positive test result)

b) P(positive test result/ not pregnant)

	Positive Test	Negative Test	Total
Pregnant	70	4	74
Not Pregnant	5	14	19
Total	75	18	93

Answers:

a) Since we know the test result was positive, we are limited to the 75 women in the first column, of which 5 were not pregnant.

P(not pregnant/ positive test result) $= \frac{5}{75} \approx 0.067$.

b) Since we know the woman is not pregnant, we are limited to the 19 women in the second row, of which 5 had a positive test.

P(positive test result/ not pregnant) $= \frac{15}{19} \approx 0.263$

The second result is what is usually called a false positive. A positive result when the woman is not actually pregnant.

We are often interested in finding the probability that one of multiple events occurs. Suppose we are playing a card game, and we will win if the next card drawn is either a heart or a king. We would be interested in finding the probability of the next card being a heart or a king. The **union of two events** E and F, written E∪F, is the event that occurs if either or both events occur.

Probability Of The Union Of Two Events

The probability of the union of two events E and F (written E∪F) equals the sum of the probability of E and the probability of F minus the probability of E and F occurring together (which is called the intersection of E and F and is written as E∩F).

$$P(E\cup F) = P(E) + P(F) - P(E\cap F)$$

Suppose the spinner in Figure 2 is spun. We want to find the probability of spinning orange or spinning a b.

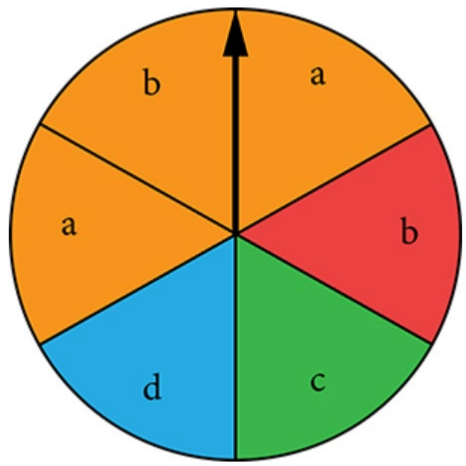


Figure 2

There are a total of 6 sections, and 3 of them are orange. So the probability of spinning orange is $\frac{3}{6} = \frac{1}{2}$. There are a total of 6 sections, and 2 of them have a b. So the probability of spinning a b is $\frac{2}{6} = \frac{1}{3}$. If we added these two probabilities, we would be counting the sector that is both orange and a b twice. To find the probability of spinning an orange or a b, we need to subtract the probability that the sector is both orange and has a b.

$$\frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{2}{3}$$

The probability of spinning orange or a b is $\frac{2}{3}$.

Computing the Probability of an Event with Equally Likely Outcomes.

The probability of an event *E* in an experiment with sample space *S* with equally likely outcomes is given by

$$P(E) = \frac{number\ of\ elements\ in\ E}{number\ of\ elements\ in\ S} = \frac{n(E)}{n(S)}$$

E is a subset of S, so it is always true that $0 =\leq P(E) \leq 1$.

Practice Exercises:

1. A ball is drawn randomly from a jar that contains 6 red balls, 2 white balls, and 5 yellow balls. Find the probability of the given event.

a. A red ball is drawn

b. A white ball is drawn

2. Suppose you write each letter of the alphabet on a different slip of paper and put the slips into a hat. What is the probability of drawing one slip of paper from the hat at random and getting:

a. A consonant

b. A vowel

3. A group of people were asked if they had run a red light in the last year. 150 responded "yes", and 185 responded "no". Find the probability that if a person is chosen at random, they have run a red light in the last year.

4. In a survey, 205 people indicated they prefer cats, 160 indicated they prefer dogs, and 40 indicated they don't enjoy either pet. Find the probability that if a person is chosen at random, they prefer cats.

5. Compute the probability of tossing a six-sided die (with sides numbered 1 through 6) and getting a 5.

6. Compute the probability of tossing a six-sided die and getting a 7.

7. Giving a test to a group of students, the grades and gender are summarized below. If one student was chosen at random, find the probability that the student was female.

	A	B	C	Total
Male	8	18	13	39
Female	10	4	12	26
Total	18	22	25	65

8. The table below shows the number of credit cards owned by a group of individuals. If one person was chosen at random, find the probability that the person had no credit cards.

	Zero	One	Two or more	Total
Male	9	5	19	33
Female	18	10	20	48
Total	27	15	39	81

9. Compute the probability of tossing a six-sided die and getting an even number.

10. Compute the probability of tossing a six-sided die and getting a number less than 3.

11. If you pick one card at random from a standard deck of cards, what is the probability it will be a King?

12. If you pick one card at random from a standard deck of cards, what is the probability it will be a Diamond?

13. Compute the probability of rolling a 12-sided die and getting a number other than 8.

14. If you pick one card at random from a standard deck of cards, what is the probability it is not the Ace of Spades?

15. Referring to the grade table from question #7, what is the probability that a student chosen at random did NOT earn a C?

16. Referring to the credit card table from question #8, what is the probability that a person chosen at random has at least one credit card?

17. A six-sided die is rolled twice. What is the probability of showing a 6 on both rolls?

18. A fair coin is flipped twice. What is the probability of showing heads on both flips?

19. A die is rolled twice. What is the probability of showing a 5 on the first roll and an even number on the second roll?

20. Suppose that 21% of people own dogs. If you pick two people at random, what is the probability that they both own a dog?

Answers:

1.a) $\frac{6}{13}$ b) $\frac{2}{13}$

3. $\frac{150}{335} = 44.8\%$

5. $\frac{1}{6}$

7. $\frac{26}{65}$

9. $\frac{3}{6} = \frac{1}{2}$

11. $\frac{4}{52} = \frac{1}{13}$

13. $1 - \frac{1}{12} = \frac{11}{12}$

15. $1 - \frac{25}{65} = \frac{40}{65}$

17. $\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

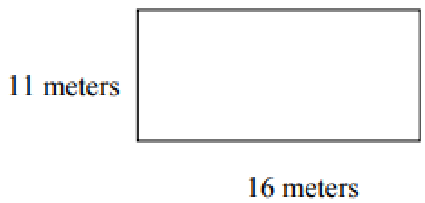
19. $\frac{1}{6} \cdot \frac{3}{6} = \frac{3}{36} = \frac{1}{12}$

Probability Word Problems https://youtu.be/vGcmjINp1x8

Probability Word Problems - Mutually Exclusive and Inclusive Events
https://youtu.be/5WVoEkScAek

Basic arithmetic - Cumulative Review

1. Find the perimeter and area of the rectangle shown below:

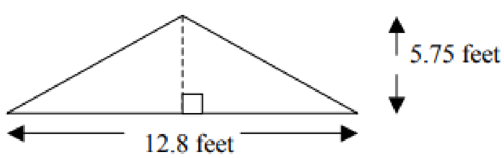


2. Find the circumference and area of a circle with radius 5 feet. Use π = 3.14.

3. A rectangle is 3.35 inches long and 7.3 inches wide. Find its area and perimeter.

4. If a circle has a diameter of 5.37 centimeters, what is the length of its radius?

5. Find the area of the triangle shown below:



6. Classify the following angles as acute, right, obtuse or straight. Then make a sketch of each angle.

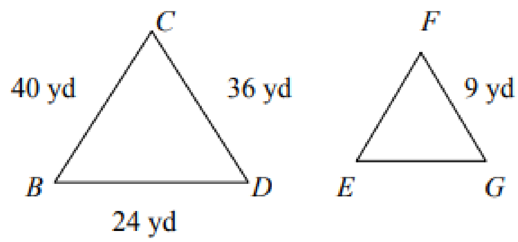
a. 65° b. 113° c. 180° d. 90°

7. The hypotenuse of a right triangle has a length of 15 inches and one of the triangle's legs has a length of 8 inches.

a. What is the length of the other leg of the triangle? Round your answer to 3 decimal places.

b. Make a drawing of the triangle labeling each side with its length.

8. Triangle BCD is similar to triangle EFG. Find EF and EG.



9. Add 3 ft. 7 in. and 8 ft. 9 in.

10. Convert 96 ounces to pounds.

11. Multiply 9036 by 309.

12. Add 0.68 + 299 + 5.2 + 0.385.

13. Subtract 3.542 from 5.82.

14. Divide 414 by 6.

15. Multiply 2.002 by 0.048.

16. Divide 9 by 0.06.

17. Multiply: $\frac{4}{33} \times \frac{3}{8}$

18. Divide:$6 \div 2\frac{1}{6}$

19. Add; write answer in simplest form:$\frac{26}{20} + \frac{36}{20} + \frac{12}{20}$

20. Add: $\frac{2}{5} + \frac{1}{15} + \frac{1}{6}$

21. Subtract:$\frac{11}{12} - \frac{1}{8}$

22. Simplify:$\frac{2}{5} + \frac{1}{2} + \frac{5}{6}$

23. Write the ratio of 48 to 70 as a fraction in simplest form.

24. Find the unknown number in the following proportion: $\frac{3}{4} + \frac{n}{52}$

25. Write 2.3 as a percent.

26. Write $\frac{5}{16}$ as a percent

27. Find 20% of $213.58. Round to the nearest cent.

28. Convert 2.3 meters to centimeters.

29. Convert 5 square yards to square feet.

30. Convert 2356 millimeters to meters.

31. Use the formula $V = \frac{4}{3}\pi r^2$ to find the volume of a sphere with a radius of 3 inches. Use 3.14 for π.

32. Find the volume of a cube with a side of 4 centimeters.

33. Simplify: $36 \div 6 \times 2 - 4 + 17$

34. Simplify: $2\ 20 - (6 + 16) + 2^2$

35. Simplify: $45 + 60 \div 15 - 3.5(-2)$

36. Subtract; write answer in simplest form: $\frac{1}{2} - \frac{1}{6}$

37. Compute $3^2 + (-5)^2$

38. Round 36,924,563 to the nearest ten thousand.

39. Round 124.36887 to the nearest thousandth.

40. Round $182.279 to the nearest cent.

41. When Marion was getting freight ready for shipment, she made a row with 10 identical boxes that were 870 centimeters long. How long was each box?

42. There are 20 people on our swim team. One fourth of the team went to a swim meet in April. How many people went to the swim meet in April?

43. Peter walked 2 miles to deliver a storage box. He stopped every $\frac{1}{3}$ mile to rest. How many times did Peter stop?

44. Sam drank $\frac{3}{5}$ of a quart of milk. Harry drank $\frac{5}{8}$ of a quarter. How much more did Harry drink than Sam?

45. Linda made a triple batch of sugar cookies. She used $5\frac{1}{8}$ cups of flour. Before she made her cookies, she had $8\frac{2}{3}$ cups of flour. How much flour does Linda have left?

46. Jane's monthly gross pay is $3014.74. If she has the following deductions, what is her net pay?

Federal Tax: $450.69	Savings Plan: $24.00	FICA: $244.38
State Tax: $112.57	Insurance: $233.16	

47. Kelly's car used 10.36 gallons of gas to go 317.33 miles. Estimate the number of miles per gallon Kelly's car gets by rounding your answer to the nearest hundredth.

48. A bus travels 90 miles on 6 gallons of gas. How many gallons will it need to travel 165 miles?

49. Lewis works at a nursery. Last fall he kept track of bulb sales and discovered that $\frac{7}{10}$ of the bulbs sold were variegated tulip bulbs. Write this fraction as a percentage.

50. Katie sold 195 chocolate bars; 40% had coconut. How many chocolate bars had coconut?

51. A coat regularly selling for $46.85 is advertised at 25% off. Find the sale price to the nearest cent.

52. The sales tax rate in a certain state is 6%. Find the total price paid for a pair of shoes that costs $48.

53. Locate the following numbers on the number line: $-2.5, 0, \frac{1}{2}$,5.5 and – 4.

54. Evaluate: $|-8+5|$

55. The temperature was -7° F at 6:00 am one day in Detroit. A cold front lowered the temperature over the next hour by 2° F. What was the temperature at 7:00am?

56. Arrange the following from smallest to largest: $\frac{3}{5}, \frac{10}{20}, \frac{5}{8}, \frac{85}{100}$

57. Arrange the following from smallest to largest: 0.073, 0.7, 0.07, 0.072, 0.0731

58. Compute 17% of 455.

BASIC ARITHMETIC - CUMULATIVE REVIEW ANSWERS

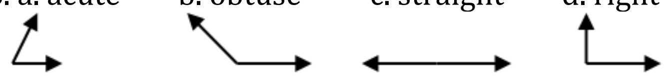
1. P = 54 meters   A = 176 square meters

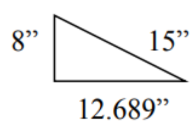
2. C = 31.4 feet   A = 78.5 square feet

3. P = 21.3 inches   A = 24.455 square inches

4. radius = 2.685 cm   5. A = 36.8 square feet

6. a. acute   b. obtuse   c. straight   d. right



7. a. 12.689 inches   b. 

8. EF = 10 yards   EG = 6 yards

9. 12 feet 4 inches	10. 6 pounds	11. 2,792,124
12. 305.265	13. 2.278	14. 69
15. 0.096096	16. 150	17. $\frac{1}{22}$
18. $\frac{36}{13}$	19. $3\frac{7}{10}\ or\ \frac{37}{10}$	20. $\frac{19}{30}$
21. $\frac{19}{24}$	22. $1\frac{19}{30} or \frac{49}{30}$	23. $\frac{24}{35}$
24. n = 39	25. 230%	26. 31.25%
27. $42.72	28. 230 cm	29. 45 square feet
30. 2.356 meters	31. 113.04 cubic inches	32. 64 cubic cm
33. 25	34. 2	
35. 56	36. $\frac{1}{3}$	37. 34
38. 36,920,000	39. 124.369	40. $182.28

41. 87 cm

42. 5 people

43. 6 times

44. $\frac{1}{40}$ quart

45. $3\frac{13}{24}$ cups

46. $1949.94

47. 30.63 miles per gallon

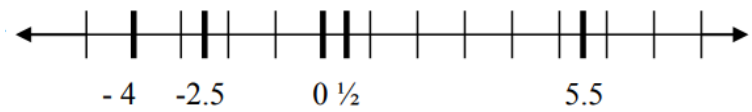
48. 11 gallons

49. 70% were tulip bulbs

50. 78 had coconut

51. $35.14

52. $50.88

53. 

54. 3

55. - 9° F

56. $\frac{10}{20}, \frac{3}{5}, \frac{5}{8}, \frac{85}{100}$

57. 0.07, 0.072, 0.073, 0.0731, 0.7

58. 77.35

Attribution:

Chapter 12: Algebraic Concepts

Algebra

In algebra, we use letters to represent numbers in equations. We call these letters variables because the numbers they represent vary. For example, let's say your salary is $10 per hour. If you worked two hours, you would be paid $10 × 2, or $20. If you worked five hours you would be paid $10 × 5, or $50. To generalize this, we can say that if you work h hours, you will be paid $10 × h.

What is an algebraic expression?

An algebraic expression is a group of **numbers**, **operation signs**, and **variables**, but NO EQUAL SIGN.

Examples are: $2x + 3$ $3 + 4 - y$ $\frac{-32}{b}$ d^2

A **variable** is a symbol that represents an **unknown number or value**. The variables above are "*x*" and "*y*". Any letter can be used as a variable.

Algebraic expressions can only be solved if you are told what the variable is.

Translating between word problems and expressions

Expressions themselves are easy to work with. However, sometimes they are presented to you in word form, which can be tricky to decipher at first. Here are some sample word problems.

Addition

*Addition is **commutative** which means the order is not important.*

Word Problem	Algebraic Expression
The **sum** of 3 and a number	n + 3 or 3 + n
7 **more than** a number	n + 7 or 7 + n
A number **plus** 2	n + 2 or 2 + n
Add 9 to a number	n + 9 or 9 + n
Khalid is 9 years **older than** Hassan who is "n" years old.	n + 9
Trish's salary, *s*, is **raised** by $120	s + 120

Practice: Fill in the missing word problems or algebraic expressions.

Word Problem	Algebraic Expression
The sum of a number and -15.	
	21 + x

Eriberta is 5 inches taller than her son, who is t inches tall	
	75 + n

Subtraction

*Subtraction is **not** commutative. The order is important.*

Word Problem	Expression	Opposite	Expression
A number **minus** 6	n – 6	6 **minus** a number	6 - n
9 **less than** a number	n - 9	A number **less than** 9	9 – n
subtract 10 from a number	n – 10	**Subtract** a number from 10	10 – n
The number **decreased** by 5	n – 5	5 **decreased** by a number	5 – n
Armando is 6 inches **shorter than** Lily who is "n" inches tall.	n - 6	Lily is "n" feet shorter **than** Joe who is 6 feet tall.	6 – n
Trish's salary, *s*, is **lowered** by $120	s - 120	Payroll of $100,000 is **lowered** by Trish's salary, *s*.	100,000 - s

Practice: Fill in the missing word problems or algebraic expressions.

Word Problem	Algebraic Expression
A number less than 25	
	32 – x
Dyna had some money and lost $10.	
	15 – n

Multiplication

*Multiplication is **commutative** which means the order is not important.*

Word Problem	**Algebraic Expression**
5 **times** a number	(5 · n) or (n ·5) or 5n
The **product** of 3 and a number	(3· n) or (n · 3) or 3n
Twice a number	(2·n) or (n · 2) or 2n
A number **doubled**	(2·n) or (n · 2) or 2n
A number **multiplied** by 9	(9·n) or (n · 9) or 9n
¼ **of** a number	(¼ ·n) or ($\frac{n}{4}$) or ¼ n

Division

*Division is **not** commutative. The order is important.*

Word Problem	Expression	Opposite	Expression
A number **divided by** 2	$\frac{n}{2}$or...n÷2	2 **divided by** a number	$\frac{2}{n}$or...2÷n
4 **divided into** a number	$\frac{n}{4}$or...n÷4	A number **divided into** 4	$\frac{4}{n}$or...4÷n
An amount **split** 3 ways	$\frac{n}{3}$or...n÷3	3 **split** a number of ways	$\frac{3}{n}$or...3÷n
Candy **shared** by 5 people	$\frac{n}{10}$ or...n÷5	10 dogs **shared** by some people	$\frac{10}{n}$ or...10÷n

Multi-Operation Expressions

As you begin to work with algebraic expressions more, you will see word problems that require you to use more than one operation. Take a look at these examples:

Word Problem	**Algebraic Expression**

John has **3 more than 4 times as many pencils, p** than Bogdan.	4p + 3 (4 times a number plus three more)
8 times a number, *n*, **decreased** by 3	8n - 3
The sum **of** a number, *n*, and 9	$\frac{2}{3}$ n + 9
Five **times** the **difference** of 7 and a number	5 (7 – n)

Remember

when you have **"the sum of"** or **"the difference of"** you may need to put parentheses around that part of the expression to show that it needs to be done first.

Expression Vocabulary

Vocabulary or key term	Definition	Example
Variable	A letter that is used to represent an unknown value	X
Constant	A number by itself in an expression	The "2" in x + 2
Coefficient	A number that is multiplied by a variable	The "2" in 2x
Term	A number, a variable, or the product or quotient of numbers and variables. A term cannot include a sum or a difference.	Terms: x, 2x, $\frac{x}{2}$ Not terms: x + 2, 2 - x
Like terms	Terms that have the same variable raised to the same power.	Like terms: x, 2x, 13x; or y^2, $4y^2$, $7y^2$ Not like terms: x, y, x^2, x^3

Guidance

Sometimes variables and numbers can be repeated within an expression. If the same variable is in an expression more than once, they can be combined by addition or subtraction. This process is called **combining like terms.**

Step 1 (optional): reorganize the expression so like terms are next to each other. **The operation always attaches to the number that <u>follows</u> it.** Ex. -1 + 4 can be reorganized as 4 + (-1) or 5 - 9 can be reorganized as -9 + 5

Step 2: combine like terms

In all examples, **bold** and colors are used to make it easier to see separate terms.

Example A

Simplify $5x - 12 - 3x + 4$

Solution: Reorganize the expression to ***<u>group together the x's and the numbers</u>***. You can either place the like terms next to each together or place parenthesis around the like terms.

$\mathbf{5x} - 12 - \mathbf{3x} + 4$

reorganize $\mathbf{5x} - \mathbf{3x} - 12 + 4$

simplify **2x** – 8

Example B

Simplify $6a - 5b + 2a - 10b + 7$

Solution: Here there are two different variables, *a* and *b*. Even though they are both variables to the same power, they are ***<u>different</u>*** **<u>variables and cannot be combined</u>**. Group together the like terms.

$6a - \mathbf{5b} + 2a - \mathbf{10b} + 7$

Reorganize $6a + 2a - \mathbf{5b} - \mathbf{10b} + 7$

Simplify $8a - \mathbf{15b} + 7$

Example C

Simplify $w^2 + 9 - 4w^2 + 3w^4 - 7w - 11$.

Solution: Here we have one variable, but there are different powers (exponents). ***<u>Like terms must have the same exponent</u>*** in order to combine them.

$\mathbf{w^2} + 9 - \mathbf{4w^2} + 3w^4 - 7w - 11$

reorganize $3w^4 + \mathbf{w^2} - \mathbf{4w^2} - 7w + 9 - 11$

simplify $3w^4 - \mathbf{3w^2} - 7w - 2$

Guided Practice:

1) $6s - 7t + 12t - 10s$

Reorganize $(6s - 10s) + (-7t + 12t)$

Simplify $-4s + 5t$

2) $7y^2 - 9x^2 + y^2 - 14x + 3x^2 - 4$

Reorganize $(-9x^2 + 3x^2) + (7y^2 + y^2) - 14x - 4$

Simplify $-6x^2 + 8y^2 - 14x - 4$

Standard Form

Mathematicians like final answers to be in a particular form.

- Terms with the largest exponents should be on the left, decreasing from left to right. (**The co-efficient value is not important**.)
- If two variables have the same exponent, the variables are listed in alphabetic order.
- The constant is always last.

Ex. $-6x^2 + 8y^2 - 14x - 4$ is written in standard form

Mixed Practice

Simplify the following expressions as much as possible. If the expression cannot be simplified, write "cannot be simplified."

1) $5b - 15b + 8d + 7d$

2) $2a - 5f$

3) $6 - 11c + 5c - 18$

4) $7p - p^2 + 9p + q^2 - 16 - 5q^2 + 6$

5) $3g^2 - 7g^2 + 9 + 12$

6) $20x - 6 - 13x + 19$

7) $8u^2 + 5u - 3u^2 - 9u + 14$

8) $8n - 2 - 5n^2 + 9n + 14$

Answers:

1)$-10b + 15d$ 2) cannot be simplified 3) $-6c - 12$ 4) $-p^2 - 4q^2 + 16p - 10$

5) $-4g^2 + 21$ 6) $7x + 13$ 7) $5u^2 - 4u + 14$

8) $-5n^2 + 17n + 12$

Simplifying Expressions Using the Distributive Property

Multiplying Unlike terms

While you cannot add or subtract unlike terms, you can multiply them. When you do this, you multiply each component separately. Remember that multiplication is commutative, we can change the order without affecting the outcome.

Example: Multiply (2ab)(4b).

1. Expanded this looks like $2 \bullet a \bullet b \bullet 4 \bullet b$
2. Multiply the coefficients: $2 \bullet 4 = 8$ which makes it $8 \bullet a \bullet b \bullet b$
3. Multiply like variables by adding exponents: $b \bullet b = b^2$ so $8 \bullet a \bullet b^2$
4. The answer is $8ab^2$ (**Note that the order in a term is coefficient then alphabetic order**.)

Remember that a variable with no exponent is a variable raised to the first power. $x = x^1$

Practice 1: Multiply the following terms.

1) (3xy)(2y)

2) (abc)(3ac)

3) (x)(x)

4) $(5x^2)(5y)$

5) (15a) (2bc)

6) $(7a^2bc^3)$ $(5ab^5c^2)$

The Distributive Property

What does the word “distribute” make you think of?

Distributive Property is used to **multiply** a term outside a set of parentheses with two or more terms inside a set of parentheses.

In the problem 5(2 + 4) the Order of Operations says you must solve parentheses first. Therefore, we add 2+ 4 first, then multiply the result (6) by 5 to get 30.

$5(2 + 4) = 5(6) = 30$

In the previous example, 2 and 4 were "like terms" so they could be added. What if the two terms were not alike, such as the next expression?

3(10x+4)

The two terms inside the parenthesis cannot be added because **they are not like terms.** In other words, the expression inside the parenthesis cannot be simplified any further.

However, we **can** simplify the whole expression using another method called Distributive Property. This method has 2 steps.

Step 1: Distribute

The Distributive Property helps us **remove the parenthesis** if the term outside the parenthesis (in this case 3) is multiplied with **each term** inside the parenthesis (10x and 4).

Simplify **3**(10x+4)

Since we can't add 10x and 4, we use the Distributive Property

Multiply **3** times 10x, then multiply **3** times 4 to get **3(10x) + 3(4)**

This is possible because 3 is still being multiplied to everything in the parenthesis, just as it would be if we could have added the numbers in the parenthesis first.

Step 2: Simplify

Now we can **simplify** the multiplication of the individual terms:

3(10x) + 3(4) becomes **30x + 12** (Make sure it is in standard form!)

Note: While we cannot add unlike terms (we cannot add 10x and 4) we can **multiply** unlike terms. This is because when we multiply, we are saying how many times that thing exists, we are not combining things.

Practice 2: Use Distributive Property to simplify these expressions:

9(x + 9) 2(4 + 9x) 7(x + -1)

12(a + b + c) 10(3 – 2x)

Distributing Negatives

What do we do if there is only a negative sign outside parenthesis?

$- (5 + x^2)$

Whenever we see a negative sign by itself in front of a parenthesis, we can assume it means *negative one*. There is one $(5 + x^2)$ and it is negative. We can rewrite the problem as **$-1(5 + x^2)$**

Next, distribute the -1 with each term inside the parenthesis.

$-1(5) + -1(x^2) \quad = \quad -5 + (-1x^2)$

And put in the standard form **$-x^2 + (-5)$**

Practice 3: Use Distributive Property to simplify with negative numbers.

$-2(x - 8)$	$-3(p - 1)$	$-10(3 + 2 + 7x)$
$-2(y - 3)$	$-(-13x + 2)$	$-1(3w + 3x + -2z)$

Distributing Variables

We can also distribute a variable with a set of parentheses. For example.

$x (y + 5)$

Use the distributive property.

$x (y) + x (5)$

Simplify.

$xy + 5x$ (The coefficient, 5, should come before the x.)

Distributing Coefficients and Variables

Note: When you add like variables you combine like terms, but when you multiply variables, you **add the exponents**. For example:

$x + x + x = 3x$ but $x(x^2) = x^3$ (in other words $x(x)(x) = x^3$)

$y + y + 2 + y + y = 4y + 2$ but $y^2(2y^2) = 2y^4$ (in other words $y(y)(2)(y)(y) = 2y^4$)

What if we want to distribute a coefficient and variable with a set of parentheses?

$3x\ (2x^2 + 1)$

Distribute:

$3x\ (2x^2\ )\ \ + 3x\ (1)$

Simplify:

$3(2)(x)(x)(x) + 3(1)(x)$ (You can skip this step. I just wanted to show what's going on)

$6x^3 + 3x$

Practice 4: Use the Distributive Property to simplify with variables and coefficients.

$y\ (1 + x)$ $x\ (4 + x^2)$ $12x\ (3x + 3)$

$x\ (x + y)$ $7xy(x + y)$ $x(9x + 9y)$

More Practice:

Use the Distributive Property to simplify the following expressions.

1. $4(x - 3)$
2. $4(2x - 3)$
3. $2(3 - 4y)$
4. $x\ (x + 1)$
5. $x\ (x - 2)$
6. $x\ (x^2 + 4x - 3)$
7. $y\ (x - y^2)$
8. $4(p + 2) + 3(2p - 3)$
9. $2(3p + 2) + 3(2p - 3)$
10. $3(2p - 5) + 2(3p - 3)$
11. $2p\ (p + 2) + 3p\ (2p - 3)$
12. $3p\ (p - 2) + 2p\ (3p - 2)$
13. $2p\ (p - 3) + 3p\ (3p - 2)$
14. $x\ (x^2 - 2y) + 3x^2(x + 2y)$
15. $-\ (x - 3)$
16. $-4(2x - 3)$
17. $-2(3 - 4y)$
18. $-x\ (x + 1)$
19. $-x\ (x - 2)$
20. $-x\ (x^2 + 4x - 3)$
21. $-y\ (x - y^2)$

Answers:

Practice 1: 1) $6xy^2$ 2) $3a^2bc^2$ 3) x^2 4) $25x^2y$ 5) 30abc 6) $35a^3b^6c^5$

Practice 2: $9(x + 9) =$ **$9x + 81$**, $2(4 + 9x) = 8 + 18x =$ **$18x + 8$**, $7(x + -1)=$ **$7x + (-7)$ or $7x – 7$**

$12(a + b + c) =$ **$12a + 12b + 12c$**, $10(3 – 2x) = 30 – 20x =$ **$-20x + 30$**

Practice 3: $-2(x – 8) =$ **$-2x + 16$**, $-3(p – 1) =$ **$-3p + 3$**, $-10(3 + 2 + 7x) = -10(5 + 7x) = -50 + (-70x)=$ **$-70x + (-50)$**

$-2(y – 3) =$ **$-2y + 6$**, $-(-13x + 2) =$ **$13x + (-2)$**, $-1(3w + 3x + -2z) =$ **$-3w + (-3x) + 2z$**

Practice 4: $y (1 + x) = y + yx = xy + y$, $x (4 + x^2) = 4x + x^3 = x^3 + 4x$,
$12x (3x + 3) = 36x^2 + 36x$

$x (x + y) = x^2 + xy$, $7xy (x + y) = 7x^2y + 7xy^2$, $x (9x + 9y) = 9x^2 + 9xy$

More Practice:

1) $4x – 12$ 2) $8x – 1$ 3) $-8y + 6$

4) $x^2 + x$ 5) $x^2 – 2x$ 6) $x^3 + 4x^2 – 3x$

7) $– y^3 + xy$ 8) $10p – 1$ 9) $12p – 5$

10) $12p – 21$ 11) $8p^2 – 5p$ 12) $9p^2 – 10p$

13) $11p^2 – 12p$ 14) $4x^3 + 6x^2y – 2xy$ 15) $–x + 3$

16) $-8x + 12$ 17) $8y – 6$ 18) $–x^2 – x$

19) $–x^2 + 2x$ 20) $–x^3 – 4x^2 + 3x$ 21) $y^3–xy$

22) $-4x + 18$

Linear Equation

What is the difference between an expression and an equation?

An expression does not have an equal sign. It cannot be solved unless you are given the value of the variable. An equation does have an equal sign. You can manipulate the equation to find the value of the variable.

Expression $2x + 7$ **Equation** $2x + 7 = 21$

The following words or phrases mean "equals" in a verbal equation.

- Is Equals To give a result of Is equal to

Equations can be written by breaking up the words and translating each part.

Ex. "A number is multiplied by three, and then nine is subtracted to give a result of twelve."

The equation above can be written using numbers as: $3x - 9 = 12$

Equations involving addition or subtraction

To solve for a variable such as *x*, you must get it by itself on one side of the equal sign. To get rid of any numbers that are next to it, perform the inverse operation. To keep the equation **equal** you must do what you just did on the other side of the equals sign as well. Otherwise, it is no longer equal.

Example: Solve $\mathbf{y + 12 = 18}$

1. Identify the operation performed on the variable. 12 is **added** to y
2. Perform the inverse operation to get rid of the number: $y + 12 \mathbf{- 12} = 18$
3. Perform the same operation on the other side to keep it equal: $y \cancel{+ 12 - 12} = 18 \mathbf{- 12}$
4. Simplify $y = 6$
5. Put your solution in the original equation to see if it is true: $\mathbf{6} + 12 = 18$ yes $18 = 18$

Example: Solve p **-18 = 54**

1. Identify the operation performed on the variable. 18 is **subtracted** from p
2. Perform the inverse operation to get rid of the number: $p - 18 \mathbf{+18} = 54$
3. Perform the same operation on the other side to keep it equal: $p \cancel{-18 +18} = 54 \mathbf{+18}$
4. Simplify $p = 72$
5. Put your solution in the original equation to see if it is true: $\mathbf{72} - 18 = 54$ yes $54 = 54$

Practice: Find the value of the variable by using addition or subtraction. Self-check your first 3 answers.

1) $x + 5 = 8$ 2) $x + 7 = 12$ 3) $y + 3 = 9$

4) $-5 + x = 10$ 5) $x - 1.5 = 3$ 6) $n - 7.5 = 9.75$

7) y – 6 = 10		8) z + .17 = .35		9) y-$\frac{1}{4}$ = $\frac{7}{4}$

10) One method to weigh a horse is to load it into an empty trailer with a known weight and reweigh the trailer. A Shetland pony is loaded onto a trailer that weighs 2200 pounds empty. The trailer is then reweighed. The new weight is 2550. Which equation represents the weight of the pony?

a. 2550 – p = 2200

b. p + 2200 = 2550

c. p – 2550 = 2200

solve for p:

11) Darrell spent $5.75 for lunch and had $2.15 left over. Which equation can be used to find the amount (m) Darrell had before lunch?

a. m - $5.75 = $2.15

b. m - $2.15 = $5.75

c. m + $2.15 = $5.75

Solve for m:

Equations involving multiplication or division

Just like with addition and subtraction, to solve for a variable such as *x*, you must get it by itself on one side of the equal sign. To get rid of any numbers that are next to it, perform the inverse operation. To keep the equation **equal** you must do what you just did on the other side of the equals sign.

Example: Solve 4**r = 20**

1. Identify the operation performed on the variable. 4 is **multiplied** to *r*
2. Perform the inverse operation to get rid of the number: 4r **÷ 4** = 20
3. Perform the same operation on the other side to keep it equal: 4r ÷ 4 = 20 **÷ 4**
4. Simplify $r = 5$
5. Put your solution in the original equation to see if it is true: **4(5)** = 20 yes 20 = 20

Example: Solve $\frac{x}{2} = 9$

Identify the operation performed on the variable. x is **divided by** 2

1. Perform the inverse operation to get rid of the number: $\frac{x}{2}(\mathbf{2}) = 9$
2. Perform the same operation on the other side to keep it equal: $\frac{x}{2}(2) = 9(\mathbf{2})$
3. Simplify $x = 18$
4. Put your solution in the original equation to see if it is true: $\frac{18}{2}$ **= 9** yes 9= 9

Practice: Find the value of the variable by using inverse operations.

12) -8k = -96 13) 8n = 48 14) 6y = 54

15) -4y = 24 16) 8.5n = 63.75 17) $\frac{x}{8}$ = 1.5

18) $\frac{x}{3}$ = 9 19) $\frac{x}{6}$ = 2.5 20) $\frac{x}{12}$ = $1.50

21) If you multiply a number n by 14, the product is 168. Which equation below can be used to find n?

a. 168n = 14 Solve for n:

b. $\frac{14}{n} = 168$

c. 14n = 168

22) Kami and two friends are sharing the cost of lunch. If the total cost is $16.45, which equation can be used to find Kami's share (called "s")?

a. $\frac{s}{3}$ = $16.45 Solve for s:

b. S = $\frac{\$16.45}{3}$

c. S = $\frac{3}{\$16.45}$

Mixed Practice

<u>Write an equation</u> to represent each statement. Use the variable *n* to represent unknown numbers. Then <u>solve your equation</u>.

1)Ten less than a number is 62

2)A number increased by 20 is equal to 30.

3) A number is multiplied by 3 and the result is 12.

4) The sum of -7 and a number is 2.

5)A number divided by 4 is equal to -5.

Solve for the given variable

6) x + 11 = 7

7) x – 1.1 = 3.2

8) 7x = 21

9) 4x = 1

10) x – $\frac{5}{6}$ = $\frac{2}{3}$

11) 0.01x = 11

12) q – 13 = -13

13) z + 1.1 = 3.0001

14) 21s = 3

15) $\frac{3}{4}$ = -$\frac{y}{2}$

16) 6r = $\frac{3}{8}$

17) -1 + x = 39

Answers:

Write the equation: 1) 10 + x = 14 2) $\frac{x}{3}$ = 11 answers may vary slightly for 3 & 4; 3) 8 times a number equals 14 4) A number less than 32 is 6.

Addition & Subtraction Practice:

1) x = 3 2) x = 5 3) y = 6 4) x = 15 5) x = 4.5 6) n = 17.25 7) y = 16 8) x = .18 9) y = 2

10) **b** the pony weighs 350 pounds 11)(**a)**, m = $7.90

Multiplication & Division Practice:

12) k = 12 13) n = 6 14) y = 9 15) y = -6 16) n = 7.5 17) x = 12

18) x = 27 19) x = 15 20) x = 18 21). c, n = 12 22) b, s = around $5.48

Mixed Practice:

1) n – 10 = 62, n = 72

2) n + 20 = 30, n = 10

3) 3n = 12, n = 4

4) -7 + n = 2, n = 9

5) $\frac{n}{4}$ = -5, n = -20

6)x = -4

7) x = 4.3

8) x = 3

9) x = ¼

10) x = 1 ½

11) x =1100

12) q = 0

13) z = 1.9001

14) s = 17

15) y = -1 ½

16) r = 116 17) x = 40

Inequalities Expressions

An **algebraic inequality** is a mathematical statement that connects two unequal expressions. You read an inequality from left to right as indicated below.

Verbs that translate into inequalities are:

$>$ "greater than"

$\geq$ "greater than or equal to"

$<$ "less than"

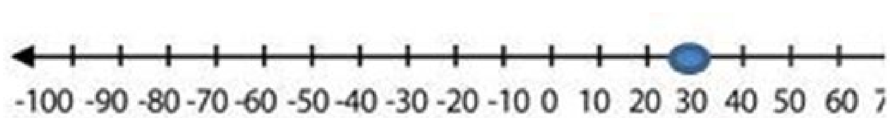
$\leq$ "less than or equal to"

$\neq$ "not equal to"

Solutions of Inequalities

Solutions to **Equations** could be graphed on a number line, but it is kind of boring, so we do not usually bother.

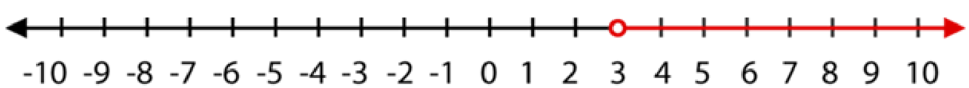
Ex. $x = 30$



Solutions to one-variable inequalities can also be graphed on a number line, and they are more meaningful.

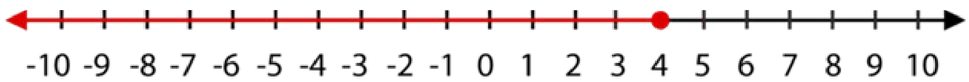
Example: Graph the solutions to $t>3$ on a number line.

Solution: The inequality is asking for all real numbers larger than 3. Since it does not include 3, we draw an open circle around the 3.



You can also write inequalities given a number line of solutions.

Example: Write the inequality pictured below.



Solution: The value of four is colored in, meaning that four is a solution to the inequality. The red arrow indicates values less than four. Therefore, the inequality is:

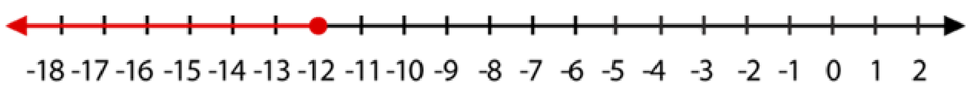
$x \leq 4$

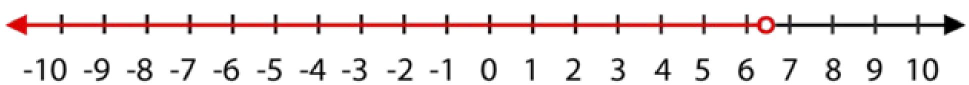
Practice 1:

Graph the solutions to the following inequalities using a number line.

1. $x < -3$
2. $x \geq 6$
3. $x > 0$
4. $x \leq 8$

Write the inequality that is represented by each graph.





Using words to describe Inequalities

It is important to both write and understand inequality statements from verbal descriptions. Certain phrases indicate inequalities.

$>$ greater than, more than, larger than

$\geq$ greater than or equal to, no less than, at least

$<$ less than, fewer than, smaller than

$\leq$ less than or equal to, no more than, at most

When writing inequalities, look for phrases that tell you which symbol to use. Also, look for key words that indicate any operation between quantities.

"A number increased by 4 **is greater than** 10."

A number increased by 4 $\quad$ $n + 4$

Is greater than 10 $n + 4 > 10$

"Two times a number is **no more than** that number plus 12."

Two times a number 2n

Is no more than $2n \leq$

That number plus 12 $2n \leq n + 12$

"Seven less than a number is **at least** 50."

Seven less than a number n – 7

Is **at least** 50 $n - 7 \geq 50$.

Practice 2:

Write the inequality given by the statement.

1. You must maintain a balance of at least $2,500 in your checking account to avoid a finance charge.
2. You must be younger than 3 years old to get free admission at the San Diego Zoo.
3. Charlie needs more than $1,800 to purchase a car.
4. The speed limit is 65 miles per hour.
5. The shelter can house no more than 16 rabbits.

Solving One-step Inequalities

The steps to solving inequalities are almost identical to solving equations. For one-step inequalities, you need to perform the inverse operation.

Just like one-step equations, the goal is to **isolate the variable,** meaning to get the variable alone on one side of the inequality symbol. To do this, you will cancel the operations using inverses.

Example: Solve for x : $x - 3 < 10$.

Solution: To isolate the variable x, you must cancel "subtract 3" using its inverse operation, addition.

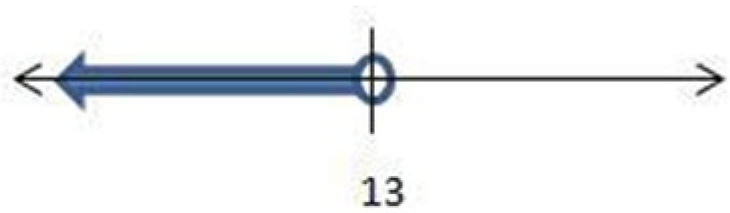
$$x - 3 + 3 < 10 + 3$$

$$x < 13$$

Now, check your answer. Choose any number less than 13 and substitute it into your original inequality. If you choose 0, and substitute it you get:

$$0 - 3 < 10 = -3 < 10$$

Graph your solution:



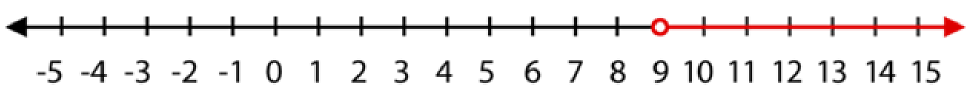
Example: Solve for $x : x + 4 > 13$

Solution:

To solve inequality $x + 4 > 13$

Subtract 4 from both sides of the inequality. $x + 4 - 4 > 13 - 4$

Simplify. $x > 9$



Practice 3:

Solve each inequality and graph the solution on a number line.

1. $x - 1 \leq -5$

2. $-20 + a \geq 14$

3. $x + 2 < 7$

4. $x + 8 \leq -7$

5. $5 + t \geq 34$

6. $x - 5 < 35$

Example: solve for x, $2x \geq 12$.

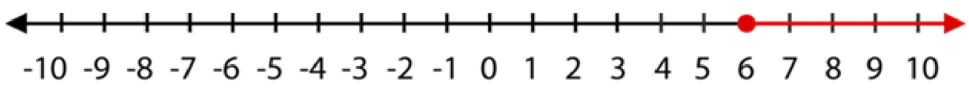
To find the solutions to this inequality, we must isolate the variable x by using the inverse operation of "multiply by 2," which is dividing by 2.

$$2x \geq 12$$

$$\frac{2x}{2} \geq \frac{12}{2}$$

$$x \geq 6$$

Graphed, it is:

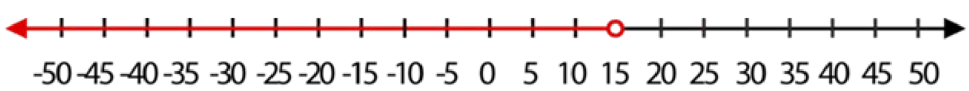


Example: *Solve for* y: $\frac{y}{5} \leq 3$.

The inequality above is read, "y divided by 5 is less than 3." To isolate the variable y, you must cancel division using its inverse operation, multiplication.

$$\frac{y}{5} \cdot \frac{5}{1} < 3 \cdot \frac{5}{1}$$

$$y < 15$$



Practice 4: Solve the inequalities

1) $75x \geq 125$

2) $\frac{x}{2} > 40$

3) $\frac{x}{25} < 32$

4) $3x \leq 6$

Special Case: Multiplying and Dividing with Negatives

When the inverse operation includes multiplying or dividing by a **negative** number, the inequality **symbol must be reversed** for the inequality to be true.

Think of it this way. When you multiply a value by –1, the number you get is negative of the original.

$$6(-1) = -6$$

Multiplying **each side** of a sentence by –1 results in the opposite of both values.

$$5x\,(-1) = 4(-1)$$

$$-5x = -4$$

When multiplying both sides by a negative, you are doing the “opposite” of everything in the sentence, including the verb.

$$x > 4$$

$$x\,(-1) > 4(-1)$$

$$-x < -4$$

Example: Solve for r: $-3r < 9$.

To isolate the variable r, we must cancel “multiply by –3” using its inverse operation, dividing by –3.

$$-\frac{3r}{-3} < \frac{9}{-3}$$

Since you are dividing by –3, everything becomes opposite, including the inequality sign.

$$r > -3$$

Practice 5:

Solve each inequality. Remember to change the sign only if multiplying or dividing by a negative number.

1) $\frac{x}{5} > -\frac{3}{10}$

2) $-10x > 250$

3) $\frac{x}{-7} \geq -5$

4) $9x > -\frac{3}{4}$

5) $\frac{x}{-15} \leq 5$

6) $620x > 2400$

7) The width of a rectangle is 16 inches. Its area is greater than 180 square inches. The formula for the area of a rectangle is $A = lw$

a. Write an inequality to represent this situation.

b. Graph the possible **lengths** of the rectangle.

Mixed Practice:

Graph the solutions to the following inequalities using a number line.

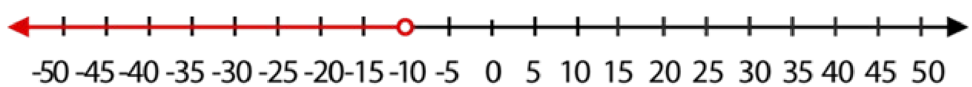
1) $x < -35$

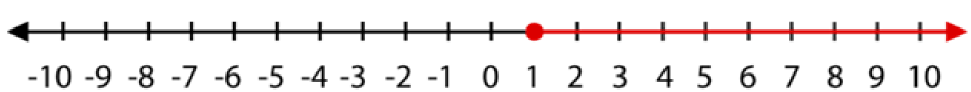
2) $x > -17$

3) $x \geq 20$

4) $x \leq 3$

Write the inequality that is represented by each graph.

5) 



Write the inequality given by the statement. Choose an appropriate letter to describe the unknown quantity.

7) You must be at least 48 inches tall to ride the "Thunderbolt" Rollercoaster.

8) Cheryl can have no more than six pets at her house.

Solve each inequality.

9) $x - 1 > -10$

10) $x - 2 \leq 1$

11)$x - 8 > -20$

12) $11 + q > 13$

13) $x + 65 < 100$

14)$x - 32 \leq 0$

15) $x + 68 \geq 75$

16) $16 + y \leq 0$

17) $\frac{x}{20} \geq -\frac{7}{40}$

18) $-0.5x \leq 7.5$

19) $\frac{x}{-3} > -\frac{10}{9}$

20) $\frac{k}{-14} \leq 1$

21) $\frac{x}{-15} < 8$

22) $\frac{x}{-3} \leq -12$

23) $\frac{x}{-7} \geq 9$

24) $238 < 14d$

25) $-19m \leq -285$

26) $-9x \geq -\frac{3}{5}$

27) $-5x \leq 21$

28) Ninety percent (0.9) of some numbers is at most 45.

a. Write an inequality to represent the situation.

b. Solve for the number.

29) Doubling Martha’s jam recipe yields at least 22 pints.

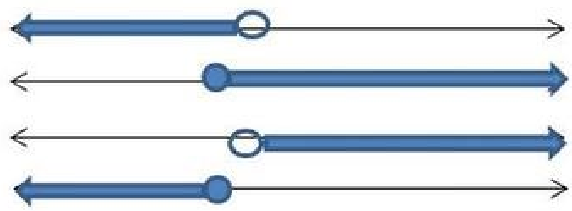
a. Write an inequality to represent the situation.

b. Martha's recipe makes at least how many pints?

Answers:

Practice 1:

1. $x < -3$
2. $x \geq 6$
3. $x > 0$
4. $x \leq 8$



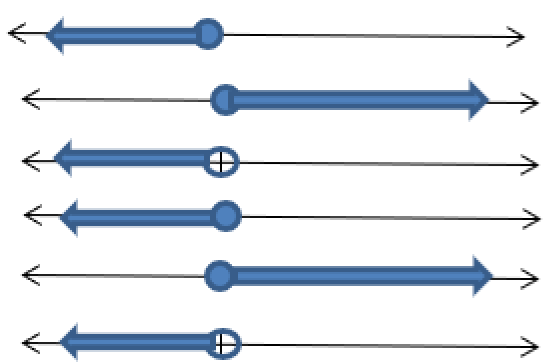
Write the inequality that is represented by the graph: 5.) x ≤ -12, 6.) x > 600

Practice 2:

1) x ≥ 2500, 2) x < 3, 3) d > 1800, 4) s ≤ 65, 5) r ≤ 1

Practice 3:

1. x ≤ 6
2. a ≥ 34
3. x < 5
4. x ≤ -15
5. t ≥ -4.25
6. x < 40



Practice 4: 1) x ≥ 1.67 2) x > 80 3) x < 37.5 4) x ≤ 2

Practice 5: 1) x > −32 2) x< -25 3) x ≤ 35 4) x > −112 5) x ≥ -75

6) x> 3.87

7) Area = length x width; a. $180 < 16l$ or $16l > 180$; $l > 11.25$ The length is greater than 11.25

Mixed Practice:

5) x < -10 6) x ≥ 1 7) h ≥ 48 8) p ≤ 6 9) x > -9 10) x ≤ 3

11) x > -12 12) q > 2 13) x < 35 14) x ≤ 32 15) x ≥ 7 16) y ≤ -16

17) x ≥ −7/2 18) x ≥ -15 19) x < 10/3 or 3 1/3 20) k ≥ -14 21) x > -120

22) x ≥ 36 23) x ≤ -63 24) 17 < d or d > 17 25) m ≥ 15

26) x ≤ 1/15 or 0.067 27) x ≥ -4.2

28)a. 0.9x ≤ 45, b. x ≤ 50 The number is at most 50.

29)a. 2r≥22, b. r ≥ 11 Her recipe makes at least 11 pints of jam.

Multi-Step Inequalities

Inequalities, like equations, can require several steps to isolate the variable. These inequalities are called **multi-step inequalities.** With the exception of the Multiplication/Division Property of Inequality, the process of solving multi-step inequalities is identical to solving multi-step equations.

Procedure to Solve an Inequality:

1. Remove any parentheses by using the Distributive Property.
2. Simplify each side of the inequality by combining like terms.
3. Use inverse operations to get all variables on the same side and constants on the other side.
4. Use inverse operations to get rid of the coefficient.

    a. Remember to reverse the inequality sign if you are multiplying or dividing by a negative number.
5. Check your solution.

The following are examples of two-step inequalities.

Example: *Solve for* x: $6x - 5 < 10$.

Begin by using the checklist above.

1. Parentheses? No

2. Like terms on the same side of inequality? yes

3. Get rid of the constant use of inverse operations:

$$6x - 5 + 5 < 10 + 5$$

Simplify.

$$6x < 15$$

4. Get rid of the coefficient using inverse operations. Are you multiplying or dividing by a negative? Not this time.

$$\frac{6x}{6} < \frac{15}{6} = x < \frac{5}{2}$$

5. Check your solution. Choose a number less than 2.5, say 0, and check using the original inequality.

$$6(0) - 5 \ < \ 10?$$

$$-5 \ < 10$$

Yes, the answer checks. $x < 2.5$

Example: Solve for x: $-9x < -5x - 15$

Begin by using the checklist above.

1. Parentheses? No

2. Like terms on the same side of inequality? No

3. Use inverse operations to get the variables on one side and constants on the other.

$$-9x + 5x < -5x + 5x - 15$$

Simplify.

$$-4x < -15$$

4. Get rid of the coefficient using inverse operations. Are you multiplying or dividing by a negative? Yes!

$$\frac{-4x}{-4} < \frac{-15}{-4}$$

Because the number you are dividing by is negative, you must reverse the inequality sign.

$$x > \frac{15}{4} \rightarrow x > 3\frac{3}{4}$$

5. Check your solution by choosing a number larger than 3.75, say 10.

$$-9(10) < -5(10) - 15?$$

$$\checkmark -90 < -65$$

Practice 1: Solve the following inequalities.

1) 2x + 4 > 6

2) 6x - 2 < 10

3) 7n – 1 > -169

4) -9x < -5x - 1

The next example is more complex.

Example: Solve for x: $4x - 2(3x - 9) \leq -4(2x - 9)$.

Begin by using the previous checklist.

1. Parentheses? Yes. Use the Distributive Property to clear the parentheses.

$$4x + (-2)(3x) + (-2)(-9) \leq -4(2x) + (-4)(-9)$$

Simplify by multiplying.

$$4x - 6x + 18 \leq -8x + 36$$

Simplify by adding or subtracting like terms.

$$-2x + 18 \leq -8x + 36$$

2. Use inverse operations to get all variables on one side:

$$-2x + 8x + 18 \leq -8x + 8x + 36$$

and constants on the other.

$$6x + 18 \; \leq \; 36$$

$$6x \; + \; 18 - 18 \; \leq \; 36 - 18$$

$$6x \; \leq \; 18$$

3. Get rid of the coefficient using inverse operations. Are you multiplying or dividing by a negative? No.

$$\frac{6x}{6} \leq \frac{18}{6} \rightarrow \; x \leq 3$$

4. Check your solution by choosing a number less than 3, say –5.

$$4(-5) \; - 2(3 \cdot -5 - 9) \; \leq -4(2 \cdot -5 - 9)$$

$$\checkmark 28 \; < \; 76$$

Practice 2: Solve the multi-step inequalities

1) 5 (g + 1) ≤ 3 (g + 2)

2) 10 – 6x ≤ 2(2x + 30)

3) 13 + 2v – 8 + 6 > -7 -v

4) –x < -x + 7(x – 2)

5) 18 ≥ 5k + 4k

6) a – 6 ≤ 15 + 8a

Identifying the Number of Solutions to an Inequality

Inequalities can have infinitely many solutions, no solutions, or a finite set of solutions. Most of the inequalities you have solved to this point have an infinite amount of solutions. By solving inequalities and using the context of a problem, you can determine the number of solutions an inequality may have.

Example: Find the solutions to $x - 5 > x + 6$.

Use inverse operations to get all variables on one side and constants on the other

$$x - x - 5 > x - x + 6$$

Simplify.

$$-5 > 6$$

This is an **untrue inequality**. Negative five is never greater than six. Therefore, the inequality $x - 5 > x + 6$ has **no solutions**.

Previously we looked at the following sentence: “The speed limit is 65 miles per hour.”

The algebraic sentence for this situation is: s ≤ 65.

Example: Find the solutions to s ≤ 65.

The speed at which you drive cannot be negative. Therefore, the set of possibilities using interval notation is [0, 65], meaning it cannot go lower than zero or higher than 65.

Solving Real-World Inequalities

Most tests rely on real-world problems rather than just numbers. When working with a real-world problem, think about the situation and closely look at the words used to determine what inequality is.

Example: In order to get a bonus this month, Leon must sell at least 120 newspaper subscriptions. He sold 85 subscriptions in the first three weeks of the month. How many subscriptions must Leon sell in the last week of the month?

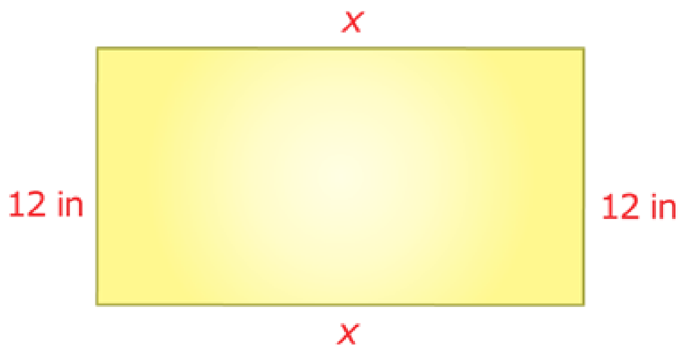
The amount of subscriptions Leon needs is “at least” 120. Choose a variable to represent the varying quantity–the number of subscriptions, say n.

The inequality that represents the situation is $n + 85 \geq 120$.

Solve using inverse operations: $n \geq 35$.

Leon must sell 35 or more subscriptions to receive his bonus.

Example: The width of a rectangle is 12 inches. What must the length be if the perimeter is at least 180 inches? (Note: Diagram not drawn to scale.)



The perimeter is the sum of all the sides.

$$12 + 12 + x + x \geq 180$$

Simplify and solve for the variable x:

$$12 + 12 + x + x \geq 180 \rightarrow 24 + 2x \geq 180$$

$$2x \geq 156$$

$$x \geq 78$$

The length of the rectangle must be 78 inches or larger.

Practice 3:

At Woodland Park Zoo, you can either pay $19.95 for the entrance fee or $49 for the yearly pass, which entitles you to unlimited admission. At most, how many times can you enter the zoo for the $19.95 entrance fee before spending more than the cost of a yearly membership?

Mixed Practice

Solve each of the inequalities.

1) $6x - 5 < 10$

2) $-9x < -5x - 15$

3) $\frac{-9x}{5} \leq 24$

4) $\frac{9x}{5} - 7 \geq -3x + 12$

5) $\frac{5x-1}{4} > 2(x + 5)$

6) $4x + 3 < -1$

7) $2x < 7x - 36$

8) $5x > 8x + 27$

9) $5x - x < 9 + x$

10) $4 - 6x \leq 2(2x + 3)$

11) $5(4x + 3) \geq 9(x - 2) - x$

12) $2(2x - 1) + 3 < 5(x + 3) - 2x$

13) $8x - 5(4x + 1) \geq -1 + 2(4x - 3)$

14) $2(7x - 2) - 3(x + 2) < 4x - (3x + 4)$

15) Proteek's scores for four tests were 82, 95, 86, and 88. What will he have to score on his last test to average at least 90 for the term?

16) Raul is buying ties and he wants to spend $200 or less on his purchase. The ties he likes the best cost $50. How many ties could he purchase?

17) Virena's charity is trying to raise at least $650 this spring. How many boxes of cookies must they sell at $4.50 per box to reach their goal?

Answers:

Review: x < -6; x > -2; x ≤ 19; x ≤ -14

Practice 1: 1) x > 1   2) x < 2;   3) n > -24;   4) x > $\frac{1}{4}$

Practice 2: 1) g ≤ 1/2   2) x ≥ -5;   3) v > -6;   4) x > 2;   5) k ≤ 2;   6) a ≥ -3

Practice 3: 19.95x ≤ 49 so x ≤ 2.46 You can enter the park 2 times before spending more than the yearly pass.

Mixed Practice:

1) x < 2.5   2) x > 3.75   3) x ≥ -13.33   4) x ≥ 3.95833   5) x > -3   6) x < -1

7) x > 7.2   8) x < -9   9) x > -2   10) x ≥ - 0.2   11) x ≥ -2.75   12) x < 14

13) x ≤ 0.1   14) x < 0.6

15) $\frac{351+x}{5} \geq 90$ so $x \geq 99$. He must score at least 99 on his last test.

16) $50x \leq 200$; $x \leq 4$ He can buy between 0 and 4 ties.

17) $4.5x \geq 650$ so $x \geq 144.44$ She must sell at least 145 boxes to reach her goal.

Solving Quadratic Equations Using the Quadratic Formula

Introduction

Until now, we have been solving equations with the degree of 1 such as $5x + 2 = 17$. What if an equation has a degree of two? Sometimes this is simple. For example, in $x^2 = 25$ we can find the square root of 25, which is 5, so $x = 5$ or $x = -5$. (Both 5^2 and -5^2 equal 25).

However, sometimes it is not so simple. What do we do if we have a variable, *x, and* that variable squared, x^2 in the same equation, such as $\mathbf{x^2 + 4x = 25}$**?** We can't get *x* and x^2 both by themselves in this situation. Fortunately, mathematicians have found a way to solve this problem. They have discovered that the **quadratic formula** can narrow it down to two possible answers.

The word **quadratic refers specifically to equations with a degree of 2. This formula does not work with equations that have other degrees.

Who uses this anyway?

According to www.montereyinstitute.org, quadratic equations are used a lot in science, business, and engineering. They are used to describe things that rise and fall, such as water in a fountain, a bouncing ball, profit and loss, or anything launched into the air that falls due to gravity. The mathematicians in the movie *Hidden Figures* were using this formula to calculate orbit.

The Quadratic Equation

In order to use the quadratic formula to solve a quadratic equation, the equation must be in the form:

$\boldsymbol{a}x^2 + \boldsymbol{b}x + \boldsymbol{c} = 0$

That means if you simplified an equation to $\mathbf{x^2 + 4x = 25}$, you would need to subtract 25 from both sides before you could solve it. This gives you $x^2 + \boldsymbol{4}x - \boldsymbol{25} = 0$

Once in this form, we should note what ***a, b,*** **and** ***c*.** We will need these in our formula.

In the example above there is no number before x^2, so ***a = 1***. Four comes before the x, so ***b = 4***. The constant is -25, so ***c = -25***. (Since the formula has a +*c*, subtraction counts as a negative.)

Practice 1:

State the value of *a, b,* and c for each of the following quadratic equations.

1. $2x^2 + 7x - 1 = 0$ $\quad a = \quad b = \quad c =$
2. $3x^2 + 2x = 7$ $\quad a = \quad b = \quad c =$
3. $9x^2 - 7 = 4x$ $\quad a = \quad b = \quad c =$
4. $2x^2 - 7 = 0$ $\quad a = \quad b = \quad c =$
5. $4 - 2x^2 = 11x$ $\quad a = \quad b = \quad c =$

The Quadratic Formula

Here is the quadratic formula. (This appears near the bottom of your formula page.)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

To solve the quadratic **formula,** take *a, b, and c* from the quadratic **equation** and put them into the formula. Use your calculator to solve it.

****IMPORTANT:** When squaring a negative number on the TI-30XS calculator, you must put parentheses around the negative and the number. For example: Typing -3^2 results in -9. (It is the negative of the result of 3^2) Typing $(-3)^2$ results in 9. (The result of -3 x -3.)

Example A

Solve the following quadratic equation using the quadratic formula: $3x^2 - 5x + 2 = 0$

Step 1: a = 3 b = -5 c = 2

Step 2:

$$\frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(2)}}{2(3)}$$

Use your calculator to solve once with "+" for the answer of **1 and** once with "- "for the answer of 23. Both answers together are called a **solution set** and written: **{**1, 23} **They** are both possible because the square root of something could be two positive or two negative numbers.

Example B

Use the quadratic formula to determine the roots of the equation **to the nearest tenth**. (Use the toggle button on your calculator TI 30XS to move from exact roots to a decimal.)

Solve the quadratic equation $2x^2 - 3x = 3$

Step 1: set the equation to equal zero $2x^2 - 3x - 3 = 0$

Step 2: locate *a, b, and c.* $a = 2 \quad b = -3 \quad c = -3$

Step 3: put *a, b, and c* into the formula and solve

$$\frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-3)}}{2(2)}$$

The solution set to the nearest tenth is {2.2, -0.7}

Example C

Solve the Equation $6x^2 - 8x = 0$

*This equation does not have a "*c*" term. The value of *c* is 0.

Step 1: locate *a, b, and c.* $a = 6 \quad b = -8 \quad c = 0$

Step 2: put *a, b, and c* into the formula and solve

$$\frac{-(-8) \pm \sqrt{(-8)^2 - 4(6)(0)}}{2(6)}$$

The solution set is {43 , 0}

Practice 2:

Determine the solution set of the following quadratic equations to the nearest tenth by using the quadratic formula.

6) $2x^2 = 8x - 7$

7) $6x = 2 - x^2$

8) $1 = 8x + 3x^2$

9) $(4x - 2)(x + 1) - (x + 3) = 0$

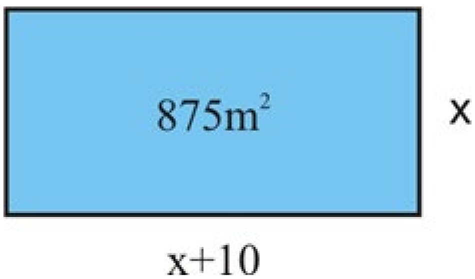
10) $x^2 - 2x - 5 = 0$

11) $3x^2 + 7x = 1$

Real Life Problems

Example: The length of a rectangular pool is 10 meters more than its width. The area of the pool is 875 square meters. Find the dimensions of the pool.

Begin by drawing a sketch. The formula for the area of a rectangle is A=l(w).



$$A = (x + 10)\,(x)$$

$$875 = x^2 + 10x$$

Now solve for x using the quadratic formula, where a = 1, b = 10, c = -875

The solution set is {25, -35} Since you cannot have a negative measurement, x = 25.

So, the length of the pool is 35 meters, and the width is 25 meters.

Practice 3: Use the quadratic formula to solve the problems.

12) The product of two consecutive integers is 72. Find the two numbers.

13) The length of a rectangle exceeds its width by 3 inches. The rectangle area is 70 square inches. Find its dimensions. Use area = length x width

Mixed Practice:

Name the following:

1) $ax^2 + bx + c = 0$

2)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

3) {2, 0}

Solve the following quadratic equations using the Quadratic Formula.

Round answers to the nearest tenth.

4) $x^2 + 4x - 21 = 0$

5) $x^2 - 6x = 12$

6) $3x^2 - 12x = 38$

7) $2x^2 + x - 3 = 0$

8) $-x^2 - 7x + 12 = 0$

9) $-3x^2 + 5x = 0$

10) $4x^2 = 0$

11) $x^2 + 6x + 2 = 0$

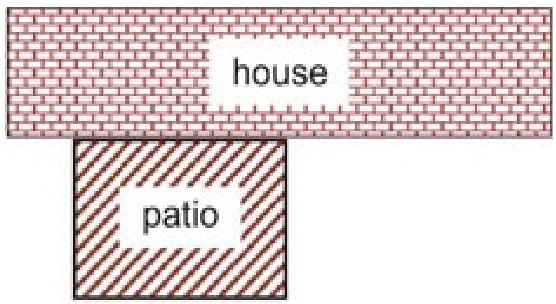
12) $x^2 - x = 6$

13) $x^2 + 7x - 18 = 0$

Solve the real-life problems:

14) The product of two consecutive odd integers is 1 less than 3 times their sum. Find the integers.

15) Mike wants to fence three sides of a rectangular patio that is adjacent to the back of his house. The area of the patio is 192 ft^2 and the length is 4 feet longer than the width. Find how much fencing Mike will need



Answers:

Practice 1:

1. $2x^2 + 7x - 1 = 0$ *a = 2* *b = 7* *c = -1*
2. $3x^2 + 2x - 7 = 0$ *a = 3* *b = 2* *c = -7*
3. $9x^2 - 4x - 7 = 0$ *a = 9* *b = -4* *c = -7*
4. $2x^2 + 0x - 7 = 0$ *a = 2* *b = 0* *c = -7*
5. $-2x^2 - 11x + 4 = 0$ *a = -2* *b = -11* *c = 4*

6. $2x^2 = 8x - 7$ equals $2x^2 - 8x + 7 = 0$ *a = 2, b = -8, c = 7;* {**2.7, 1.3**}	7. $6x = 2 - x^2$ equals $x^2 + 6x - 2 = 0$ *a = 1, b = 6, c = -2;* {**0.3, -6.3**}
8. $1 = 8x + 3x^2$ equals $-3x^2 - 8x + 1 = 0$ *a=-3, b = -8, c = 1;* {**-2.8, 0.1**}	9. $(4x - 2)(x + 1) - (x + 3) = 0$ Simplified is $4x^2 + x - 5 = 0$ *a=4, b = 1, c = -5; {1,* **-1.3**}

10. $x^2 - 2x - 5 = 0$ *a = 1, b = -2, c = -5;* **{3.4, -1.4}**	11. $3x^2 + 7x = 1$ equals $3x^2 + 7x - 1 = 0$ *a = 3, b = 7, c = -1;* **{0.1, -2.5}**

Practice 3:

12) one integer is x, the other is (x + 1). Set up the equation x (x + 1) = 72. Simplify to

$x^2 + x = 72$.

Set to quadratic equation $x^2 + x - 72 = 0$; a = 1, b = 1, c = -72 This gives a solution set of {8, -9}

The two consecutive numbers must be **8, 9 OR -9, -8.**

13) Let the width be *w* and the length be *w + 3*. If area = length x width,

then 70 = w (w + 3) this simplifies to $70 = w^2 + 3w$.

In quadratic form it is $w^2 + 3w - 70 = 0$; a = 1, b = 3, c = -70 for a solution set of {7, -10}. Since you cannot have a negative measurement, the width is 7 and the length is 10.

Mixed Practice:

1. Quadratic equation, 2. Quadratic formula, 3. Solution set

4) {3, -7} 5) {7.6, -0.3} 6) {0.4, -0.3} 7) {1, -1.5} 8) {-8.4, 1.4} 9) {0, 1.7}

10) 0 11) {-0.4, -5.6} 12) {3, -2} 13) {1, 7.4}

14) let the first odd number be x and the consecutive odd number be (x + 2). The problem

states that x (x + 2) = 3 (x + x +2) - 1 Simplified and put in quadratic equation form it is $x^2 - 4x - 5 = 0$ with a solution set of {5, -1} The consecutive numbers are either **5, 7 or -1, 1**

15) Let the width be *w* and the length be *w + 4*. If area = length x width, then 192 = w (w + 4). This simplifies to $192 = w^2 + 4w$. In quadratic form it is $w^2 + 4w - 192 = 0$ with a solution set of {12, -16}. Since you cannot have a negative measurement, the width must be 12. This means the length is 16. The question is about fencing. The diagram shows 2w plus a length, or 2(12) + 16 which equals 40. **Mike will need to buy 40 feet of fencing.**

Solve Quadratics using Factoring when a = 1

Quadratic Equations can be used to describe the motion of objects. They are called "quadratic" because "quadra" means "square," and the first term of a quadratic equation is always squared.

Solving a quadratic equation can tell you when an object will land on the ground after being thrown, or even how high it will be off the ground during its travel. Which careers do you think would be interested in this information?

Objective: Solve a quadratic equation in the form $\mathbf{ax^2 + bx + c}$ **when a = 1**, by factoring.

Does the formula above look familiar? We have been using the quadratic formula to solve this question, but there is another way to solve quadratic formulas, called *factoring. Sometimes this method is easier than the quadratic formula.*

Factoring is when we find out which numbers multiply to be another number. (If your times tables are not strong, you can work backwards by dividing the number in question by one, then two and so on. We are only looking for whole numbers right now.)

Example: Find the factors of 18

They are 1 x 18, 2 x 9, 3 x 6 and their negatives –1 x -18, -2 x -9, -3 x -6.

Practice 1: Find the factors.

1) 24:

2) 36:

3) 15:

Feel like you have the hang of that? (If not, search online for "factoring worksheets") We will use this skill when factoring quadratics and take it a bit further.

When you factor quadratics, you are looking for **two numbers that add to one number and multiply to be another.** For example, 2 and -3 add to be -1 and multiply to be -6. When factoring quadratics, however, we work backwards.

Example: Find two numbers whose sum is 10 and product is 16.

> Start by factoring the product 16: 1 x 16, 2 x 8, 4 x 4, -1 x-16, -2 x-8, -4 x-4
>
> Look at these sets. Do any of them add to be 10? Yes 2+8 = 10

The numbers we are looking for are 2 and 8 ($2 \times 8 = 16$: $2 + 8 = 10$)

Practice 2: Find the two numbers that fit the sums and products given.

4) sum 10; product 21

5) sum 11; product 30

6) sum 4; product -5

Factoring When $a = 1$

Now that you are familiar with those skills, we can look more closely at quadratic equations. A **quadratic equation** has the form $ax^2 + bx + c = 0$. For all quadratic equations, 2 is the largest and only exponent.

In this lesson, we are going to focus on factoring **when $a = 1$,** or when there is no number in front of x^2 . These are the simplest quadratic equations to factor.

Previously, we learned to multiply binomials using the FOIL method.

Example: multiply (y + 3) (y + 2)

1. Use Distributive Property (FOIL)

    F = multiply the first terms of each binomial $y(y) = \mathbf{y^2}$

    O = multiply the outer terms $y(2) = 2\mathbf{y}$

    I = multiply the inner terms $3(y) = 3\mathbf{y}$

    L = multiply the last terms of each binomial $3(2) = \mathbf{6}$

2. Add the results together $\mathbf{y^2} + 2\mathbf{y} + 3\mathbf{y} + 6$
3. Simplify if necessary: $\mathbf{y^2} + 5\mathbf{y} + 6$

The result is a quadratic expression! When we factor a quadratic expression, we are undoing FOIL to get back to the original binomials.

Notice that the numbers in the original binomial, (y + 3) (y + 2), add to be 5 (the b in our quadratic expression) and multiply to be 6 (the c in our quadratic expression).

The official rule states:

$$(x + m)(x + n) = x^2 + bx + c$$

$$\text{where } m + n = b \text{ and } mn = c$$

In other words:

1. The **constant** term, (called *c)*, is equal to the **product** of the constant numbers inside each factor.
2. the **coefficient** in front of *x*, (called *b)*, is equal to the **sum** of these numbers.

Let's put this all together.

Example: Factor $x^2 + 5x + 6$

Step 1: We are looking for two numbers that multiply to be 6 and add to be 5.

Step 2: Factor the constant (c), 6: 1x6, 2x3, -1x-6, -2x-3

Step 3: Which set adds to be the coefficient *(b),* 5? -1 + 6 = 5

Step 4: The number set we want is -1 and 6.

Step 5: Put them into the binomial (x - 1) (x + 6).

Answer: Since multiplication is commutative, the order does not matter. (x + 6) (x -1) is also correct.

To check, multiply the binomials. You should end up with the quadratic expression you started with.

Example: Factor $x^2 + 6x + 8$

Step 1: Factor *c*, 8: 1 x 8, 2 x 4, -1 x -8, -2 x-4

Step 2: Which adds to be *b*, 6? 2 and 4

Step 3: Put them into the binomial: (x + 2) (x + 4)

Answer: x^2 + 6x + 8 factors to (x + 2) (x + 4)]

Example: Factor $x^2 - 4x$

This is an example of a quadratic expression that has no *c*, or constant term. We could use the above method, looking for two numbers that multiply to 0 and add to be -4. They would be 0 and -4 and give us (x + 0) (x - 4), or x (x - 4), but there is a simpler way:

To factor this, we will simply factor out the x by dividing both terms by x.

$\frac{x^2-4x}{x} = \frac{x^2}{x} = \frac{4x}{x}$ gives us $(x - 4)$

To put it back together we would multiply the x back in x (x - 4) These are our factors.

Example: Factor $x^2 + 1x - 3$

Step 1: Factor *c*, -3: 1 x-3, -1 x3

Step 2: Which add to be *b*, 1? none of them

Answer: We say this is **not factorable.**

Practice 3: Factor the following quadratic expressions, if possible:

7) $x^2 - 9x + 20$

8) $x^2 + 7x - 30$

9) $x^2 + x + 6$

10) $x^2 + 10x$

Solving a Quadratic Equation

Now that we can factor, it is one simple step to solving the equation. In order to solve the quadratic equation, we need to set the factors to equal zero and use the zero-product property.

The zero-product property says that if $ab = 0$ then either $a = 0$ or $b = 0$.

Example: what is the solution set for the factors (x + 3) (x+ 7)?

1) Set the factors to zero: (x + 3) (x+ 7) = 0

2) Apply zero-product property:

If the first binomial = 0, then **x must be -3 because** -3 + 3 = 0. If the second binomial = 0, then **x must be -7** because -7 + 7 = 0.

3) Our solution set is {-3, -7}.

Example: Solve $x^2 - 9x + 18 = 0$

1) Factor: (x - 6) (x - 3) = 0

2) Apply zero product property to solve: $x = \{6, 3\}$

3) Optional: check your answer

$6x^2 - 9(6) + 18 = 0$ or $3^2 - 9(3) + 18 = 0$

$36 - 54 + 18 = 0$ **(Correct)** or $9 - 27 + 18 = 0$ (**Correct)**

Practice 4: go back to questions 7 and 8 (renumbered 11 and 12) and find the solution sets.

11) $x^2 - 9x + 20$ 12) $x^2 + 7x - 30$

13) Find the solution set for $x^2 - 5x = 6$

Mixed Practice:

Factor the following quadratic expressions. If one cannot be factored, write *not factorable.*

1) $x^2 - x - 2$ 2) $x^2 + 2x - 24$

3) $x^2 - 6x$ 4) $x^2 + 6x + 9$

5) $x^2 + 8x - 10$ 6) $x^2 - 11x + 30$

Solve the following quadratic equations by factoring.

7) $x^2 + 13x - 30 = 0$ 8) $x^2 + 11x + 28 = 0$

9) $x^2 - 8x + 12 = 0$ 10) $x^2 - 7x - 44 = 0$

11) $x^2 + 4x + 3 = 0$ 12) $x^2 - 8x - 20 = 0$

Answers:

Practice 1: 1) 24: 1 x 24, 2 x 12, 3 x 8, 4 x 6 and their negatives; 2) 36: 1x36, 2x18, 3x12, 4x8, 6 x 6 and their negatives; 3) 15: 1x15, 3x5, -1 x-15, -3 x-5

Practice 2: 4)3 and 7 5) 6 and 5 6) -1 and 5

Practice 3: 7) $(x - 4)(x-5)$ 8) $(x - 3)(x + 10)$ 9) not factorable 10) $x(x + 10)$

Practice 4: 11) $\{4, 5\}$ 12) $\{3, -10\}$ 13) set to 0 so $x^2 - 5x - 6 = 0$, factor $(x - 6)(x + 1) = 0$, solve $\{6, -1\}$

Mixed practice: 1) $(x - 2)(x + 1)$; 2) $(x + 6)(x - 4)$ 3) $x(x - 6)$

4) (x + 3) (x + 3)   5) not factorable   6) (x – 6) (x - 5)

7) (x + 15) (x - 2) so **{-15, 2}**   8) (x + 7) (x + 4) so **{-7, -4}**   9) (x – 6) (x - 2) so **{6, 2}**

10) (x – 11) (x + 4) so **{11, -4}**   11) (x + 3) (x + 1) so **{-3, -1}**   12) (x – 10) (x + 2) so **{10, -2}**

Solve Quadratics by Factoring when a > 1

Remember that quadratic equations are in the form $ax^2 + bx + c = 0$. In the last lesson, *a* = 1 so it was like nothing was there. In this lesson, however, *a* will be greater than one, which adds more steps to our solving process.

Remember also what it is like to multiply binomials using FOIL. This time, the variables have coefficients.

Example: multiply (2y + 3) (4y + 2)

1. Use Distributive Property (**FOIL**)

   F = multiply the first terms of each binomial 2y(4y) = $\mathbf{8y^2}$

   O = multiply the outer terms 2y (2) = **4y**

   I = multiply the inner terms 3(4y) = **12y**

   L = multiply the last terms of each binomial 3(2) = **6**

2. Add the results together $\mathbf{8y^2 + 4y + 12y + 6}$
3. Simplify if necessary: $\mathbf{8y^2 + 16y + 6}$

Now we have a quadratic expression with *a* > 1. To factor it back to its original binomials, we must undo it. Let's walk through an example remembering $\boldsymbol{ax^2 + bx + c}$

This time we are looking for two numbers that multiply to be ***ac*** (not just *c*) and add to be ***b***.

Example: Factor $6x^2 – x – 2$

1. Find two numbers that multiply to be *ac*, 6 x -2 = -12 and add to be *b*, -1.

   Factor *ac*, **-12**: 1x-12, -1x12, 2x-6, -2x6, 3x-4, -3x4

   Which set adds to be *b*, **-1**? 3 + -4 = -1 Our numbers are **-4** and **3**.

2. Now things have changed a little. Instead of putting these numbers into a binomial as we did when $a = 1$, we need to expand our quadratic expression by replacing the middle term with our found numbers as coefficients.

   $6x^2 - x - 2$ becomes $6x^2$ **- 4**x **+ 3**x – 2

   These two expressions are equal because **- 4**x **+ 3**x = -x

3. Next, group the first two terms together and the last two terms together

   $6x^2$ **- 4**x **+ 3**x – 2

   ($6x^2$ **- 4**x) **+ (3**x – 2)

4. Now we need to look at each binomial to see if we can pull out any common factors. A common factor is a number and/or variable that goes in to both the first and the second term.

In the first binomial we could divide both terms by 2x. 2x (3x - 2) + (3x – 2)

We can rewrite this as: 2x (3x - 2) + 1(3x – 2)

5. Take a good look at this. Do you see anything interesting? The terms inside the parentheses are the same! Since both 2x and 1 are multiplied to (3x – 2), we can rewrite it.

   2x (3x - 2) + 1(3x – 2) is the same as (2x + 1) (3x – 2)

   The factors of $6x^2 - x - 2$ are **(2x + 1) (3x – 2)**

   Since multiplication is commutative, (3x – 2) (2x + 1) is also correct.

Let's look at another example:

Example: Factor $4x^2 + 8x - 5$.

1. Find two numbers that multiply to be *ac*, 4 x -5 = **-20** and add to be *b*, **8**.

The factors of -20 that add up to 8 are **10 and -2.**

2. Rewrite the trinomial with the x-term expanded, using the two factors from Step 1.

   $4x^2 + 10x - 2x - 5$

3. Group the first two and second two terms together and factor where possible.

$(4x^2 + 10x) + (-2x - 5)$ The negative stays with 2x.

$2x(2x + 5) - 1(2x + 5)$ Factor 2x out of the first binomial and -1 out of the second. Note that this causes the -5 to become a +5.

$(2x + 5)(2x - 1)$ Rewrite as two binomials.

Question: What if we had written $4x^2 - 2x + 10x - 5$, reversing the middle terms?

Good question. Let's check it out. Go back to step 3:

4. Group the first two and second two terms together and factor where possible.

$(4x^2 - 2x) + (10x - 5)$

$2x(2x - 1) + 5(2x - 1)$ Factor 2x out of the first binomial and 5 out of the second.

$(2x - 1)(2x + 5)$ Rewrite as two binomials.

Answer: It does not matter. The binomials end up in a different order, but that is ok because multiplication is commutative: order does not matter.

Practice 2: Factor the following quadratics, if possible.

4) $15x^2 - 4x - 3$

5) $3x^2 - 7x + 2$

6) $5x^2 + 18x + 9$

7) $4x^2 + 8x + 3$

8) $10x^2 - x - 3$

9) $2x^2 + 7x + 3$

Solving Quadratic Equations when a > 1

We have been busy factoring quadratic expressions, now it's time to solve them.

Example: Solve for x: $(2x + 1)(3x - 2)$

1. Set the equation to equal 0. $(2x + 1)(3x - 2) = 0$

2. The zero product principle states either $(2x + 1)$ or $(3x - 2)$ must be equal to 0.

If $(2x + 1) = 0$, then we solve for x: $2x = -1$ and $x = - 12$

If $(3x - 2) = 0$, then we solve for x: $3x = 2$, $x = 23$

4. Our solution set is {-12, 23}

Practice 3: Find the solution sets for Practice 2, numbers 4 – 9. (Renumbered 10 – 15 below.)

10) $15x^2 - 4x - 3$

11) $3x^2 - 7x + 2$

12) $5x^2 + 18x + 9$

13) $4x^2 + 8x + 3$

14) $10x^2 - x - 3$

15) $2x^2 + 7x + 3$

Mixed Practice:

Factor and find the solution set for each quadratic equation:

1) $16x^2 - 6x - 1 = 0$

2) $4x^2 + 4x - 3 = 0$

3) $15x^2 - 14x - 8 = 0$

4) $4x^2 - 17x + 4= 0$

5) $6x^2 + 7x - 49= 0$

6) $6x^2 + 37x + 6= 0$

7) $6x^2 + 25x + 25= 0$

8) $2x^2 - 3x - 5= 0$

9) $5x^2 - 14x - 3= 0$

Answers:

Practice 1: 1) (b + 7) (b + 1) 2) (m – 9) (m + 10) 3) (n – 1) (n – 9) {1, 9}

Practice 2: 4) ac = -45, b = -4. The set -9, 5 work for both. Expanded is $15x^2 - 9x + 5x - 3$ so $(15x^2 - 9x) + (5x - 3)$ and 3x(5x-3) + 1 (5x-3) which is **(5x – 3) (3x +1)**

5) ac = 6, b = -7. The set -6, -1 work for both. Expanded is $3x^2 - 6x - 1x + 2$; $(3x^2 - 6x) + (-1x + 2)$ and 3x(x-2) – 1 (x – 2) which is **(x – 2) (3x – 1)**

6) (5x + 3) (x + 3), 7) (2x + 1) (2x + 3) 8) (5x – 3) (2x + 1) 9) (2x + 1) (x + 3)

Practice 3: 10) (5x – 3) (3x +1) = 0; $\{\frac{3}{5}, -\frac{1}{3}\}$ 11) (x – 2) (3x – 1) = 0; $\{2, \frac{1}{3}\}$

12)(5x + 3) (x + 3), $\{-\frac{3}{5}, 3\}$ 13) (2x + 1) (2x + 3); $\{-\frac{1}{2}, -\frac{3}{2}\}$

14) (5x – 3) (2x + 1) = 0; $\{\frac{3}{5}, -\frac{1}{2}\}$ 15) (2x + 1) (x + 3) = 0; $\{-\frac{1}{2}, -3\}$

Mixed Practice: 1) (8x + 1) (2x – 1); $\{-\frac{1}{8}, \frac{1}{2}\}$ 2) (2x – 1) (2x + 3); $\{\frac{1}{2}, -\frac{3}{2}\}$

3) (5x + 2) (3x – 4); $\{-\frac{2}{5}, \frac{4}{3}\}$ 4) (x – 4) (4x – 1); $\{4, \frac{1}{4}\}$
5) (3x – 7) (2x + 7); $\{\frac{7}{3}, -\frac{7}{2}\}$ 6) (x + 6) (6x + 1); $\{-6, -\frac{1}{6}\}$
7) (2x + 5) (3x + 5); $\{-\frac{5}{2}, -\frac{5}{3}\}$ 8) (x + 1) (2x – 5); $\{-1, \frac{5}{2}\}$ 9) (x – 3) (5x + 1); $\{3, -\frac{1}{5}\}$

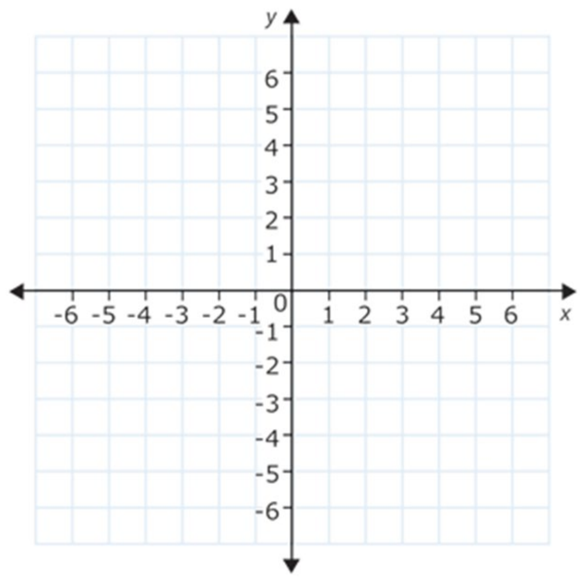
Introduction to the Coordinate Plane

Judy is on vacation in Portland and is trying to find a restaurant where she will meet her friends for lunch. Her phone died, so she got a map from her hotel that is formatted like a coordinate grid. The hotel is at the origin of the map, and she was told that the restaurant is at the point (4,5) on the map grid. How can Judy find this location on the map so she can make it to the restaurant?

In this lesson, you will learn how to name, and graph ordered pairs of integer coordinates in a coordinate plane.

Guidance

The **coordinate plane** is a grid created by a horizontal number line, the x-**axis**, intersecting a vertical number line, the y-**axis**. The point of intersection, where the two lines cross, is called the **origin**.



[Figure 2]

The coordinate plane allows you to describe locations in two-dimensional space.

Each point on the coordinate plane can be named by a pair of numbers called an **ordered pair** in the form (x, y).

- The first number in an ordered pair identifies the x-**coordinate** of the point. This coordinate describes how far from the origin a number is horizontally.
- The second number in an ordered pair identifies the y-**coordinate** of the point. This coordinate describes how far from the origin a number is vertically.

Each square also has a name. The top right square is quadrant one (I). The top left square is quadrant two (II). The bottom left square is quadrant three (III). The bottom right square is quadrant four (IV). They are usually numbered in roman numerals.

Here is an example.

Plot the **point** $(3, -4)$ on the coordinate plane.

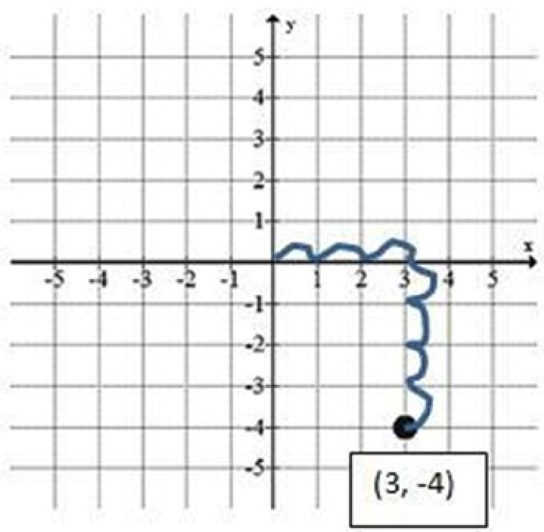
This point has an x-coordinate of 3 and a y-coordinate of -4.

To plot this point, first **start at the origin** (0, 0).

Then, **find the location of the** x-**coordinate**. Because the x-coordinate is positive, you will be moving to the right. Move to the right along the x-axis 3 units until you find 3.

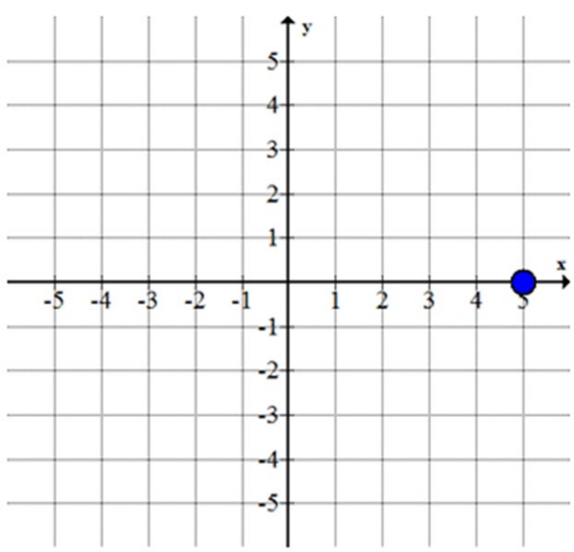
Next, **look at your y-coordinate**. Because the y-coordinate is negative, you will be moving down. Move down from the 3 on the x-axis 4 units until you are lined up with the -4 on the y-axis.

The answer is shown plotted on the coordinate plane below. It is in quadrant IV (four).



Here is another example.

Give the ordered pair for the point plotted below.



[Figure 4]

To write the ordered pair you need both the x-coordinate and the y-coordinate.

1. Start at the origin. You need to figure out how far to the right/left you need to move and then how far up/down you need to move to reach your point.

2. First notice that you need to **move 5 units to the right** from the origin to reach the point. This means the x-coordinate is 5.

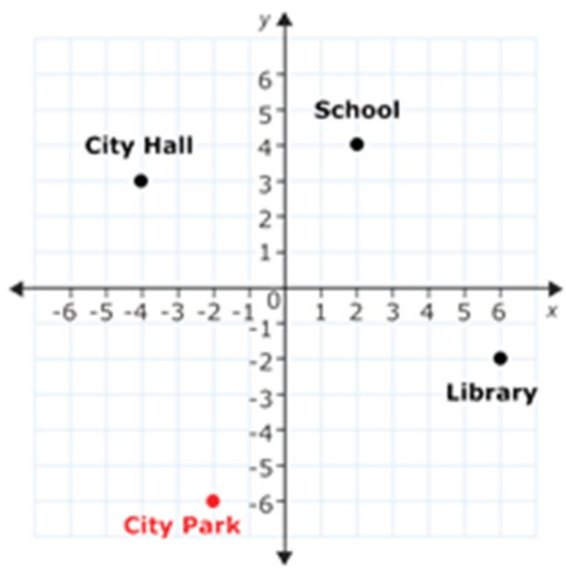
3. Next notice that **you do not need to move up or down** from the x-axis at all to reach the point. This means the y-coordinate is 0.

The **answer** is that the ordered pair is (5,0). What quadrant is it in?

It is not in any quadrant because it is on the x-axis.

Guided Practice

This coordinate grid shows locations in Mohammed's city. Name the ordered pair that represents the location of the city park and the quadrant it is in.



[Figure 5]

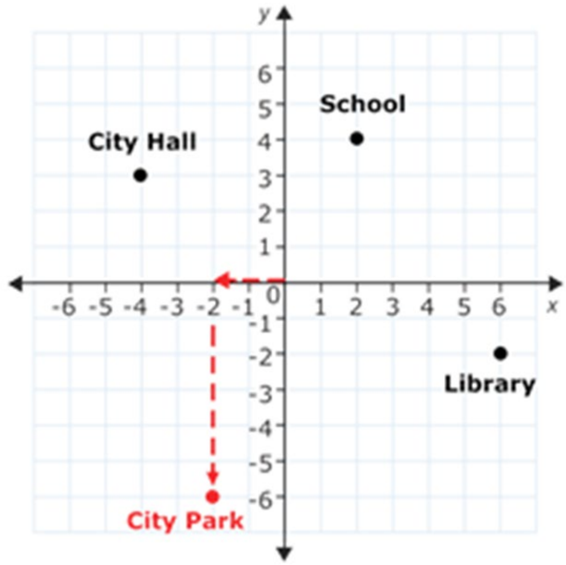
To write the ordered pair you need both the x-coordinate and the y-coordinate.

1.**Start at the origin.**

2. First you need to move 2 units to the left from the origin to be exactly above the point of the city park. This means the x-coordinate is -2.

3. Next you need to move down 6 units from the -2 on the x-axis to reach the point. This means that the y-coordinate is -6.

The arrows below show how you should have moved your finger to find the coordinates.



[Figure 6]

The **answer** is that the ordered pair for the city park is (−2,6). It is in quadrant 3.

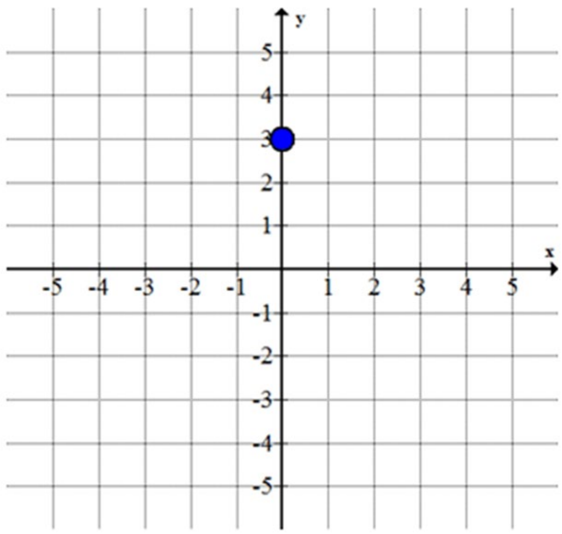
Example 1

Plot the point (0,3) on the coordinate plane.

This point has an x-coordinate of 0 and a y-coordinate of 3.

1) **To plot** this point, first start at the origin.
2) **Then**, find the location of the x-coordinate. Because the x-coordinate is 0, you do not need to move to the right or to the left from the origin. Stay at the origin.
3) **Next,** look at your y-coordinate. Because the y- coordinate is positive you will be moving up. Move up from the origin 3 units until you are lined up with the 3 on the y-axis.

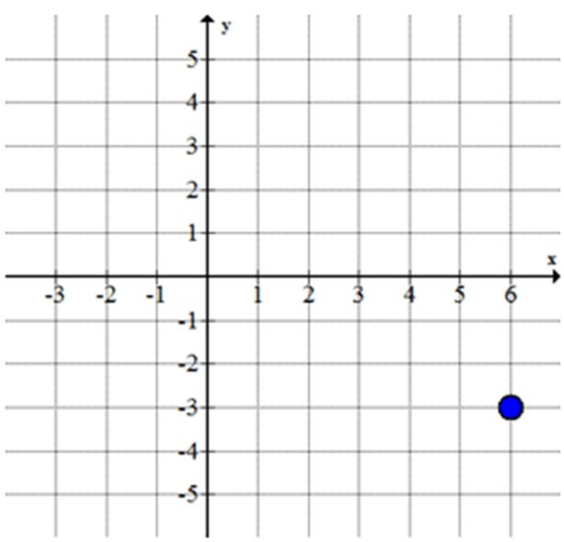
The **answer** is shown plotted on the coordinate plane below.



[Figure 7]

Example 2

Give the ordered pair for the point plotted below.



[Figure 8]

To write the ordered pair you need both the x-coordinate and the y-coordinate.

1) **Start** at the origin.
2) **First** notice that you need to move 6 units to the right from the origin to be exactly above the point. This means the x-coordinate is 6.

3) **Next** notice that you need to move down 3 units from the 6 on the x-axis to reach your point. This means your y-coordinate is -3.

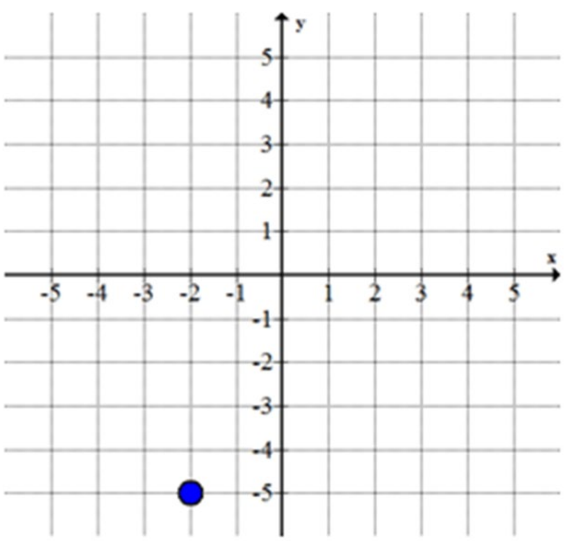
The **answer** is that the ordered pair is $(6, -3)$. It is in quadrant IV.

Example 3

Plot the point $(-2, -5)$ on the coordinate plane.

1) **To plot** this point, first start at the origin.
2) **Then,** find the location of the x-coordinate. Because the x-coordinate is negative, you will be moving to the left. Move to the left 2 units until you reach the -2 on the x-axis.
3) **Next**, look at your y-coordinate. Because the y-coordinate is negative, you will be moving down. Move down 5 units from the -2 on the x-axis.

The answer is shown plotted on the coordinate plane below.



[Figure 9]

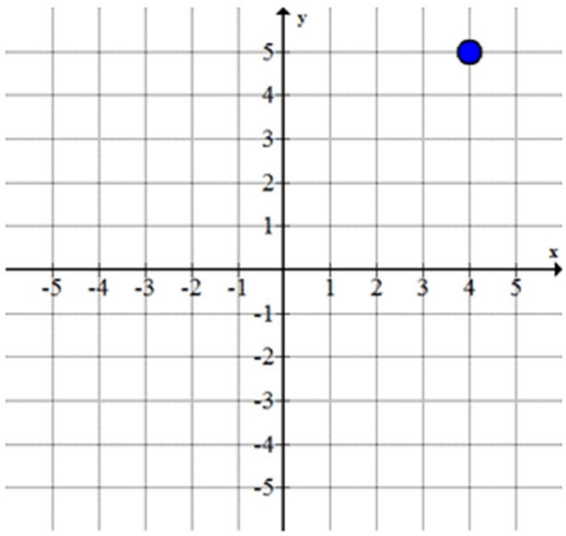
Follow Up

Remember Judy on vacation in Rome? She is at her hotel and trying to use a map to find the location of the restaurant where she will be meeting her friends for lunch. The restaurant is at the point (4,5) on the map.

1) To find this point, Judy should **look at the origin** on the map (her hotel).
2) **Then**, she should find the location of the x-coordinate. Because the x-coordinate is positive, she will be moving to the right. She needs to move to the right 4 units until she reaches the 4 on the x-axis.

3) **Next**, Judy should look at her y-coordinate. Because the y-coordinate is positive, she will be moving up. She should move up 5 units from the 4 on the x-axis.

The **answer** is shown plotted on the coordinate plane below. If each number represents a city block, she has directions to the hotel.



[Figure 10]

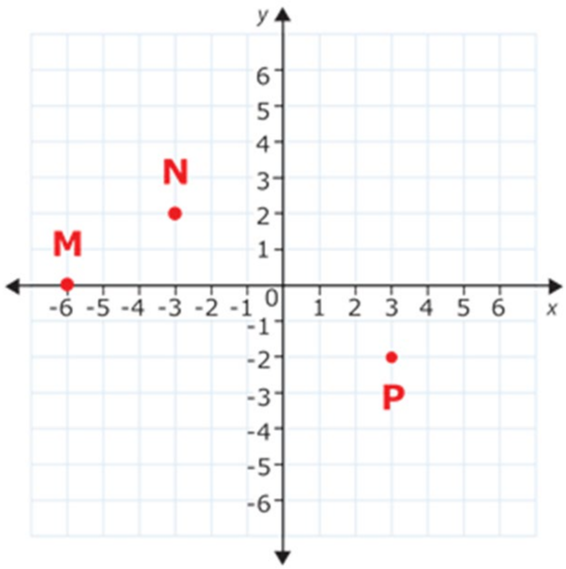
Video Review

The video below reviews ordered pairs and the coordinate plane.

https://youtu.be/s7NKLWXkEEE

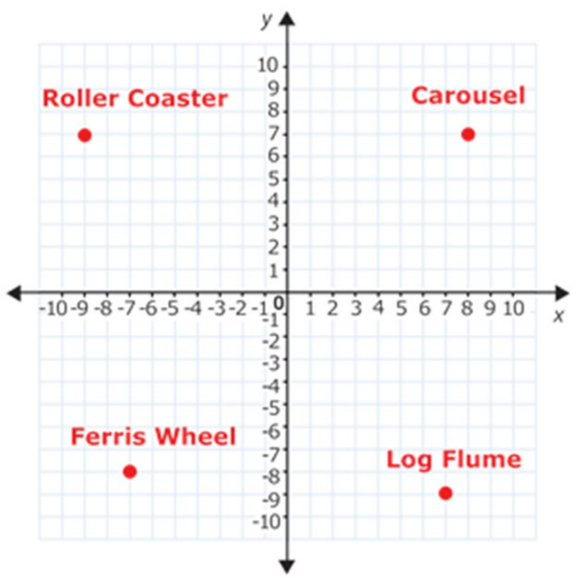
Practice

1. Name the ordered pair that represents each of these points on the coordinate plane.



[Figure 11]

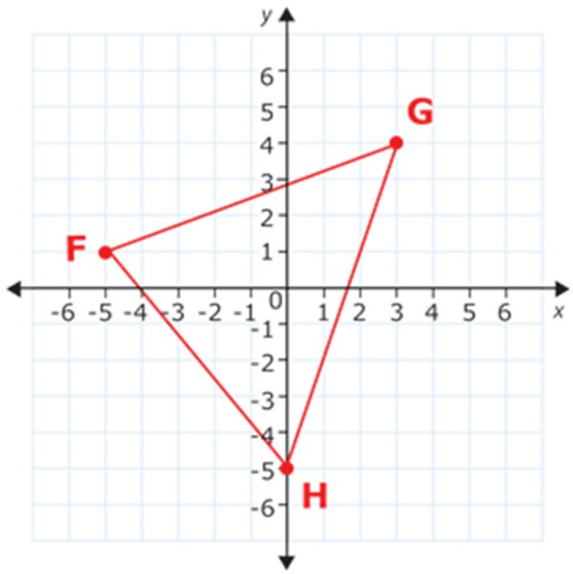
2.Below is a map of an amusement park. Name the ordered pair that represents the location of each of these rides.



[Figure 12]

- Roller coaster
- Ferris wheel
- Carousel
- Log flume

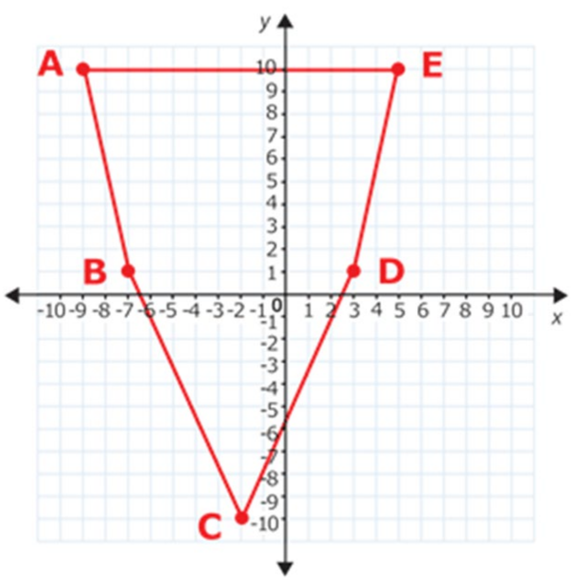
3. Name the ordered pairs that represent the vertices of triangle FGH.



[Figure 13]

F
G
H

4. Name the ordered pairs that represent the vertices of pentagon ABCDE.



[Figure 14]

A

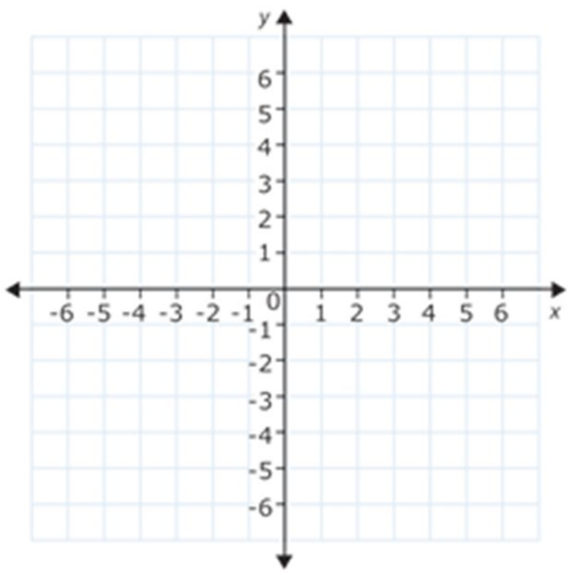
B

C

D

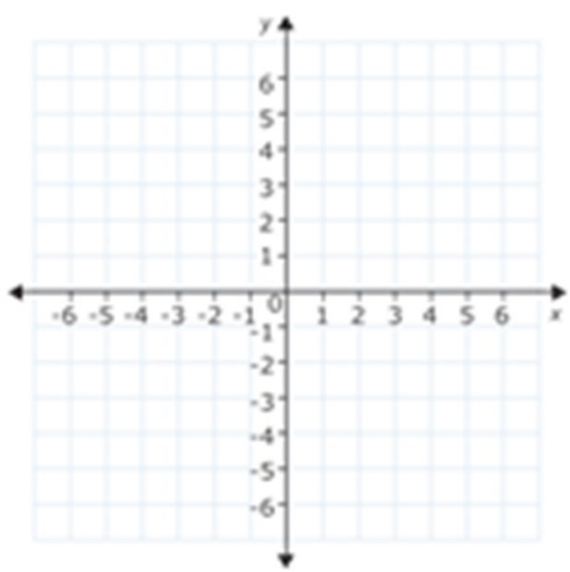
E

5. On the grid below, plot point V at $(-6,4)$.



[Figure 15]

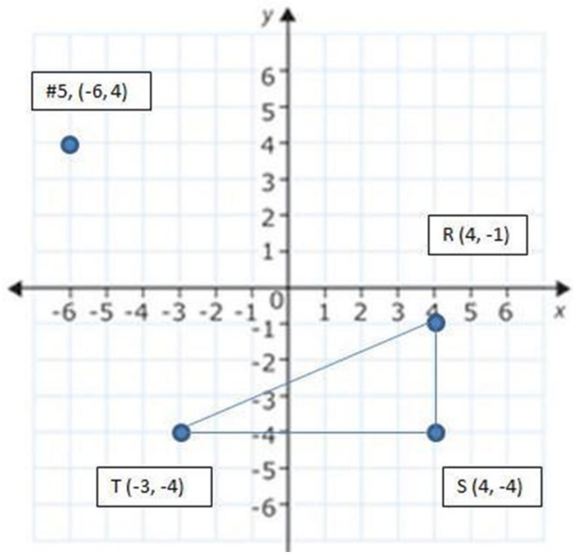
6. On the grid below, plot point a triangle with vertices $R(4,-1)$,$S(4,-4)$ and $T(-3,-4)$.



[Figure 16]

Answers:

Answers: 1. M (-6, 0) N (-3,2) P (3, -2)   2. Roller Coaster (-9,7) Q. II, Carousel (8,7) Q. I, Ferris Wheel (-7, -8) Q.III, Log Flume (7,-9)Q.IV. 3.F (-5,1) Q. II, G (3,4) Q. I, H (0, -5) no quadrant. 4.A (-9,10) B (-7,1) C (-2, -10) D (3,1) E (5, 10) 5 & 6 are on the same grid, below.



[Figure 17]

Graph Equations Using Tables

We have learned how to solve equations with one variable. The answer was of the form *variable = some number*. For example, x = 5.

In this lesson, you will learn how to solve equations with two variables. Below are several examples of two-variable equations:

$P = 20(h)$ $\quad$ $m = 8.25h$ $\quad$ $y = 4x + 7$

The solutions of these equations are not one value because there are *two variables*. The solutions to these equations are ordered pairs. Each ordered pair can be put on a table and graphed in a Coordinate Plane.

Example

A taxi fare costs more the further you travel. Taxis usually charge a fee on top of the per-mile charge. In this case, the taxi charges $3 as a set fee and $0.80 per mile traveled. Find all the possible solutions to this equation.

(miles)	(cost $)
0	3
10	11
20	19
30	27
40	35

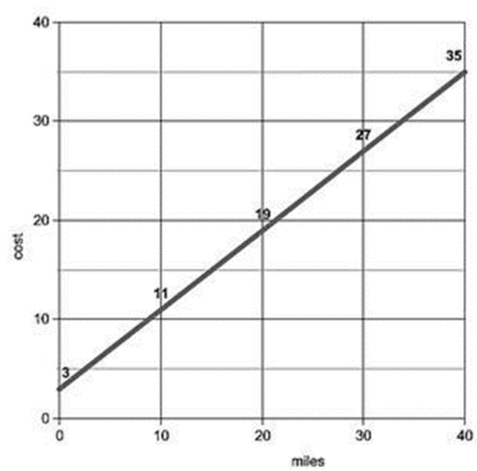
Solution: Here is the equation linking the cost in dollars (**y**) to hire a taxi and the distance traveled in miles (**x**): $\mathbf{y = 0.8\,x + 3}$

This is an equation in two variables. We can create a table by choosing **any number** for x and solving the equation to find the matching y.

The x and y's in the table make up ordered pairs. For example, (0,3) is an ordered pair from the first row in the table below. Now we can graph these ordered pairs to find the solutions.

X (miles)	Y (cost $)
0	3
10	11
20	19
30	27
40	35

The solutions to the taxi problem are located on the line graphed below. To find any cab ride cost, you just need to find the y-coordinate of the desired x-coordinate.



Practice 1:

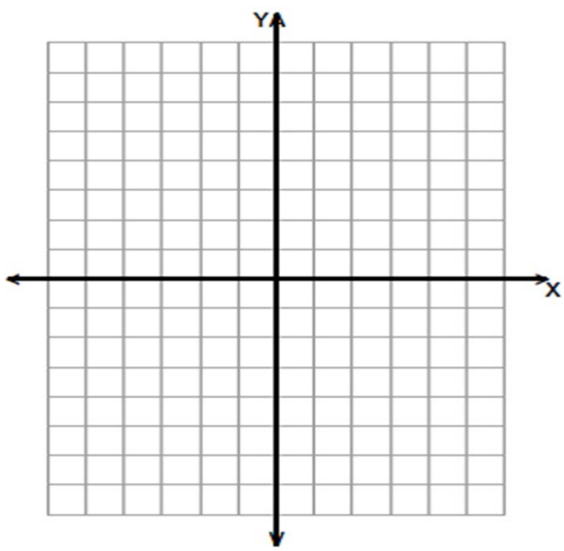
1. Cherry is thinking of being a nursing assistant but wants to know how much she will make. The information she has says that she will make $13/hour. She wants to know how much that is a day and a week.

Write an equation with two variables to describe how much she would earn in an hour:

Fill in the x/y table.

Hours worked (*X*)	Pay (*Y*)
1	
8	
40	

Plot the (x, y) coordinates below or on graph paper. Connect the points to show the result.



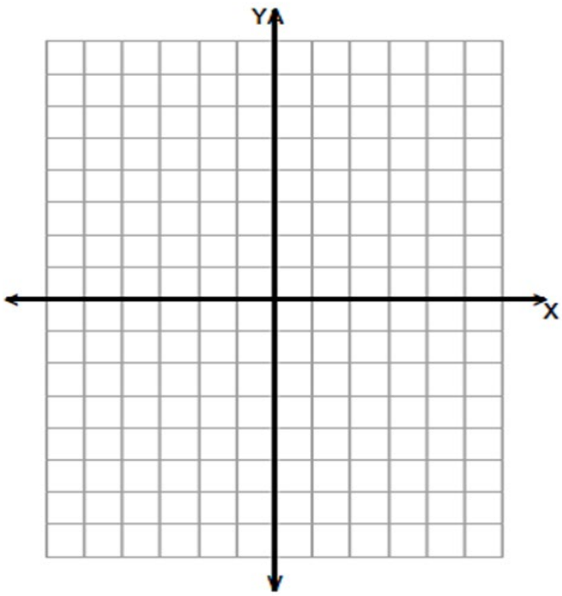
2. Mark landed a job selling cars. He will make $100/day plus $125 for every car he sells.

Write an equation with two variables to describe how much he could earn in a day:

Fill out a table and draw a graph on your graph showing how much Mark could make depending on how many cars he sells.

Cars sold (x)	Pay (y)
1	
3	
10	

Plot the (x, y) coordinates below or on graph paper. Connect the points to show the result.



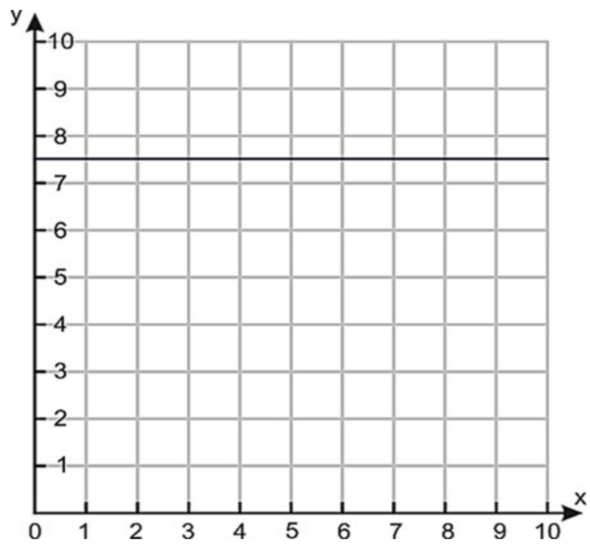
Graphs of Horizontal and Vertical Lines

Not all graphs are slanted. Some are horizontal or vertical. Read through the next situation to see why.

Example: "Mad-cabs" have an unusual offer going on. They are charging $7.50 for a taxi ride of any length within the city limits. Graph the function that relates the cost of hiring the taxi to the length of the journey in miles.

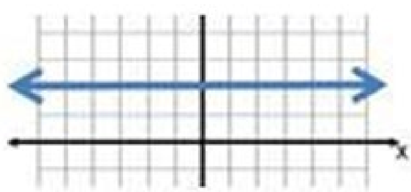
Solution: No matter the mileage, your cab fare will be $7.50. To see this visually, create a graph. You can also create a table to visualize the situation.

# Of miles	Cost
0	7.50
10	7.50
15	7.50
25	7.50
60	7.50

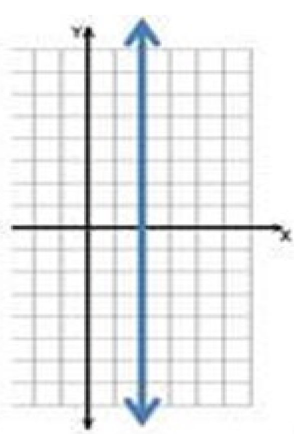


Because the mileage can be anything, the equation should relate only to the restricted value, in this case, y. The equation that represents this situation is: $y = 7.50$

Whenever there is an equation of the form ***y*= constant**, the graph is a horizontal line that intercepts the y-axis at the value of the constant. The graph below represents $y = 2$.



Similarly, if there is an equation of the form **x = constant**, the graph is a vertical line that intercepts the x-axis at the value of the constant. The graph below represents $x = 2$.

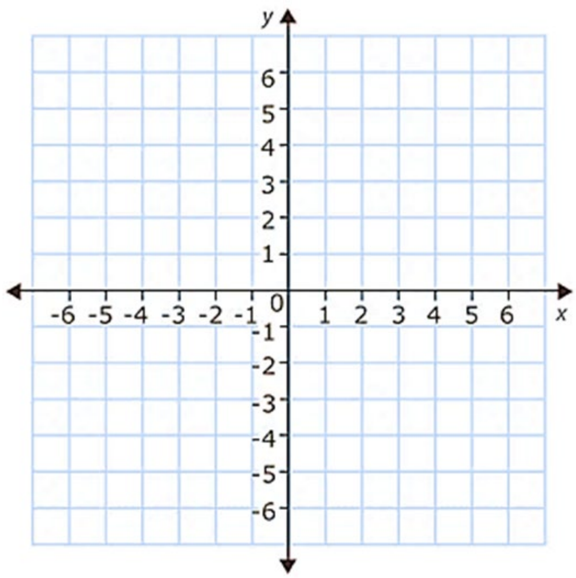


Practice 1:

Plot the following on the graph.

(Answers at the end of the lesson.)

(a) y = 4   (b) y = -4   (c) x = 4   (d) x = -4

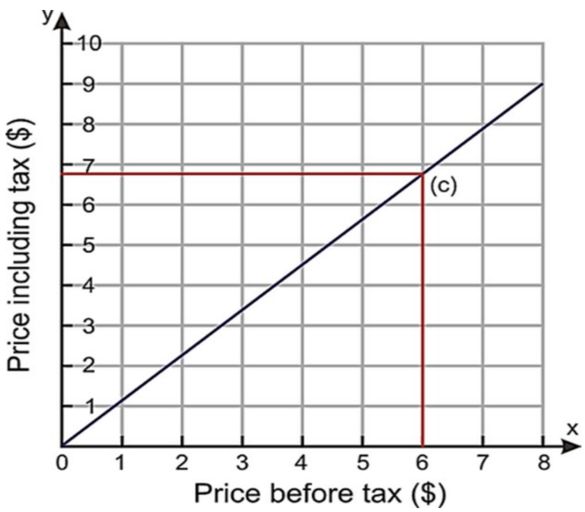


Analyzing Linear Graphs

Situation 1.

Analyzing linear graphs is a part of life – whether you are trying to decide to buy stock, figure out if your blog readership is increasing, or predict the temperature from a weather report. Although linear graphs can be quite complex, many are basic to analyze.

The graph below shows the solutions to the price before tax and the price after tax at a particular store. Determine the price after tax of a $6.00 item.



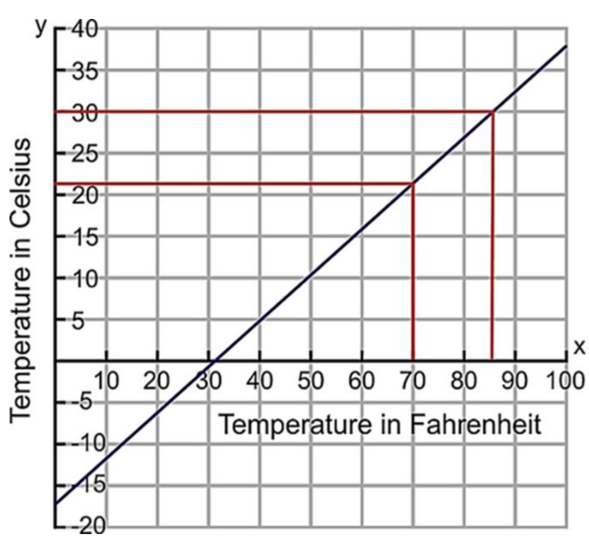
1. Find 6 on the *price before tax* axis.

2. Travel straight up until you reach the graph line.

3. From the graph line, travel left until you reach a number. In this case, the price after tax is about $6.80.

Situation 2.

The graph below shows the linear relationship between Celsius and Fahrenheit temperatures. Using the graph, convert 70°F to Celsius.



By finding the temperature of 70°F and locating its appropriate Celsius value, you can determine that 70°F ≈ 22°C.

Practice 2:

1. Using the tax graph from situation 1, determines the net cost of an item costing $8.00 including tax.

2. Using the temperature graph above, determine the following:

a. The Fahrenheit temperature of 0°C

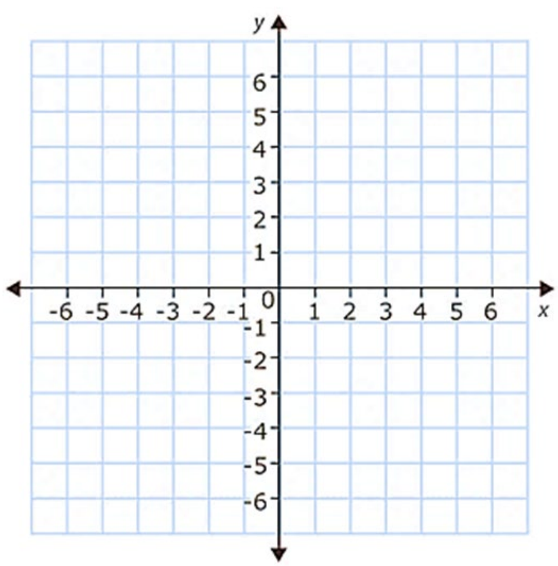
b. The Fahrenheit temperature of 30°C

c. The Celsius temperature of 0°F

Mixed Practice

1) Graph the following equations on **the** coordinate plane below.

y = -2 7 = x 4.5 = y x = - 6



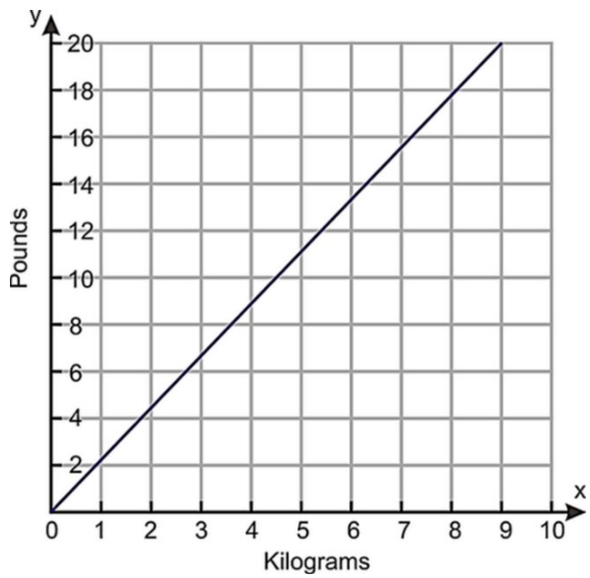
2) The graph below shows a conversion chart for converting between the weight in kilograms to weight in pounds. Use it to convert the following measurements.

a. 4 kilograms into weight in pounds

b. 9 kilograms into weight in pounds

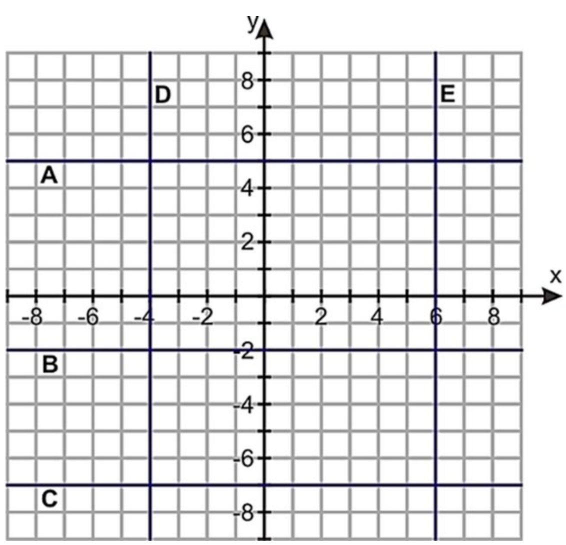
c. 12 pounds into weight in kilograms

d. 17 pounds into weight in kilograms



3) Write the equations for the graphed lines pictured below.

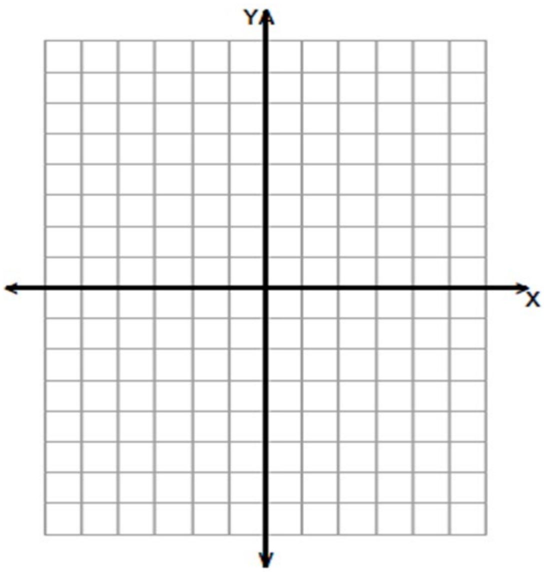
a. b. c. d. e.



4) At the airport, you can change your money from dollars into Euros. The service costs $5, and for every additional dollar you get 0.7 Euros. Make a table to show total cost depending on the number of dollars you exchange and plot the information on a graph. Use your graph to determine how many Euros you would get if you gave the exchange office $50.

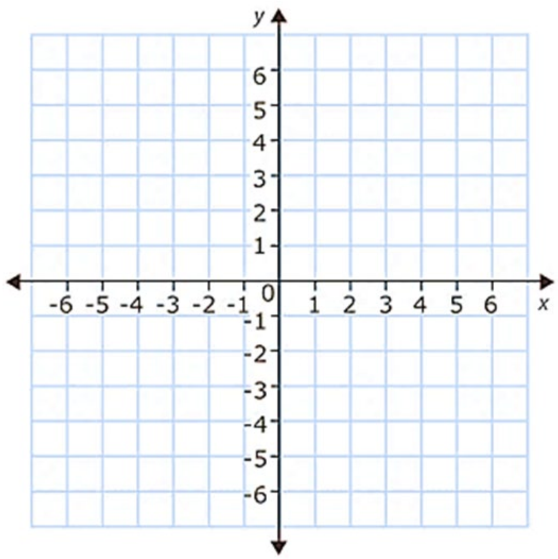
Equation Table:

# of dollars (x)	cost (y)
	.
	.
	.

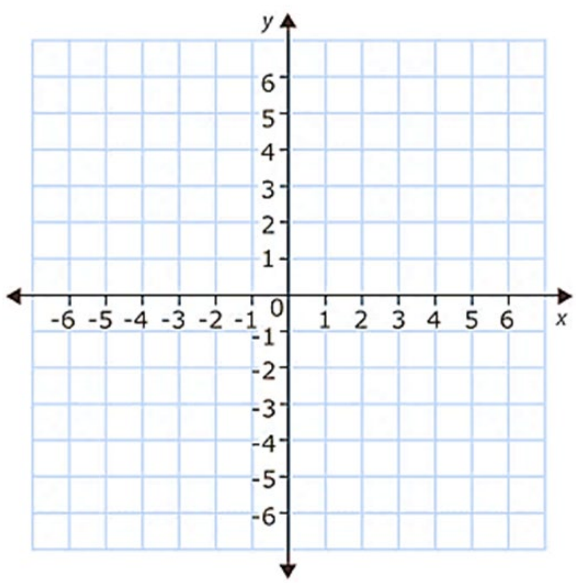


5) Think of a number. Triple it, and then subtract seven from your answer. Make a table of values and plot the line that this sentence represents.

number (x)	result (y)



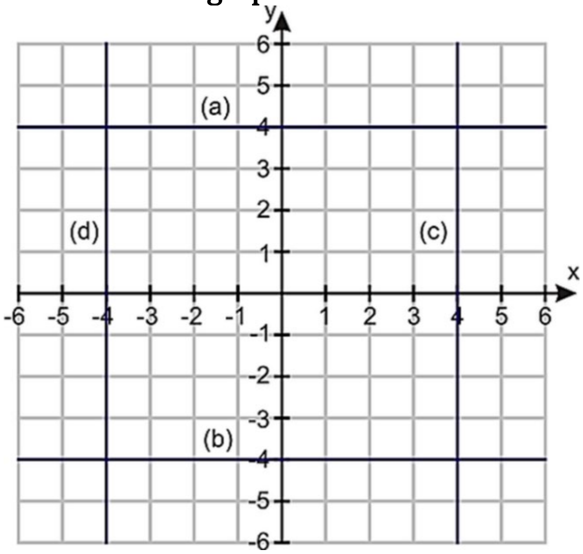
6) Find the solutions to each equation by making a table and graphing the coordinates on the same page.
   a. y = 2x + 3
   b. y = 0.5x – 1
   c. y = 6 –2x



Answers:

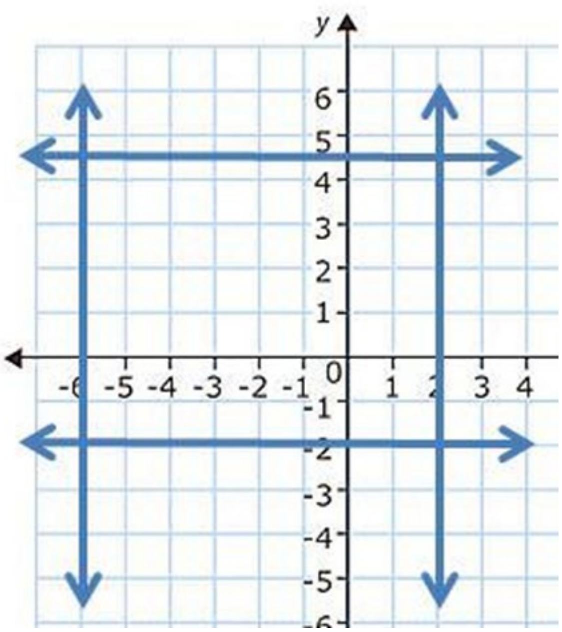
Practice 1: 1) y = 13x 2) y = 100 + 125x

Practice **2: see graph below**



Practice 3: 1) $7.00, 2) a.32F, b.85F, c.-17C

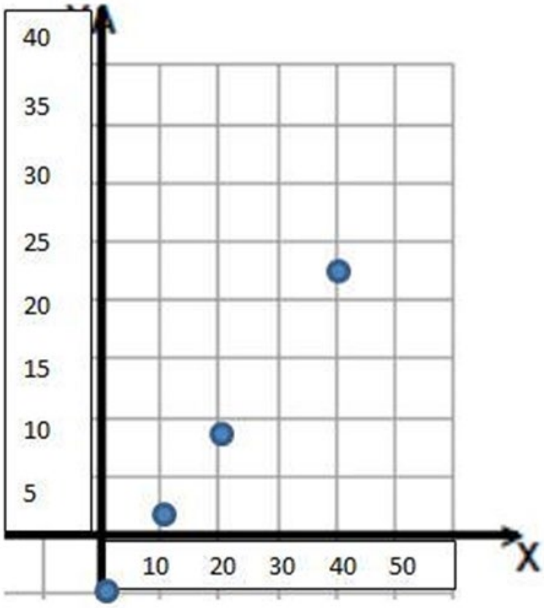
Mixed Practice: 1) below



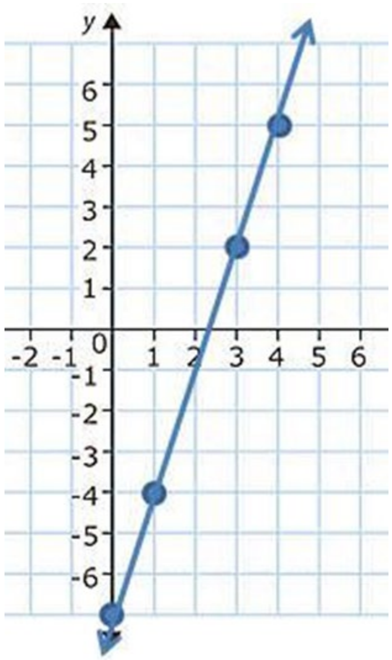
2) a.9 *lb*, b. 20 *lb*, c. 5.5 *lb*, d. 7.5 *lb*

3) a.y = 5, b.y = -2, c.y = -7 d. x = -4, e. x = 6

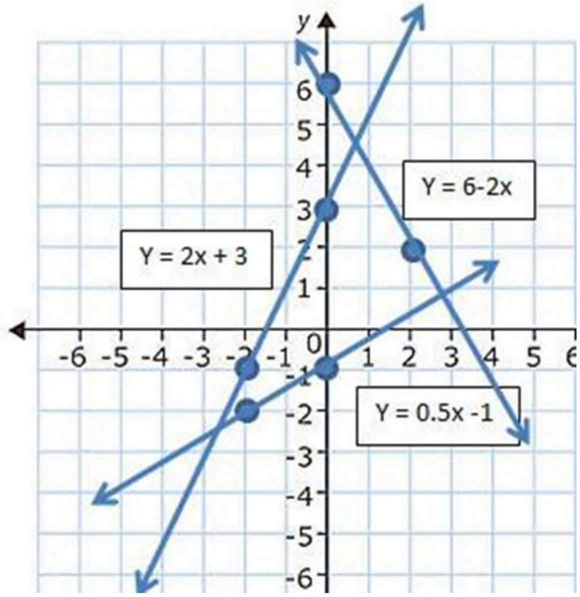
4) ex. (0, -5) (40, 23); see below



5)ex (0, -7) (4, 5); see below



6)



Graphing Equations Using Intercepts

Making tables to graph solutions to an equation of two variables can be time-consuming. There are other ways to graph solutions. This lesson will focus on **graphing a line by finding its intercepts.**

In geometry, there is a rule that states, "Two points determine a line." In other words, to draw any line, all you need is two points to connect. These two points could be the **x and y intercepts**.

An **intercept** is the point at which a graphed equation crosses an axis.

The x–**intercept** is an ordered pair at which the line crosses the x–axis (the horizontal line). Its ordered pair has the form $(x,0)$.

The y–**intercept** is an ordered pair at which the line crosses the y–axis (the vertical line). Its ordered pair has the form $(0, y)$

By finding the intercepts of an equation, you can quickly graph all the possible solutions to the equation.

Finding Intercepts Using Substitution

The Substitution Property allows the replacement of a variable with a numerical value or another expression. You can use this property to help find the intercepts of an equation. To do this, both variables need to be on one side of the equal's sign.

Example 1: Graph $2x + 3y = -6$ using its intercepts.

The ***x*****—intercept** has an ordered pair (x,0). In this case the *y*—coordinate has a value of zero. By substituting zero for the variable of *y*, the equation becomes: $2x+3(0) =-6$

$2x + 0 = -6$ $2x = -6$ $x = -3$

When y is 0, x will be -3.

The *x*—intercept has an ordered pair of (–3, 0).

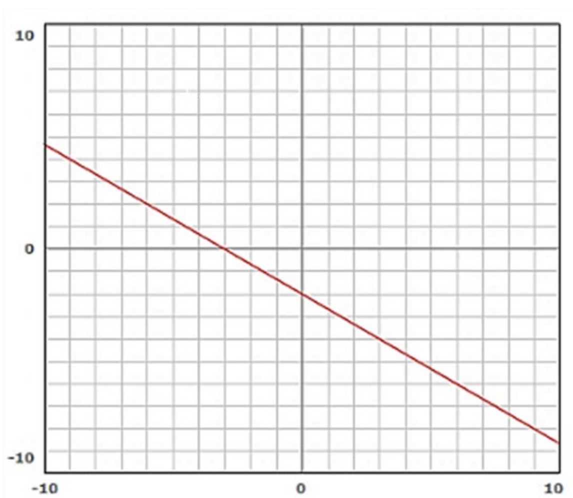
Repeat the process to find the ***y*****—intercept.** The ordered pair of the *y*—intercept is (0, y). Substitute 0 for x,

$2(0) + 3y = -6$ $3y = -6$ $y = -2$

When x is 0, y will be -2.

The *y*—intercept has the ordered pair (0, –2).

To graph the line formed by the solutions of the equation $2x + 3y = -6$, graph the two intercepts (–3, 0) and (0, –2) and connect them with a straight line, as below.



Example 2: Graph 4x – 2y = 8 using its intercepts.

Determine the *x*—intercept by substituting 0 for the variable *y*.

$4x - 2(0) = 8$ $4x = 8$ $x = 2$

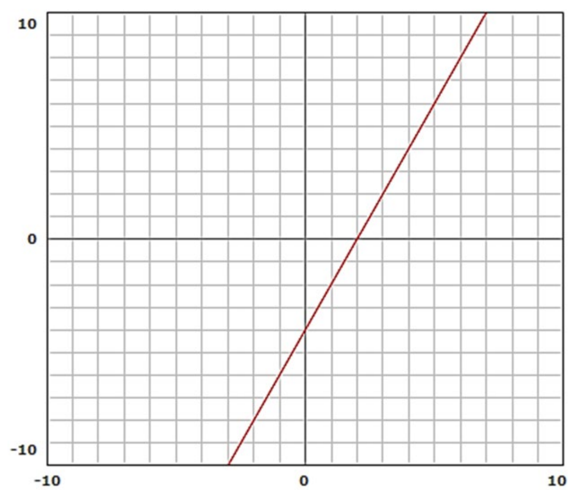
The ordered pair of the *x*—intercept is (**2, 0**).

Repeat the process for y by substituting 0 for x.

$4(0) - 2y = 8 \quad -2y = 8 \qquad y = -4$

The *y*–intercept has the ordered pair (**0, –4**).

Graph these two ordered pairs and connect with a line, as below.



By finding an intercept, you are substituting the value of zero in for one of the variables.

To find the *x*–**intercept**, substitute 0 for the *y*-**value**.

To find the *y*–**intercept**, substitute 0 for the *x*-**value**.

Practice 1

Find the intercepts for the following equations using substitution. You may need to change the equations to standard form first.

1) $y - 3x = -6$

2) $y + 2x = 4$

3) $y = 7x - 21$

4) $y = 6 - 3x$

Finding Intercepts Using the Cover-Up Method

In the previous two examples the equations are written in **standard form.** Standard form equations are always written "**coefficient** times *x* plus (or minus) **coefficient** times *y* equals **value**". In other words, they look like this: $ax + by = c$

There is a simple method for finding intercepts in standard form, often referred to as the cover-up method.

To solve each intercept, we realize that at the intercepts the value of **either** x **or** y is zero, and so any terms that contain that variable effectively drop out of the equation. To make a term disappear, simply cover it (a finger is an excellent way to cover up terms) and solve the resulting equation.

Example 3: Graph $-7x - 3y = 21$ using its intercepts.

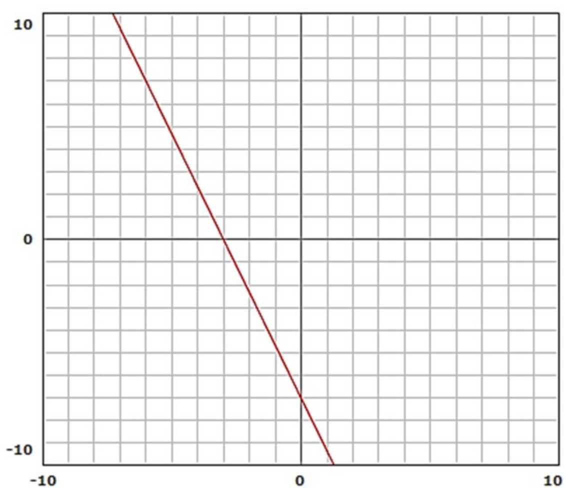
Solution: To solve for the y–intercept we set $x = 0$ and cover up the x term:

$-3y = 21$ $\quad$ $-3y = 21$ $\quad$ $y = -7$, so $(0, -7)$ is the y-intercept.

To solve for the x–intercept, cover up the y–variable and solve for x:

$-7x \quad = 21$ $\quad$ $-7x = 21$ $\quad$ $x = -3$ so $(-3, 0)$ is the x-intercept.

Now graph by first plotting the intercepts then drawing a line through these points, as below.



Example 4: Jose has $30 to spend on food for a class barbeque. Hot dogs' cost $0.75 each (including the bun) and burgers cost $1.25 (including bun and salad). Plot a graph that shows all the combinations of hot dogs and burgers he could buy for the barbecue, spending exactly $30.

Solution: Begin by translating this sentence into an algebraic equation. Let y = *the number of hot dogs* and x = *the number of burgers*.

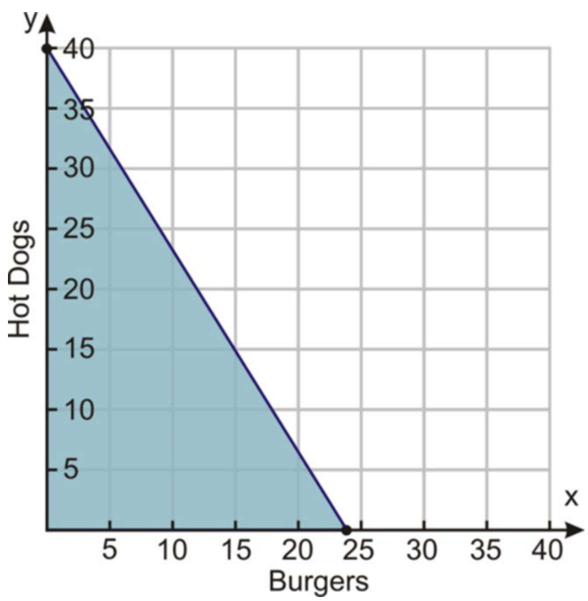
$1.25x + 0.75y = 30$

Find the intercepts of the graph. This example will use the Cover-Up Method. Feel free to use Substitution if you prefer.

$+ 0.75y = 30 \quad 0.75y = 30$, so y = 40 and the y$-$*intercept is* (0,40)

$1.25x + \quad = 30 \quad 1.25x = 30$ so x = 24 and the $x-$*intercept is* (24,0)

By graphing Jose's situation, you can determine the combinations of hot dogs and burgers he can purchase for exactly $30.00.



Practice 2:

Find the intercepts of the following equations using the Cover-Up Method.

5) $5x - 3y = 15$

6) $3x - 4y = -12$

7) $2x + 7y = -28$

8) $5x + 10y = 30$

Mixed Practice:

In 2 – 13, use any method (tables, substitution, or cover up) to *find the intercepts* and then *graph the equation*. You may put more than one line on a piece of graph paper. (As you will see in the answers.)

2) $y = 2x + 4$

3) $6(x-1)=2(y+3)$

4) $x-y=5$

5) $x+y=8$

6) $4x+9y=0$

7) $12x+4y=12$

8) $x-2y=4$

9) $7x-5y=10$

10) $4x-y=-3$

11) $x-y=0$

12) $5x+y=5$

13) $6x-2y=-6$

14) Does the equation $y=5$ have both an x—intercept and a y—intercept? Explain your answer.

15) What needs to be done to the following equation **before** you can use either method to find its intercepts? $3(x+2)=2(y+3)$

16) At the local grocery store, strawberries cost \$3.00 per pound and bananas cost \$1.00 per pound. If I have \$9 to spend between strawberries and bananas, *draw a graph* to show what combinations of each I can buy and spend exactly \$9.

17) In football, touchdowns are worth 6 points, field goals are worth 3 points, and safety are worth 2 points. Suppose there were no safeties and the team scored 36 points. *Graph the situation* to determine the combinations of field goals and touchdowns the team could have had.

18) A movie theater charges \$7.50 for adult tickets and \$4.50 for children. If the theater takes in \$900 ticket sales for a particular screening, *draw a graph* that depicts the possibilities for the number of adult tickets and the number of child tickets sold.

Answers:

Practice 1: 1) (0, -6) and (2, 0); 2) (0, 4) and (2, 0); 3) (0, -21) and (3, 0); 4) (0,6) and (2, 0)

Practice 2: 5) (0, -5) and (3,0); 6) (0, 3) and (-4, 0); 7) (0, -4) and (-14, 0) 8) (0, 3) and (6, 0)

Mixed Practice: 1) Answers will depend on student preference. 2 – 13) intercepts below, graphs follow with 2 – 7 on one graph, 8 – 13 on another, and 16 – 17 on a third, 18 alone.

2) (0, 4) and (-2, 0); 3) (0, -6) and (2, 0); 4) (0, -5) and (5, 0); 5) (0, 8) and (8, 0)

6) (0, 0) and (0, 0); 7) (0, 3) and (1, 0)   8) (0, -2) and (4, 0)   9) (0, -2) and (1.43, 0)

10) (0, 3) and (-.75, 0) 11) (0, 0) and (0, 0) 12) (0, 5) and (1, 0)   13) (0, 3) and (-1, 0)

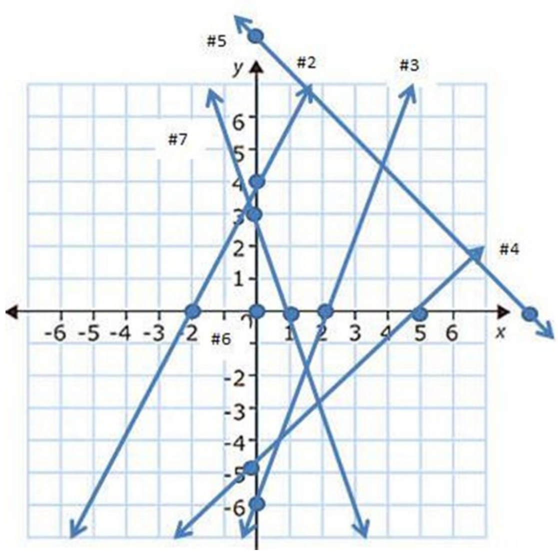
14) There is no x-intercept because the line only passes through the y-axis.

15) You need to distribute the parenthesis and get all variables on one side.

16) 3x + 1y = 9 (0, 9) and (3, 0)   17) 6x + 3y = 36 (0, 12) and (6, 0)

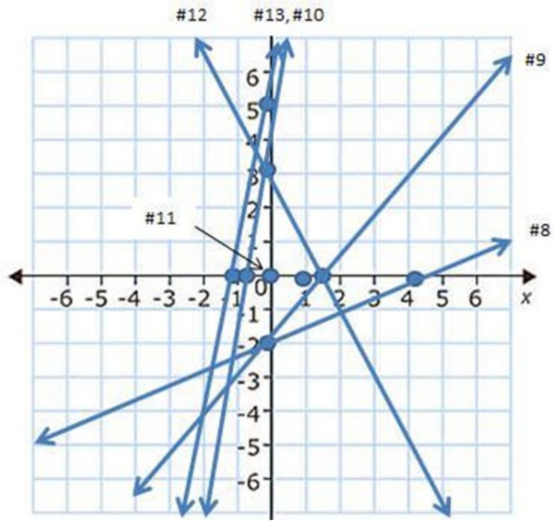
18) 7.50x + 4.50y = 900 (0, 200) and (120, 0)

2 - 7

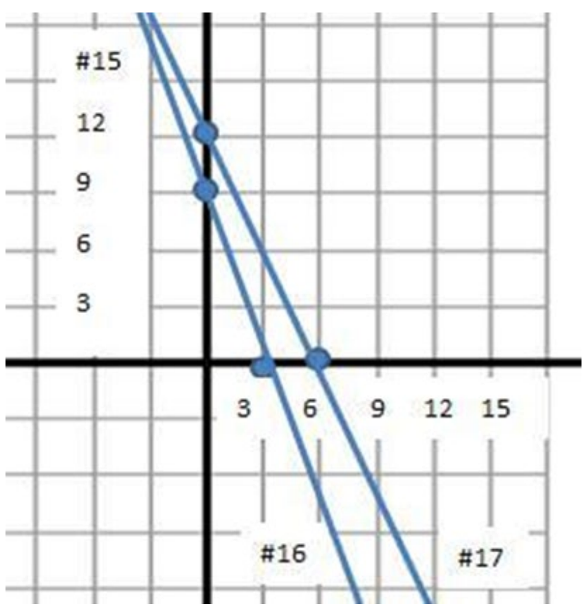


[Figure 5]

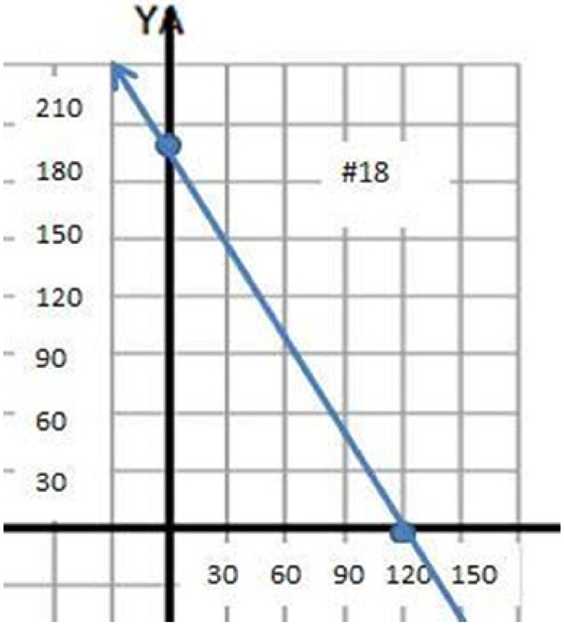
8 - 13



16-17



18



Finding Slope from a Graph

The pitch of a roof, the slant of a ladder against a wall, the incline of a road, and even your treadmill incline are all examples of slope.

The **slope** of a line measures its steepness (either negative or positive).

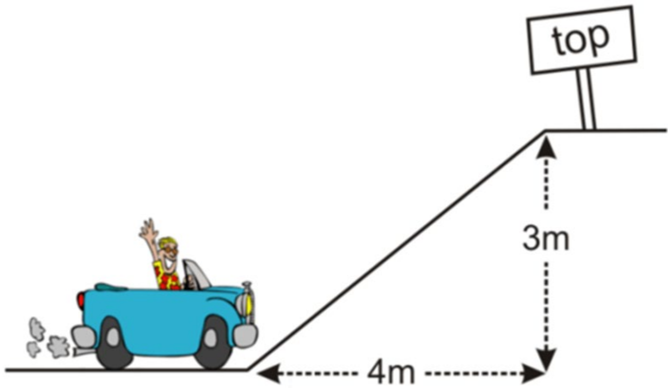
For example, if you have ever driven through a mountain range, you may have seen a sign stating, "10% incline." The percentage tells you how steep the hill is. You have seen this on a treadmill too. The incline on a treadmill measure how steep you are walking uphill. Below is a more formal definition of slope.

The **slope** of a line is the <u>vertical change (rise)</u> divided by the <u>horizontal change (run)</u>.

In the figure below, a car is beginning to climb up a hill. The height of the hill is 3 meters, and the length of the hill is 4 meters. Using the definition above, the slope of this hill can be written as

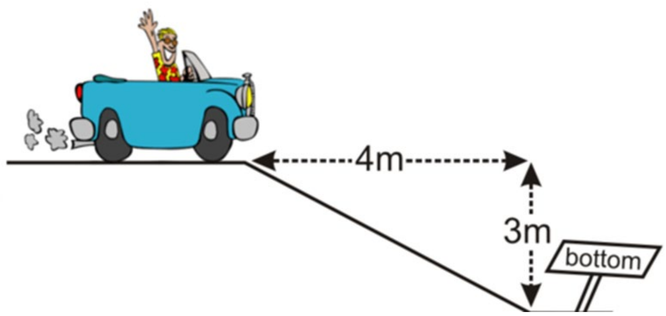
$\frac{3\ meters}{4\ meters}$ or $\frac{3}{4}$ Since $\frac{3}{4} = 0.75\% = 75\%$

we can also say this hill has a **75% positive slope**.



[Figure 1]

Similarly, if the car begins to descend *down* a hill, you can still determine the slope.



[Figure 2]

Figures 1-2:

$$Slope = \frac{vertical\ change\ (rise)}{horizontal\ change\ (run)} = \frac{-3}{4}$$

The slope in this instance is **negative** because the car is traveling downhill.

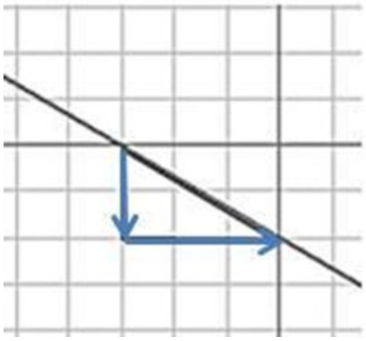
We can shorten this formula to:

Slope= $\frac{rise}{run}$

(Memory Tip: you must rise before you can run.)

When graphing an equation, slope is a powerful tool. It provides directions for how to get from one ordered pair to another. To determine slope using a graph, it is helpful to draw a ***slope-triangle***.

Using the following graph, choose any two ordered pairs on the line. It is helpful if these pairs land on crosshairs. For example, (–3, 0) and (0, –2). Now draw in the slope triangle by connecting these two points as shown below.



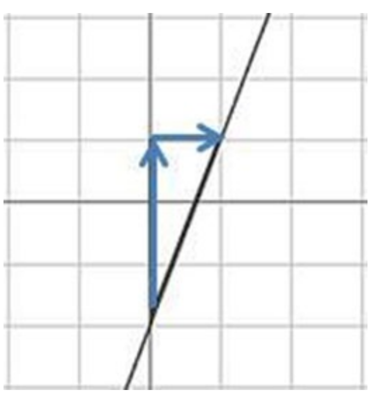
The vertical (up and down) leg of the triangle represents the ***rise*** of the line, and the horizontal (sideways) leg of the triangle represents the ***run*** of the line.

To find the slope using a graph, **start at the left-most coordinate**, count the number of vertical units (rise) and horizontal units (run) it takes to **get to the right-most** coordinate. For the graph above:

$$Slope = \frac{rise}{run} = \frac{down\ 2}{right\ 3} = \frac{-2}{3}$$

(While you can do this in a different order, it is best to start by always using the same order to avoid confusion.)

Example 1: Find the slope of the line graphed below.



Begin by finding two ordered pairs on the line such as (0, –2) and (1, 1).

Draw in the slope triangle.

Count the number of vertical units to get from the left ordered pair to the right.

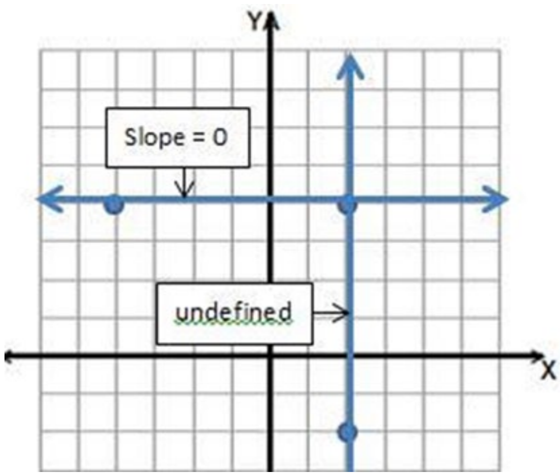
Count the number of horizontal units to get from the left ordered pair to the right.

$Slope = \frac{rise}{run} \qquad = \frac{up\ 3}{right\ 1} \qquad = \frac{3}{1}$

NOTE:

If a line is flat, like y = 4, it has a slope of 0.

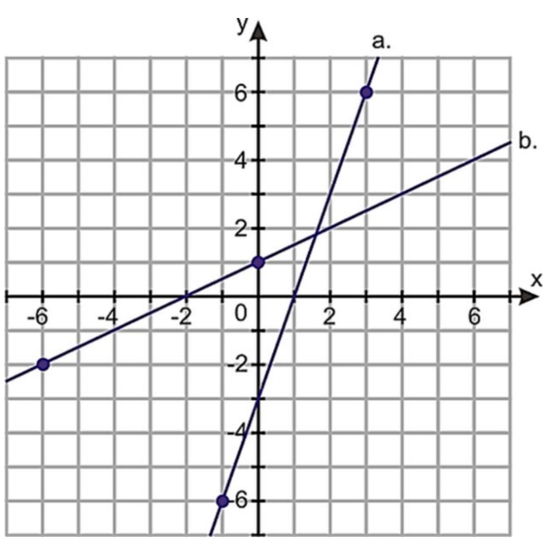
If a line is vertical, like x = 2, its slope is “undefined.”



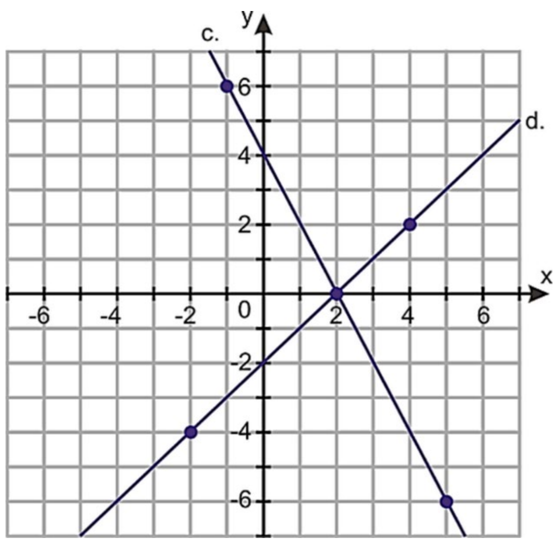
Practice

Using the graphed coordinates, find the slope of each line.

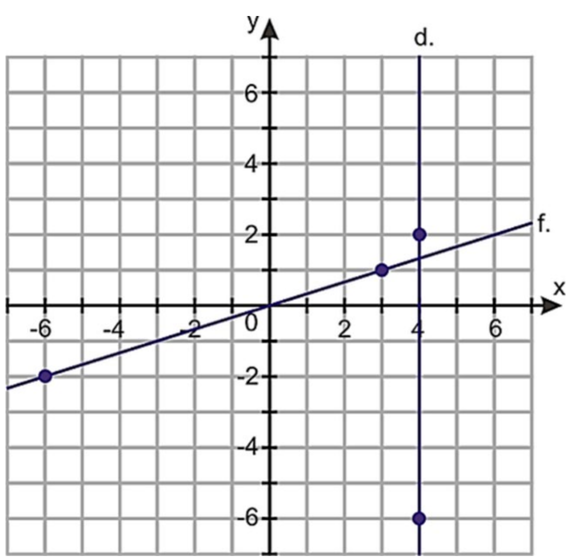
1.slope a.   slope b.



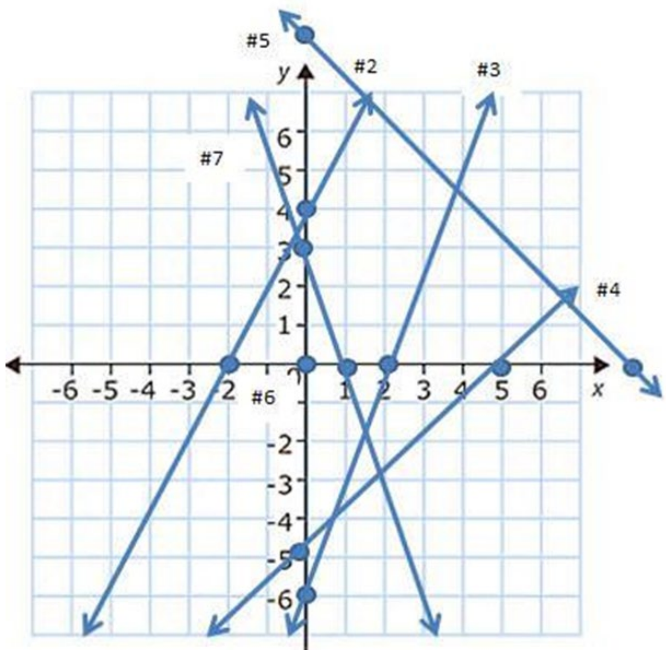
2.slope c slope d.



3. slope d. slope f.



4.Find the slopes for numbers 2 – 5 & 7. Assume each point is on a crosshair.



[Figure 9]:

Answers:

1. **a.** $\frac{12}{4}$ or 3 b. $\frac{3}{6}$ *or* $\frac{1}{2}$ **2.** c. $-\frac{6}{3}$ or -2 d. $\frac{4}{4}$ or 1 **3.**d. undefined f. $\frac{3}{9}$ *or* $\frac{1}{3}$

4.2) 2, 3) 3 4) 1 5) -1 7) -3

Using the Slope Formula

We have practiced finding slope by using a graph. Another way to determine a slope is by using a formula. The **slope formula** can be found on your formula page.

The slope between a point (x_1, y_1) and a point to its right (x_2, y_2) is

Slope $= \frac{y^2 - y1}{x^2 - x1} = \frac{rise}{run}$

(x_1, y_1) represents one of the two ordered pairs and (x_2, y_2) represents the other. The following example helps show this formula.

Example 1: Using the slope formula, determine the slope of a line from the points (1, 1) to (0, –2).

Since (1, 1) is written first, it can be called (x_1, y_1). That means (0, -2) = (x_2, y_2)

Use the formula:

$$\textit{Slope} = \frac{y2-y1}{x2-x1} = \frac{-2-1}{0-1} = \frac{-3}{-1} = 3$$

slope = 3

If we were to graph this slope, we would start at one point and go up three spaces and right one space to the next point. We could also go down 3 and 1 left.

The slope is the same whether you use a graph or a formula. If the ordered pairs are fractional or spaced far apart, it is easier to use the formula than to draw a slope triangle.

Practice 1:

Find the slope between the two given points.

1) (–5, 7) and (0, 0)

2) (–3, –5) and (3, 11)

3) (3, –5) and (–2, 9)

4) (9, 9) and (–9, –9)

5) (3, 5) and (–2, 7)

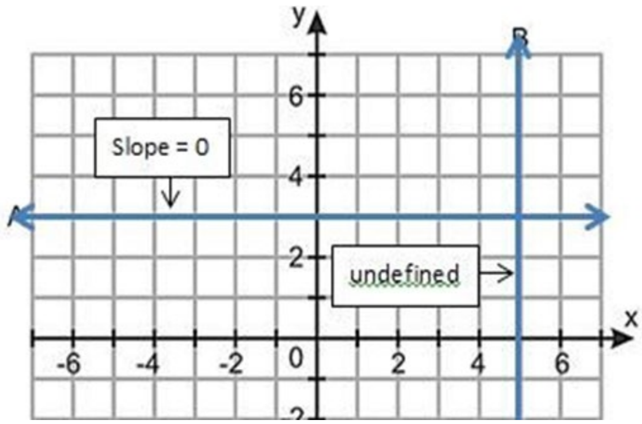
Slope Types

Slope comes in four different types: negative, zero, positive, and undefined.

A graph going down from left to right has a **negative** slope. A graph going up from left to right has a **positive** slope. A flat line has **zero** slope, and vertical **undefined** slopes cannot be computed.

> Any line with a slope of **zero** will be a horizontal line with equation y = some number.
>
> Any line with an **undefined** slope will be a vertical line with equation x = some number.

The graph below can help illustrate these definitions.



To determine the slope of line A you need to find two ordered pairs on the line.

Choose one ordered pair to represent $(x_{1,}\ y_1)$ and the other to represent $(x_2, y_2\ )$. For

example, you could choose (−4, 3) *and* (1, 3). Now apply the formula:

Slope= $\frac{y_2 - y_1}{x_2 - x_1} = \frac{3-3}{1—4} = \frac{0}{5} = 0$

To determine the slope of line B, you need to choose two ordered pairs on this line and apply the formula. For example, you could choose (5, 1) and (5, −6)

Slope= $\frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-1}{5-5} = \frac{-7}{0} =$ undefined

It is impossible to divide by zero, so the slope of line B cannot be determined and is called **undefined**.

Practice 2:

Find the slope represented by each set of ordered pairs.

6) (−5, 7) and (−5, 11)

7) (−17, 11) and (4, 11)

Mixed Practice:
Find the slope between the two given points.

1) (−2, 3) and (4, 8)

2) (31, 2) and (31, −19)

3) (0, −3) and (3, −1)

4) (2, 7) and (7, 2)

5) Determine the slope of y = 16.

6) Determine the slope of x = -99.

7) (3, 15) and (3, 200)

8) (0.5, 6.2) and (1.5, 10.4)

9) (325, 1525) and (500, 2050)

10) (-14, $\frac{3}{4}$) and -5.75, $\frac{3}{4}$)

11) (10, 1984) and (43, 2017)

12) (5.8, 6.2) and (29, 17.8)

13) (18, -7) and (21.5, -14)

14) (56, 72) and (269, 924)

15) 83, 26) and (203, -214)

Answers:

Practice 1:

1) $\frac{0-7}{0-(-5)} = \frac{-7}{5}$

2) $\frac{11-(-5)}{3-(-3)} = \frac{16}{6} = \frac{8}{3}$

3) $\frac{9-(-5)}{-2-3} = \frac{14}{-5}$

4) $\frac{-9-9}{-9-9} = \frac{-18}{-18} = 1$

5) $\frac{7-5}{-2-3} = \frac{2}{-5}$

Practice 2:

6) $\frac{11-7}{-5-(-5)} = \frac{4}{0}$ **= undefined**

7) $\frac{11-11}{-4-(-17)} - = \frac{0}{21}$ **= 0 slope**

Mixed Practice

1) $\frac{8-3}{4-(-02)} = \frac{5}{6}$

2) $\frac{-19-2}{31-31} = \frac{4}{0}$ =**undefined**

3) $\frac{-1-(3)}{3-0} = \frac{2}{3}$

4) $\frac{2-7}{7-2} = \frac{-5}{5} = -1$

5) 0 slope 6) undefined 7) undefined 8) 4.2 9) 3 10) 0 slope

11) 1 12) 0.5 13) -2 14) 4 15) -2

Slope Intercept Form

So far, we have learned how to graph the solutions to an equation in two variables **by making a table** and by **using its intercepts**. The last lesson introduced formulas **for slope**. This lesson will combine intercepts and slope into a new formula.

You have seen different forms of the slope formula several times. Below are several examples.

$y = 2x + 5$ $y = \frac{-1}{3}x + 11$ $d = 60(h) + 45$

The proper name given to each of these equations is **slope-intercept form** because each equation tells the slope and the y-intercept of the line.

Slope-Intercept Form

The **slope-intercept form of an equation** is: $y =$ (slope)$x +$ (y-intercept)

OR **$y =$ (m)$x +$ b** where **m stands for slope** and **b stands for y-intercept**.

This equation makes it quite easy to graph the solutions to an equation of two variables because it gives you two necessary values:

1. The starting position of your graph (the y-intercept)

2. The directions to find your second coordinate (the slope)

Example 1: Determine the slope and the y-intercept of the first two equations in the review.

Using the definition of slope-intercept for

$y = 2x + 5$ has a slope of 2 and a y-intercept of 5.

$y = \frac{-1}{3}x + 11$ has a slope of $\frac{-1}{3}$ and a y-intercept of 11.

Slope-intercept form applies to many equations, even those that do not look like the "standard" equation.

Example 2: Determine the slope and y-intercept of $7x = y$.

At first glance, this does not look like the "standard" equation. However, we can substitute values for the slope and y-intercept.

$7x + 0 = y$ OR $y = 7x + 0$ This means the slope is 7 and the y-intercept is 0.

Example 3: Determine the slope and y-intercept of **y = 5** .

Using what you learned in the last lesson, the slope of every line of the form y = some number is **zero** because it is a horizontal line. Rewriting our original equation to fit slope-intercept form yields:

$y = (0)x + 5$ Therefore, the slope is zero and the y-intercept is 5.

Practice 1:

Identify the slope and y-intercept of the formulas.

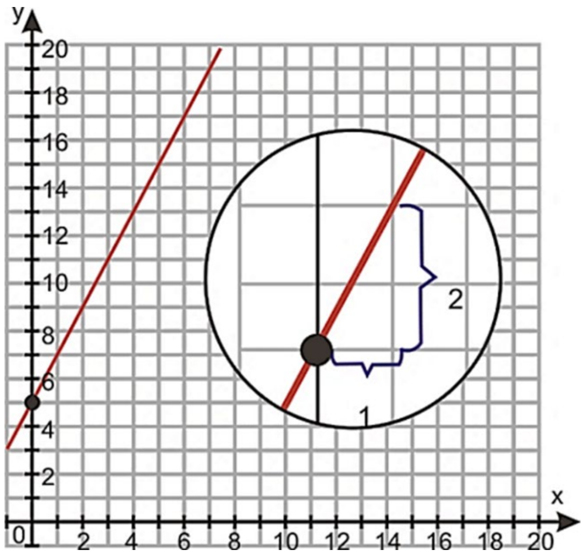
1) $y = 4x - 4$ 2) $y = 3.75$

Graphing an Equation Using Slope-Intercept Form

Working the other way, if we know the slope and y-intercept of an equation, it is quite easy to graph the solutions.

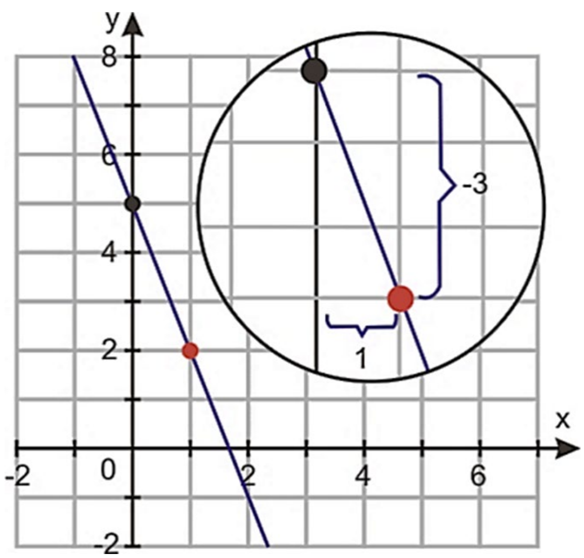
Example 4: Graph the solutions to the equation **y = 2x + 5**

The equation is in slope-intercept form. To graph the solutions to this equation, you should start at the y-intercept (5). Then, using the slope (2), find a second coordinate. (Slope of 2 means up two, right one.) Finally, draw a line through the pairs ordered as below.



Example 5: Graph the solutions to the equation **y = -3x + 5**

This equation has a *y*-intercept of 5 and a slope of -3 (down three, right one.)



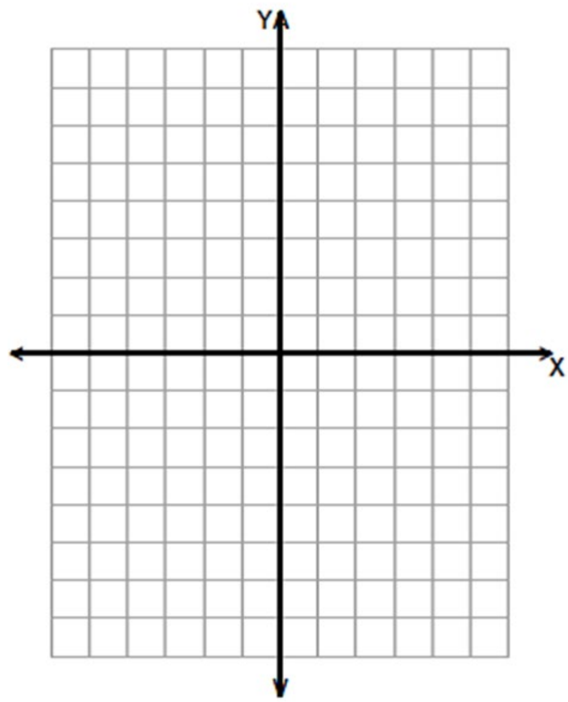
Practice 2:

Plot the following lines on the graph.

2) y = 2x + 5

3) $y = -2x + 3$

4) y = - x



Slopes of Parallel Lines

Parallel lines will never intersect, or cross. The only way for two lines never to cross is if the method of finding additional coordinates is the same.

Therefore, it's true that **parallel lines have the same slope.**

Example 6: Determine the slope of any line parallel to y = -3x + 5

Because parallel lines have the same slope, the slope of any line parallel to y = -3x + 5 must also be –3.

Practice 4:

State the slope of the line parallel to the line given.

1) y = 2x + 5

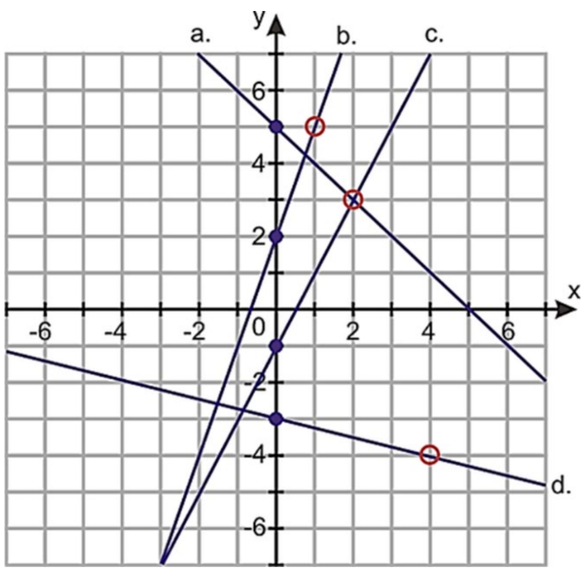
2) y= -2x + 3

3) y = -x

Writing Slope-Intercept Form from a Graph

You may be asked to use a graph to determine the slope intercept form.

Example 7: Determine the slope-intercept form of the lines graphed below.



First, find the y-intercept. For line a, you can see the graph crosses the *y*-axis at **(0, 5) so** we know that y = mx + **5**

From this point, find a second coordinate on the line, such as (2,3) then use any method to calculate slope.

Line a: The y-intercept (0.5). The line also passes through (2.3). Find the slope using the graph (count down two and right two. $\frac{-2}{2} = -1$), the formula, or rise over run (below).

$Slope = \frac{3-5}{2-0} = -1$ so, slope-intercept form is y= -1x + 5

Line b: The y-intercept is (0,2). The line passes through (1,5). Find slope by counting up three, right one $\frac{3}{1} = 3$, rise over run, or: Slope=$\frac{5-2}{1-0} = 3$ to get $y = 3x + 2$

Practice 2: Write the slope intercept form of lines 1) c and 2) d

Mixed Practice:

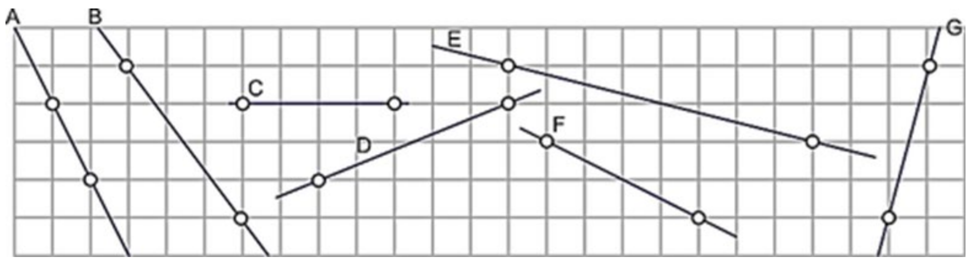
In 8 -10, identify the slope and *y*-intercept for the equation.

8) $\frac{2}{3}x - 9 = y$

9) y = - 0.01x + 10,000

10) $7 + \frac{3}{5}x = y$

In 11-17, identify the slope of the following lines using any method.



11) F

12) C

13) A

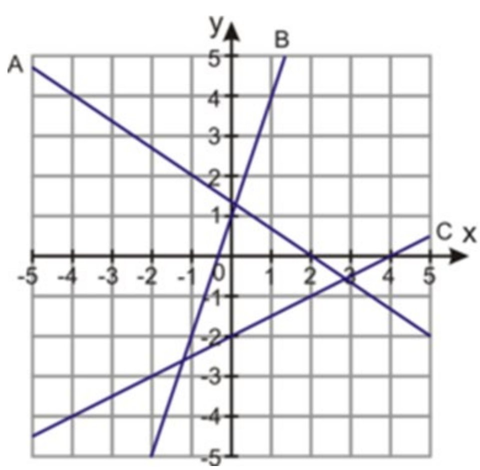
14) G

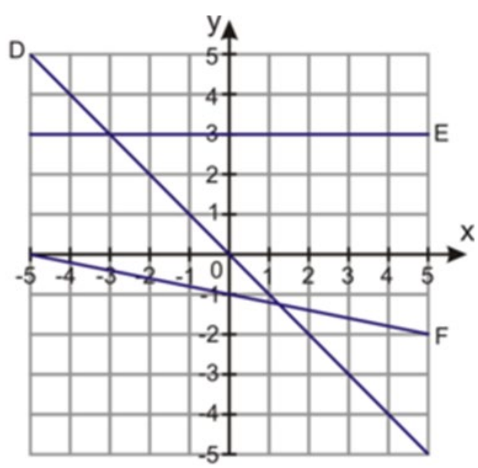
15) B

16) D

17) E

In 18-23, identify the slope and *y*-intercept for the following lines.





18) D

19) A

20) F

21) B

22) E

23) C

Draw the following lines on a graph.

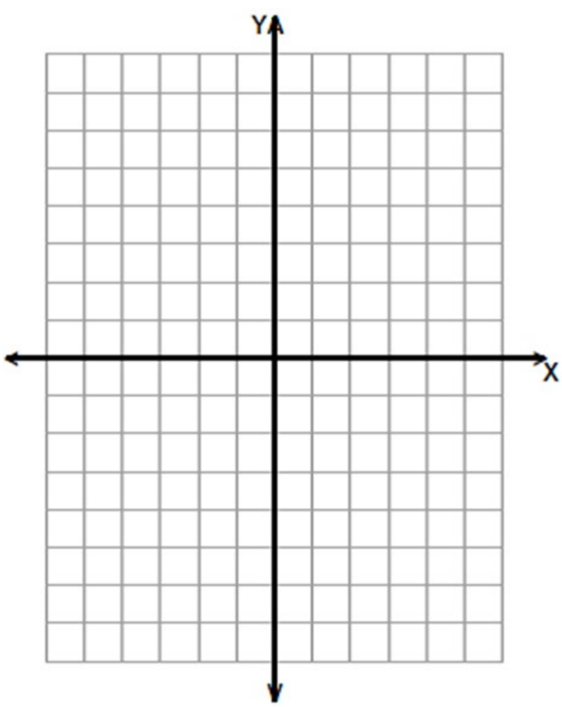
26) y = 3.75

27) $\frac{2}{3}x - 4 = y$

28) y = -4x + 3

29) $-2 + \frac{3}{2}x = y$

30) $y = ½ + 2x$



State the slope of the line parallel to the line given.

31) $y = x - 11$

32) $y = -5x + 5$

33) $y = -3x + 11$

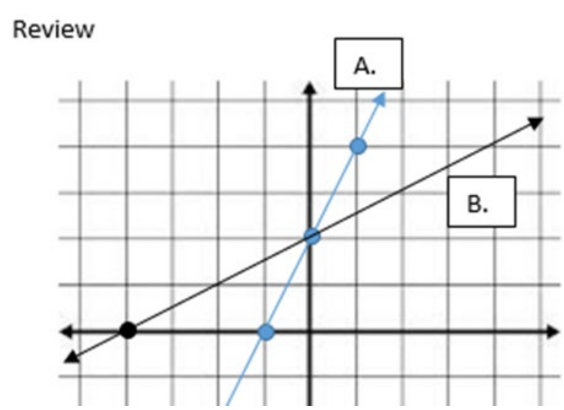
34) $y = 3x + 3.5$

Answers:

Review: A. Graph the solution to the equation $y = 2x + 2$ by making a table.

X	y
-1	0
0	2
1	4

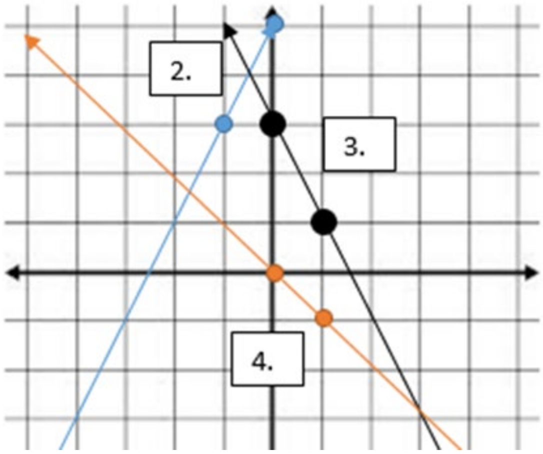
B. Points are (0, 2) and (-4, 0)



Slope	The steepness of a line
x-intercept	Where a line crosses the x-axis
y-intercept	Where a line crosses the y-axis
Slope formula	$Pendiente = \frac{y2 - y1}{x2 - x1}$

Practice 1: 1) slope 4, y-intercept -4 2) slope 0, y-intercept 3.75

Practice 2: see graph below



Practice 3: 1) 2, 2) -2, 3) -1

Practice 4: c. $y = 2x - 1, d. y = \frac{1}{4}x - 3$

Mixed Practice: 8) slope $\frac{2}{3}, y - intercept\ - 9$ 9) slope -0.01, y-intercept 10,000 10) $slope \frac{3}{5}, y - intercept$ 7 11) $F = \frac{-2}{4} or \frac{-1}{2}$ 12) C 0 $slope$ 13) A -2 14) G 4

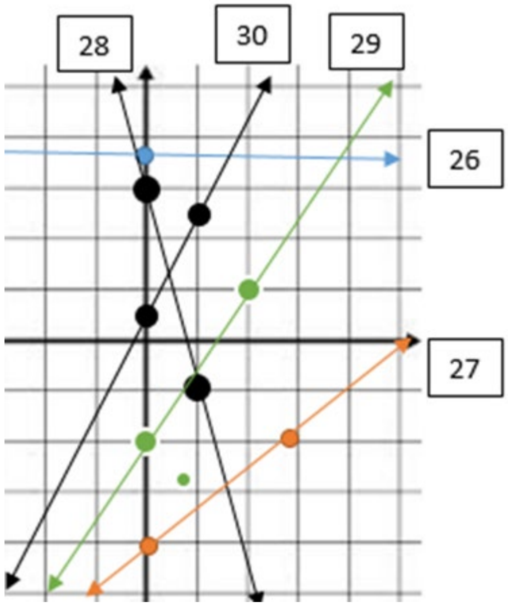
15) $B \frac{-4}{3}$ 16) $D \frac{2}{5}$ 17) $E \frac{-2}{8}$ or $\frac{-1}{4}$

18) D slope -1, y-intercept 0 19) $A\ slope\ -\frac{2}{3}, y - intercept\ 1.5ish$

20) $F\ slope\ -\frac{1}{5}, y - intercept\ - 1$ 21) B slope 3, y-intercept 1

22) E slope0, y-intercept 3 23) $C\ slope \frac{1}{2}, y - intercept\ - 2$

26) line should be flat. 27 - 30 see below



31) $-\frac{1}{5}$ 32) -5 33)-3 34) 3

Introduction to Systems of Equations

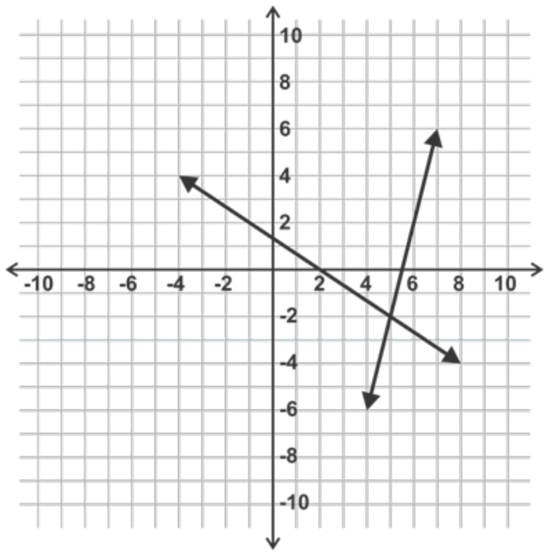
A system of linear equations consists of the equations of two lines. The solution to a system of linear equations is the point which lies on both lines.

In other words, the solution is the point where the two lines intersect.

To verify whether a point is a solution to a system or not, we will either determine whether it is the point of intersection of two lines on a graph (Example A) or we will determine whether the point lies on both lines algebraically (Example B.)

Example A

Is the point (5, -2) the solution of the system of linear equations shown in the graph below?



[Figure 2]

Solution: Yes, the lines intersect at the point (5, -2) so it is the solution to the system.

Example B

Is point (-3, 4) the solution to the system given below?

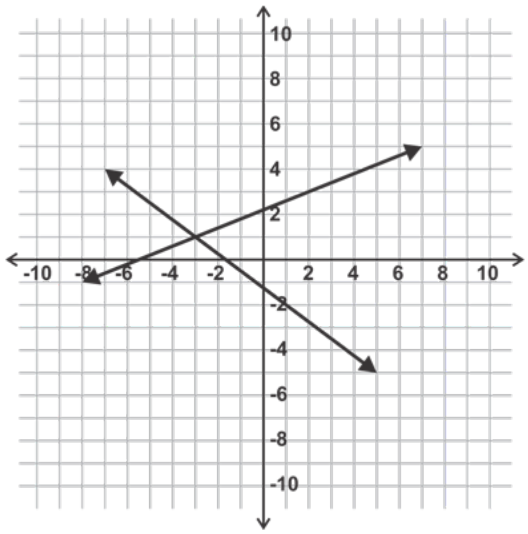
$$2x - 3y = -18$$

$$x + 2y = 6$$

Solution: No, (-3, 4) is not the solution. If we replace the x and y in each equation with -3 and 4 respectively, only the first equation is true. The point is not on the second line; therefore, it is not the solution to the system.

Guided Practice

1. Is point (-2, 1) the solution to the system shown below?



[Figure 3]

2. Verify algebraically that (6, -1) is the solution to the system shown below.

$$3x - 4y = 22$$

$$2x + 5y = 7$$

3. Explain why the point (3, 7) is the solution to the system:

$$y = 7$$

$$x = 3$$

Answers:

1. No, (-2, 1) is not the solution. The solution is where the two lines intersect which is the point (-3, 1).
2. By replacing x and y in both equations with 6 and -1 respectively (shown below), we can verify that the point (6, -1) satisfies both equations and thus lies on both lines.

$$3(6) - 4(-1) = 18 + 4 = 22$$

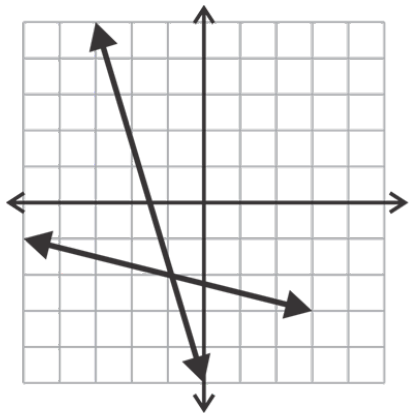
$$2(6) + 5(-1) = 12 - 5 = 7$$

3. The horizontal line is the line containing all points where the y-coordinate is 7. The vertical line is the line containing all points where the x—coordinate is 3. Thus, the point (3, 7) lies on both lines and is the solution to the system.

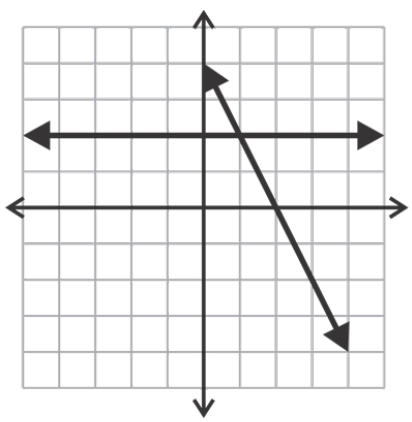
Problem Set

Match the solutions with their systems.

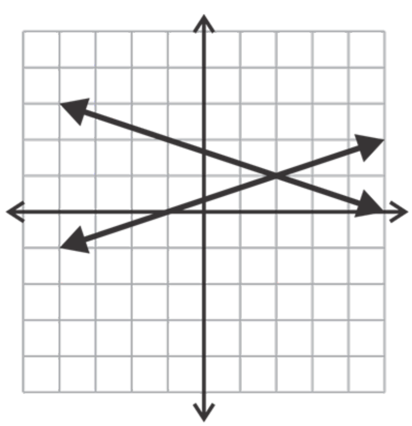
1. (1, 2)

a. 

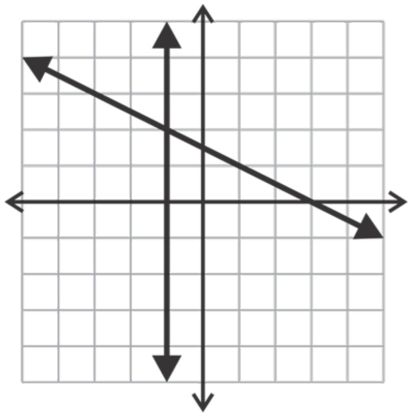
[Figure 4]

b. 

[Figure 5]

c. 

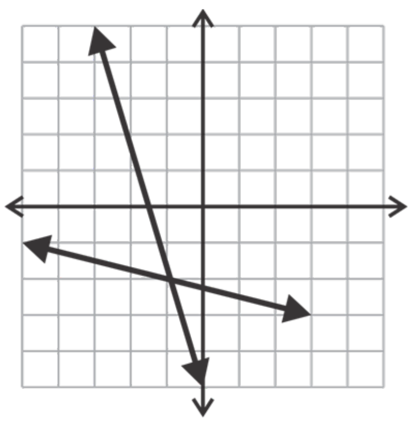
[Figure 6]



d.

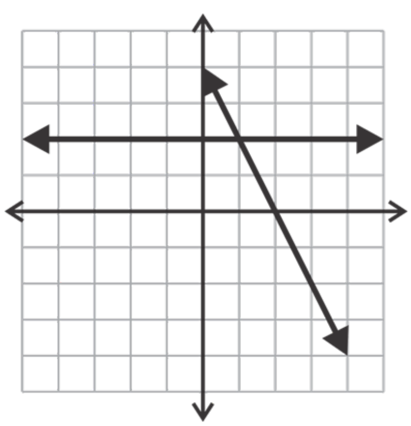
[Figure 7]

2. (2, 1)



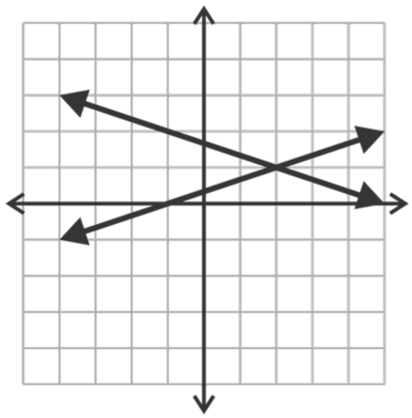
a.

[Figure 8]

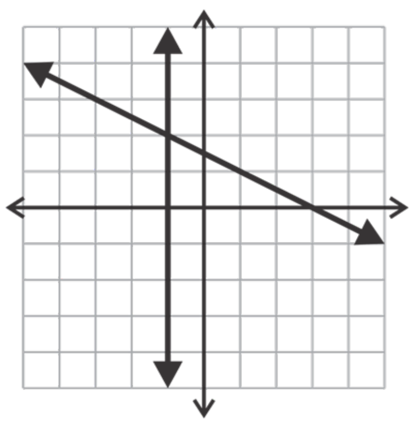


b.

[Figure 9]

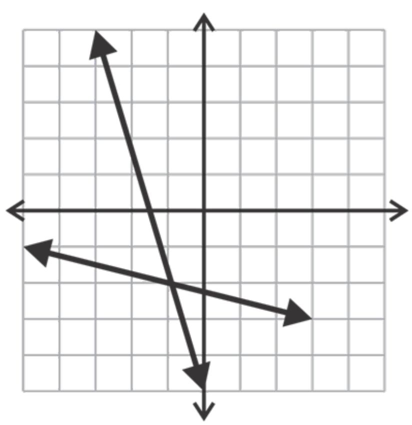
c. 

[Figure 10]

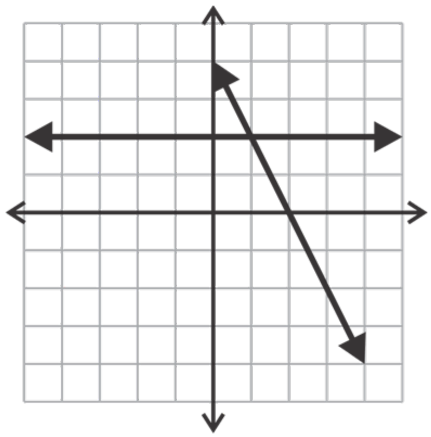
d. 

[Figure 11]

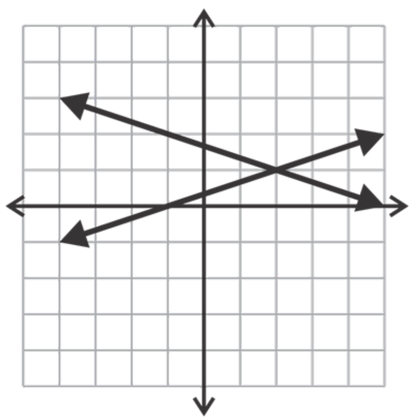
3. (-1, 2)

a. 

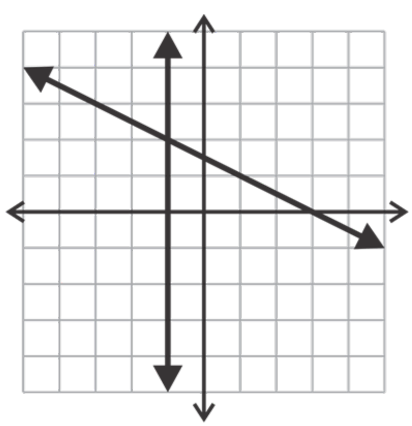
[Figure 12]

b. 

[Figure 13]

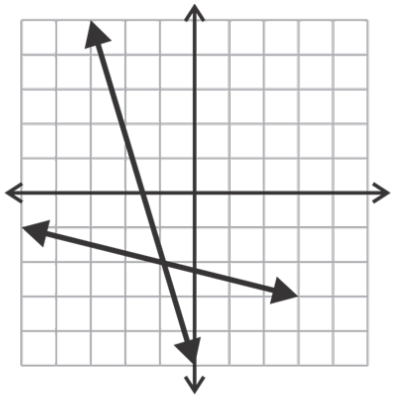
c. 

[Figure 14]

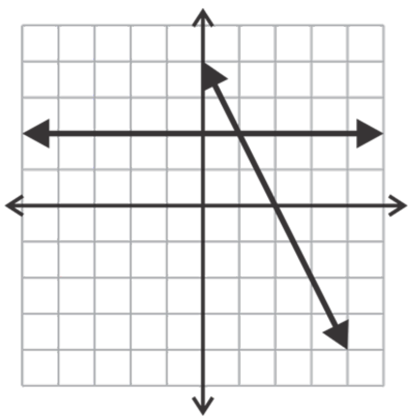
d. 

[Figure 15]

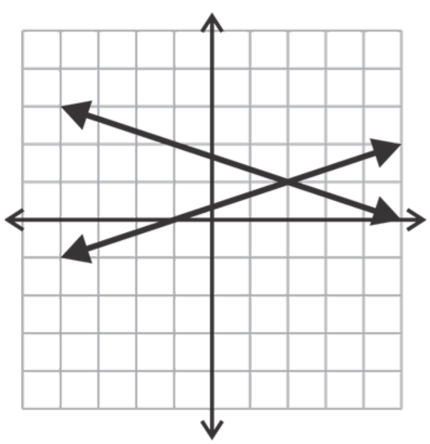
4. (-1, -2)

a. 

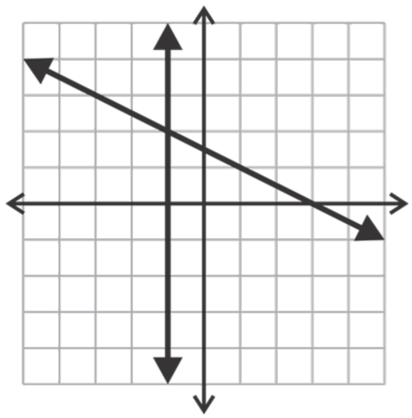
[Figure 16]

b. 

[Figure 17]

c. 

[Figure 18]



d.

[Figure 19]

Determine whether each ordered pair represents the solution to the given system.

5. Is (-3,8) the solution for 4x + 3y = 12   and   5x + 2y = 1?

6. Is (5, -2) the solution for 3x - y = 17   and 2x + 3y = 5?

7. (1, 0) for 7x - 9y = 7 and x + y = 1

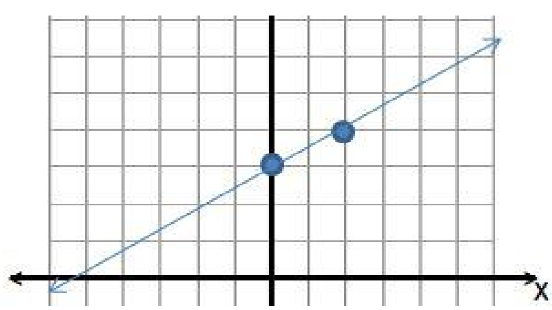
8. (5, -9) for x + y = -4 and x - y = 4

9. (11, 10) for x = 11 and y = 10

10. (15, -5) for x + 3y = 0 and y = -5

Answers:

Review: 1.



2. $y = \frac{4}{3}x - 2$

3. x = 5

Problem Set

1. b 2. c 3. d 4. a

5. yes 6. no 7. yes 8. no 9. yes 10. Yes

Solving Linear Systems by Graphing

In this lesson we will be using various techniques to graph the pairs of lines in systems of linear equations to identify the point of intersection or the solution to the system. It is important to use graph paper and a straight edge to graph the lines accurately.

Example A

Graph and solve the system:

$$y = -x + 1$$

$$y = \frac{1}{2}x - 2$$

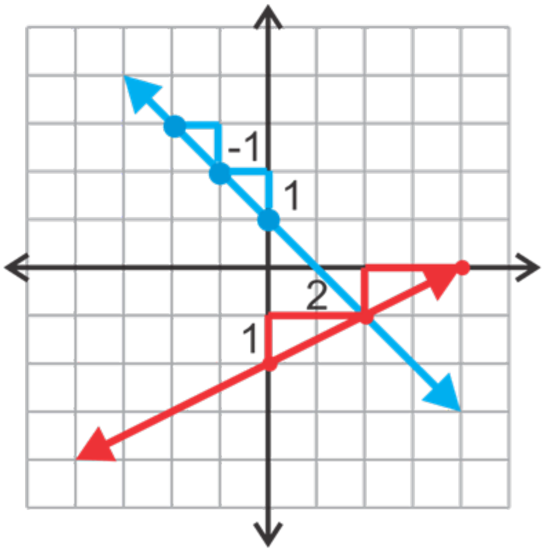
Solution:

Since both of these equations are written in slope intercept form, we can graph them easily by plotting the y−intercept point and using the slope to locate additional points on each line.

The equation, y= −x + 1, graphed in blue, has y−intercept 1 and slope $-\frac{1}{1}$.

The equation, $y = \frac{1}{2}x - 2$, graphed in red, has y−intercept -2 and slope $\frac{1}{2}$.

Now that both lines have been graphed, the intersection is observed to be the point (2, -1).



[Figure 1]

Option: You can check this solution algebraically by substituting the point into both equations.

Equation 1: y=− x + 1, making the substitution gives: $(-1) = (-2) + 1$. ✓

Equation 2: $y = \frac{1}{2}x - 2$, making the substitution gives: $-1 = \frac{1}{2}(2) - 2$. ✓

(2, -1) is the solution to the system.

Example B

Graph and solve the system:

$$3x + 2y = 6$$

$$y = -\frac{1}{2}x - 1$$

Solution: This example is similar to the first example. The only difference is that equation 1 is not in slope intercept form.

We can either solve for y by putting it in slope-intercept form or we can use the intercepts to graph the equation. To review using intercepts to graph lines, we will use the latter method.

Recall that the x−intercept can be found by replacing y with zero and solving for x:

$$3x + 2(0) = 6$$

$$3x = 6$$

$$x = 2$$

Similarly, the y—intercept is found by replacing x with zero and solving for y:

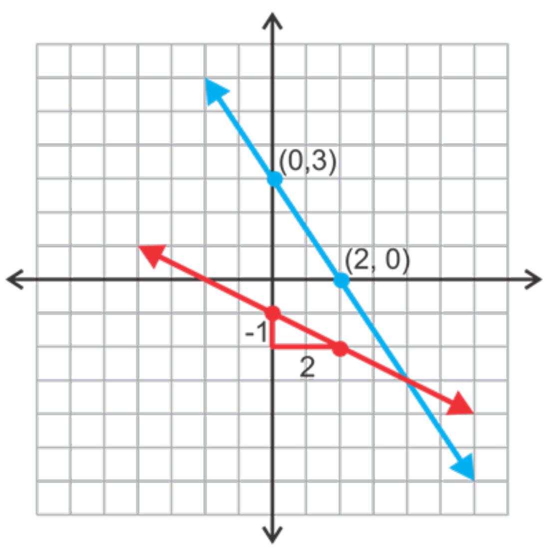
$$3(0) + 2y = 6$$

$$2y = 6$$

$$y = 3$$

We have two points, (2, 0) and (0, 3) to plot and graph this line. Equation 2 can be graphed using the y—intercept and slope as shown in Example A.

Now that both lines are graphed, we observe that their intersection is the point (4, -3).



[Figure 2]

You can check this solution by substituting it into each of the two equations.

Equation 1: $3x + 2y = 6$; $3(4) + 2(-3) = 12 - 6 = 6$ ✓

Equation 2: $y = x - 1$; $-3 = (4) - 1$ ✓

Guided Practice

Solve the following systems by graphing.

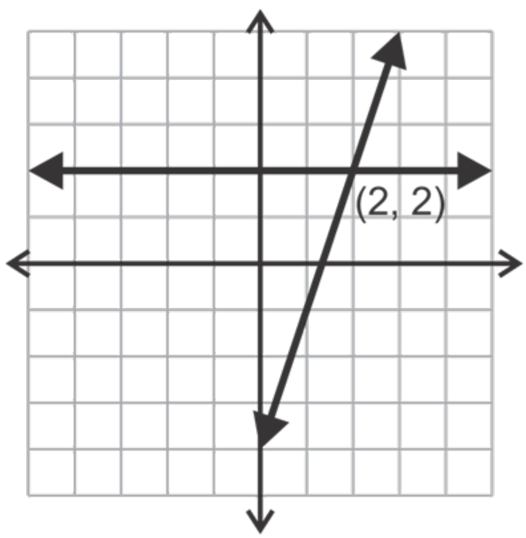
1. $y = 3x - 4$ and $y = 2$

2. $2x - y = -4$ and $2x + 3y = -12$

3. $5x + y = 10$ and $y = \frac{2}{3}x - 7$

Answers

1. The first line is in slope intercept form and can be graphed accordingly. The second line is a horizontal line through (0, 2). The graph of the two equations is shown below. From this graph the solution appears to be (2, 2).



[Figure 3]

Checking this solution in each equation verifies that it is indeed correct.

Equation 1: $2=3(2)-4$ ✓

Equation 2: $2=2$ ✓

2. Neither of these equations is in slope intercept form. The easiest way to graph them is to find their intercepts as shown in Example B.

Equation 1: Let $y=0$ to find the $x-$intercept.

$$2x - y = -4$$

$$2x - 0 = -4$$

$$x = -2$$

Now let x=0, to find the y—intercept.

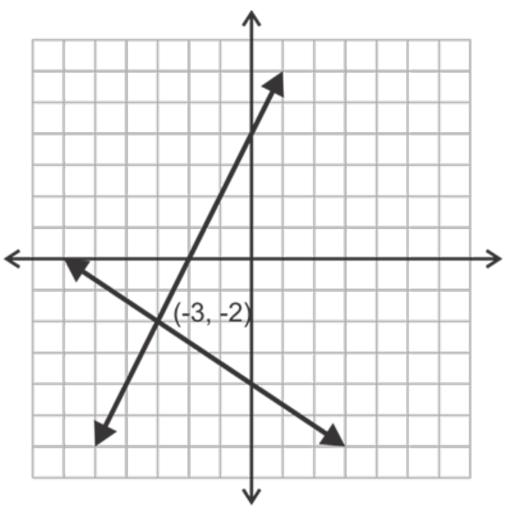
$$2x - y = 4$$

$$2(0) - y = -4$$

$$y = -4$$

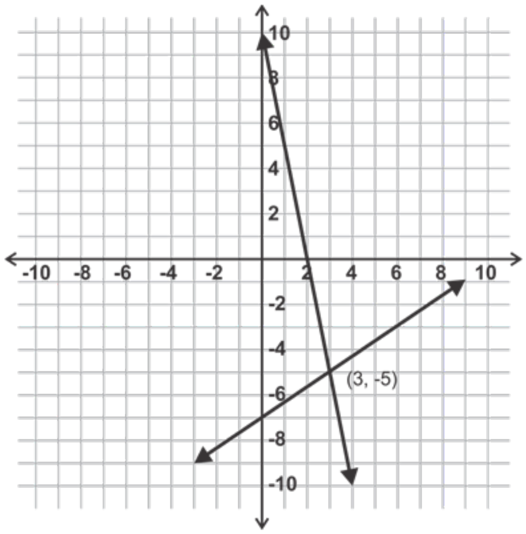
Now we can use (-2, 0) and (0, 4) to graph the line as shown in the diagram below. Using the same process, the intercepts for the second line can be found to be (-6, 0) and (0, -4).

Now the solution to the system can be observed to be (-3, -2). This solution can be verified algebraically as shown in the first problem.



[Figure 4]

3. The first equation here can be rearranged to be y=—5x+10. The second equation can be used as is.The graph below shows the solution is (3, -5).



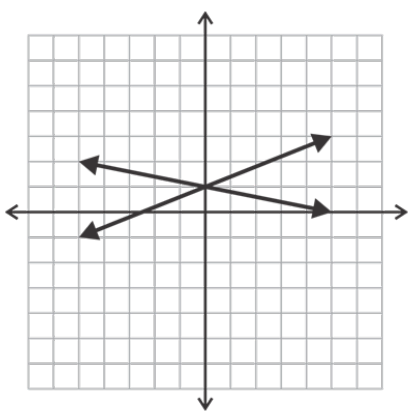
[Figure 5]

Problem Set

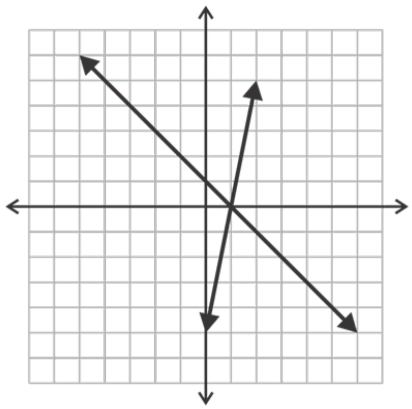
Match the system of linear equations to its graph and state the solution.

1. 3x + 2y = -2 and x - y = -4

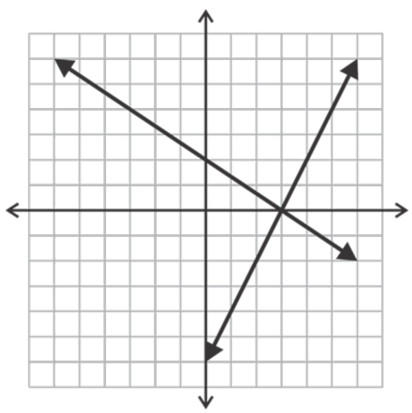
1.

a. 

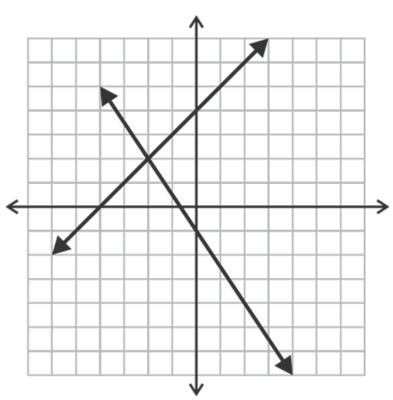
[Figure 6]

b. 

[Figure 7]

c. 

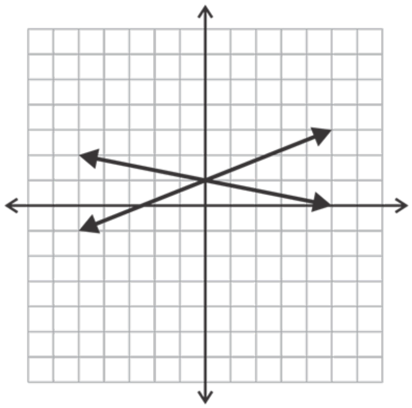
[Figure 8]

d. 

[Figure 9]

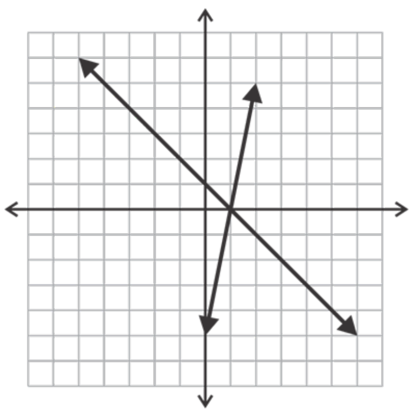
2. $2x - y = 6$ and $2x + 3y = 6$

a.



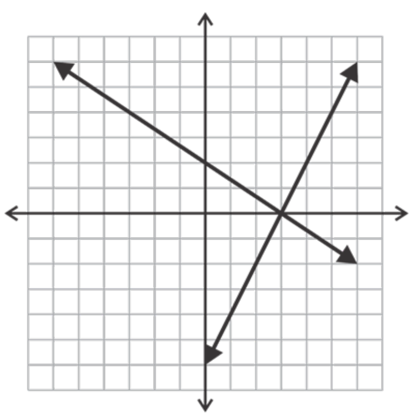
[Figure 10]

b.

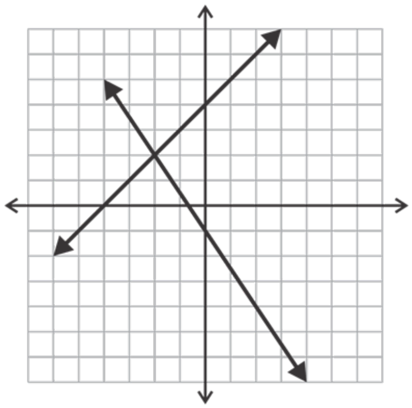


[Figure 11]

c.

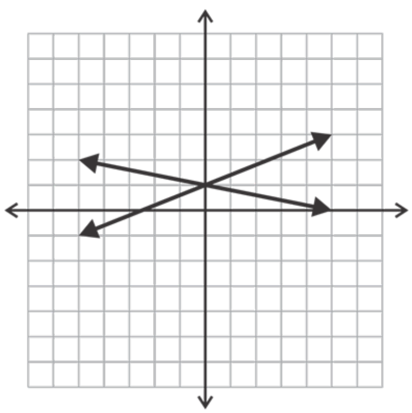


[Figure 12]

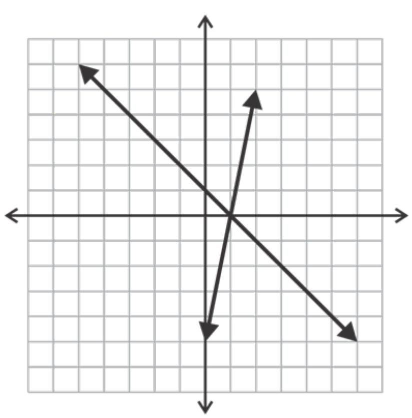
d. 

[Figure 13]

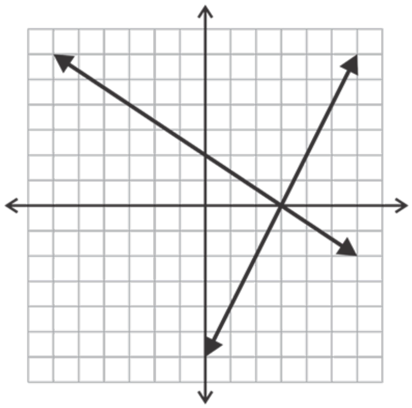
3. 2x - 5y = -5 and x + 5y = 5

a. 

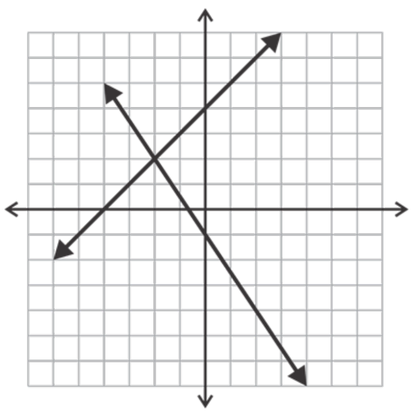
[Figure 14]

b. 

[Figure 15]

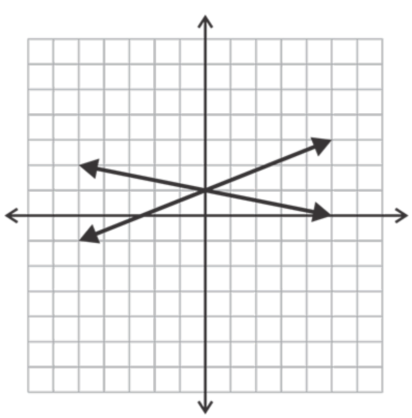
c. 

[Figure 16]

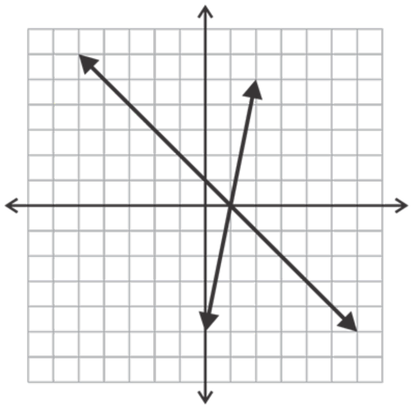
d. 

[Figure 17]

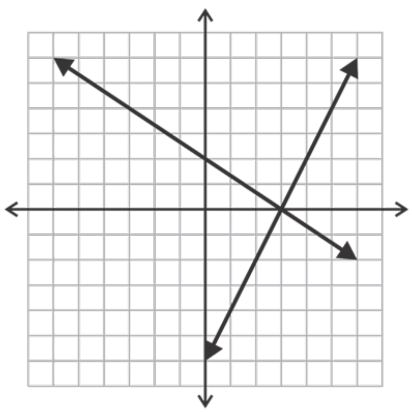
4. y = 5x - 5 and y = -x + 1

a. 

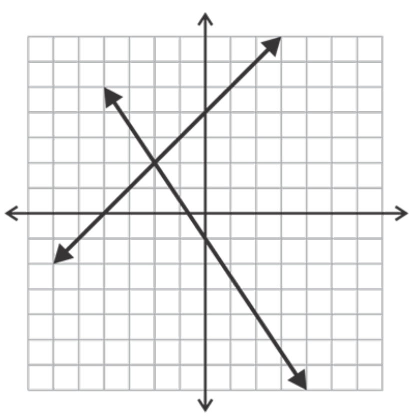
[Figure 18]

b. 

[Figure 19]

c. 

[Figure 20]

d. 

[Figure 21]

Solve the following linear systems by graphing. Use graph paper and a straight edge to insure accuracy. You are encouraged to verify your answer algebraically.

5. $y = \frac{-2}{5}x + 1$ and $y = \frac{3}{5}x - 4$

6. $y = \frac{2}{3}x + 4$ and $y = 3x - 7$

7. $y = -2x + 1$ and $x - y = -4$

8. $3x + 4y = 12$ and $x - 4y = 4$

9. $7x - 2y = -4$ and $y = -5$

10. $x - 2y = -8$ and $x = -3$

Use the following information to complete exercises 11-17.

Clara and her brother, Carl, are at the beach for a vacation. They want to rent bikes to ride up and down the boardwalk. One rental shop, Bargain Bikes, advertises rates of $5 plus $1.50 per hour. A second shop, Frugal Wheels, advertises a rate of $6 plus $1.25 an hour.

11. How much does it cost to rent a bike for one hour from each shop? How about 10 hours?

12. Write equations to represent the cost of renting a bike from each shop. Let x represent the number of hours and y represent the total cost.

13. Solve your system to figure out when the cost is the same.

14. Clara and Carl want to rent the bikes for about 3 hours. Which shop should they use?

Answers:

Problem Set

1. d 2. c 3. a 4. b

5. (5, -1) 6. (3, 2) 7. (-1, 3) 8. (4, 0) 9. (-2, -5) 10. (-3, 2.5)

11. 1 hour: BB = $6.50, FW = $7.25 10 hours: BB = $20, FW = $18.50

12. BB $y = 1.50x + 5$, FW $y = 1.25x + 6$

13. (4, 11) 14. BB is better because they charge $9.50 for three hours and FW charges $11

Solving Linear System by Substitution

In the substitution method we will be looking at the two equations and deciding which variable is easiest to solve for so that we can write one of the equations as x = or y =. Next, we will replace either the x or the y accordingly in the *other* equation. The result will be an equation with only one variable that we can solve

Example A

Solve the system using substitution:

$$2x + y = 12$$

$$-3x - 5y = -11$$

The first step is to look for a variable that is easy to isolate. In other words, does one of the variables have a coefficient of 1? Yes, that variable is the y in the first equation. So, start by isolating or solving for y: $y = -2x + 12$

This expression can be used to replace the y in the other equation and solve for x:

$$-3x - 5(-2x + 12) = -11$$

$$-3x + 10x - 60 = -11$$

$$7x - 60 = -11$$

$$7x = 49$$

$$x = 7$$

Now that we have found x, we can use this value in our expression to find y:

$$y = -2(7) + 12$$

$$y = -14 + 12$$

$$y = -2$$

Recall that the solution to a linear system is a point in the coordinate plane where the two lines intersect. Therefore, our answer should be written as a point: (7, -2).

You can check your answer by substituting this point into both equations to make sure that it satisfies them:

$$2(7) + -2 = 14 - 2 = 12$$

$$-3(7) - 5(-2) = -21 + 10 = -11 \checkmark$$

Example B

Solve the system using substitution:

$$2x + 3y = 13$$

$$x + 5y = -4$$

In the last example, y was the easiest variable to isolate. Is that the case here? No, this time, x is the variable with a coefficient of 1. It is easy to fall into the habit of always isolating y since you have done so much to write equations in slope-intercept form. Try to avoid this and look at each system to see which variable is easiest to isolate. Doing so will help reduce your work.

Solving the second equation for x gives: $x = -5y - 4$.

This expression can be used to replace the x in the other equation and solve for y:

$$2(-5y - 4) + 3y = 13$$

$$-10y - 8 + 3y = 13$$

$$-7y - 8 = 13$$

$$-7y = 21$$

$$y = -3$$

Now that we have found y, we can use this value in our expression to find x:

$$x = -5(-3) - 4$$

$$x = 15 - 4$$

$$x = 11$$

So, the solution to this system is (11, -3). Do not forget to check your answer:

$$2(11) + 3(-3) = 22 - 9 = 13$$

$$11 + 5(-3) = 11 - 15 = -4 \checkmark$$

Example C

Solve the system using substitution:

$$4x + 3y = 4$$

$$6x - 2y = 19$$

In this case, none of the variables have a coefficient of 1. So, we can just pick one to solve for. Let's solve the x in equation 1:

$$4x = -3y + 4$$

$$x = -\frac{3}{4}y + 1$$

Now, this expression can be used to replace the x in the other equation and solve for y:

$$6\left(\frac{-3}{4}y + 1\right) - 2y = 19$$

$$-\frac{18}{4}y + 6 - 2y = 19$$

$$-\frac{9}{2}y - \frac{4}{2y} = 13$$

$$-\frac{13}{2}y = 13$$

$$\left(-\frac{2}{13}\right)\left(-\frac{13}{2}\right)y = 13\left(-\frac{2}{13}\right)$$

$$y = -2$$

Now that we have found y, we can use this value in our expression find x:

$$x = \left(-\frac{3}{4}\right)(-2) + 1$$

$$x = \frac{6}{4} + 1$$

$$x = \frac{3}{2} + \frac{2}{2}$$

$$x = \frac{5}{2}$$

So, the solution is (52, -2). Check your answer:

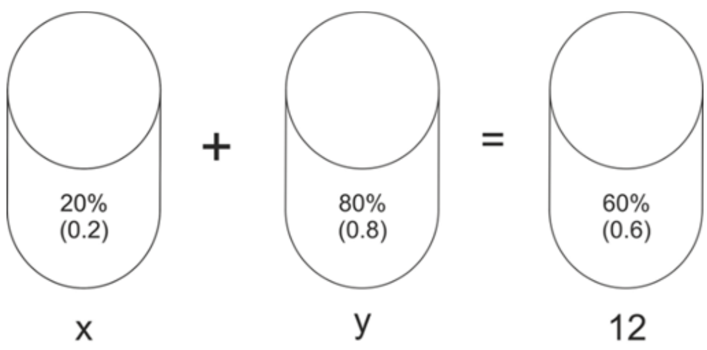
$$4\left(\frac{5}{2}\right)+3(-2)=10-6-4$$

$4\left(\frac{5}{2}\right)+3(-2)=10-6-4$ (Correct)

Example D

Rex and Carl are making a mixture in science class. They need to have 12 ounces of a 60% saline solution. To make this solution they have a 20% saline solution and an 80% saline solution. How many ounces of each do they need to make the correct mixture?

This type of word problem can be daunting for many students. Let's try to make it easier by organizing our information into a "picture" equation as shown below:



[Figure 1] **License:** CC BY-NC

In this picture, we can see that we will be mixing x ounces of the 20% solution with y ounces of the 80% solution to get 12 ounces of the 60% solution. The two equations are thus:

$$0.2x + 0.8y = 0.6(12)$$

$$x + y = 12$$

Now we can solve the system using substitution. Solve for y in the second equation to get: $y = 12 - x$.

Now, substitute and solve in the first equation:

$$0.2x + 0.8(12-x) = 0.6(12)$$

$$0.2x + 9.6 - 0.8x = 7.2$$

$$-0.6x = -2.4$$

$$x = 4$$

Now we can find y:

$$y = 12 - x$$

$$y = 12 - 4$$

$$y = 8$$

Therefore, Rex and Carl need 4 ounces of the 20% saline solution and 8 ounces of the 80% saline solution to make the correct mixture.

Guided Practice

Solve the following systems using the substitution method.

1. 3x + 4y = -13 and x = -2y - 9

2. -2x - 5y = -39 and x + 3y = 24

3. $y = \frac{1}{2}x - 21$ and y = -2x + 9

Answers:

1. In this problem, the second equation is already solved for x so we can use that in the first equation to find y:

$$3(-2y - 9) + 4y = -13$$

$$-6y - 27 + 4y = -13$$

$$-2y - 27 = -13$$

$$-2y = 14$$

$$y = -7$$

Now we can find x:

$$x = -2(-7) - 9$$

$$x = 14 - 9$$

$$x = 5$$

Therefore, the solution is (5, -7).

2. This time the x in the second equation is the easiest variable to isolate:

$x = -3y + 24$. Let's use this in the first expression to find y:

$$-2(-3y + 24) - 5y = -39$$

$$6y - 48 - 5y = -39$$

$$y - 48 = -39$$

$$y = 9$$

Now we can find x:

$$x = -3(9) + 24$$

$$x = -27 + 24$$

$$x = -3$$

Therefore, the solution is (-3, 9).

3. In this case, both equations are equal to y. Since y=y, by the Reflexive Property of Equality, we can let the right-hand sides of the equations be equal too. This is still a substitution problem; it just looks a little different.

$$\frac{1}{2}x - 21 = 2x + 9$$

$$2(\frac{1}{2}x - 21 = -2x + 9)$$

$$x - 42 = -4x + 18$$

$$5x = 60$$

$$X = 12$$

Now we can find y=

$y = \frac{1}{2}(12) - 21$ or $y = -2(12) + 9$

$y = 6 - 21$ $y = -24 + 9$

$y = -15$ $y = -15$

Problem Set

Solve the following systems using substitution. Remember to check your answers.

1) $x + 3y = -1$ and $2x + 9y = 7$

2) $7x + y = 6$ and $x - 2y = -12$

3) $5x + 2y = 0$ and $y = x - 7$

4) $2x - 5y = 21$ and $x = -6y + 2$

5) $y = x + 3$ and $y = 2x - 1$

6) $x + 6y = 1$ and $-2x - 11y = -4$

7) $2x + y = 18$ and $-3x + 11y = -27$

8) $2x + 3y = 5$ and $5x + 7y = 8$

9) $-7x + 2y = 9$ and $5x - 3y = 3$

10) $2x - 6y = -16$ and $-6x + 10y = 8$

11) $2x - 3y = -3$ and $8x + 6y = 12$

12) $5x + y = -3$ and $y = 15x + 9$

Set up and solve a system of linear equations to answer each of the following word problems.

13) Alicia and Sarah are at the supermarket. Alicia wants to get peanuts from the bulk food bins and Sarah wants to get almonds. The almonds cost \$6.50 per pound and the peanuts cost \$3.50 per pound. Together they buy 1.5 pounds of nuts. If the total cost is \$6.75, how much did each girl get? Solve using substitution.

14) Marcus goes to the department store to buy some new clothes. He sees a sale on t-shirts (\$5.25) and shorts (\$7.50). Marcus buys seven items and his total, before sales tax, is \$43.50. How many of each item did he buy?

15) Jillian is selling tickets for the school play. Student tickets are \$3 and adult tickets are \$5. If 830 people buy tickets and the total revenue is \$3104, how many students attended the play?

Answers:

1) (-10, 3) 2) (0, 6) 3) (2, -5) 4) (8, -1) 5) (4, 7) 6) (13, -2) 7) (9, 0)

8) (-11, 9) 9) (-3, -6) 10) (7, 5) 11) $(\frac{1}{2}, \frac{4}{3})$ 12) $(\frac{-3}{5}, 0)$

13) 3.50p + 6.50a = 6.75 and p + a = 1.5; a = 0.5 and p = 1; Alicia got 1 pound of peanuts and Sarah got half a pound of almonds.

14) 5.25t + 7.50s = 43.40 and t + s = 7; s = 3, t = 4; Marcus bought 3 shorts and 4 t-shirts.

15) 3s + 5a = 3104 and s + a = 830; a = 307 and s = 523; 523 students attended the play.

Practice Exams

Complete the two math practice exams in the websites links listed below:

https://www.hisetpracticetest.org/hiset-math-practice-test/

https://www.ets-cls.org/hiset/

Additional Resources – Instructional Videos

Pre-Algebra Lessons: https://youtube.com/playlist?list=PLDAA144326A49CF77

Algebra Lessons: https://youtube.com/playlist?list=PL7ED8B390FED947F6

Geometry Lessons: https://youtube.com/playlist?list=PLCF3EF8DD295C5FF6

Polynomials:
https://youtube.com/playlist?list=PLndfEMRm03EUKFc65JMg1oKiju97R20s1

GED/HiSET Math Skills:
https://youtube.com/playlist?list=PLcYMsa5lmTSzjxXx1TPl0KCugPoO5tg7_

Math Roundtable Recaps: https://youtube.com/playlist?list=PLcYMsa5lmTSxGuQa8S-5VZMzSYCk2v2-U

How to Simplify Radicals: https://youtu.be/C_e2SW-Bl3E

Radical Operations: https://youtu.be/4Gq3LPORQ-U

Linear Equations: https://youtu.be/Ft2_QtXAnh8

Algebra Formulas

Slope of a Line	$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$
Distance between two points	$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
Slope Intercept Form	$y = mx + b$
Point Slope	$y - y_1 = m(x - x_1)$
Standard Form of Quadratic Formula	$y = ax^2 + bx + c$
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Line of Symmetry	$x = -\frac{b}{2a}$

Pythagorean Theorem $a^2 + b^2 = c^2$

Simple Interest Formula I= Pr t

Distance Formula= $d = rt$

Total Cost Formula= Number of Units x Cost Per Unit

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